Title:
Kirchhoff's laws and quantum walks

Abstract:
We consider the Szegedy walk on graphs adding infinite length tails to a finite internal graph. We assume that on these tails, the dynamics is given by the free quantum walk. We set the uniformly bounded initial state so that the internal graph receives time independent input from the tails at every time step. We show that the response of the Szegedy walk to the input to the tails in the long time limit, depends on the reversibility of the underlying random walk.

If the underlying random walk is reversible, we show that the scattering matrix on the surface of the internal graph is described by the Szegedy matrix determined by only its boundary.The Szegedy matrix, which is the local quantum coin, determines the local dynamics at each time step and at each vertex; it can also describe such global dynamics. On the other hand, if the underlying random walk is non-reversible, the scattering matrix on the surface is described by a diagonal matrix. Therefore we show the scattering of a quantum walker towards only the input tails in the long time limit.

Moreover in the interior of the internal graph, the stationary state is a convex combination of an electric current and the reversible measure if the underlying random walk is reversible, while the stationary state is similar to an electric current but it satisfies a different type of Kirchhoff's law if the underlying random walk is non-reversible.