Title: Electric quantum walks and Bloch oscillations

Abstract:

Bloch oscillations appear when an electric field is superimposed on a quantum particle that evolves on a lattice with a tight-binding Hamiltonian (TBH), i.e., evolves via what we will call an electric TBH ; this phenomenon will be referred to as TBH Bloch oscillations. A similar phenomenon is known to show up in so-called electric discrete-time quantum walks (DQWs); this phenomenon will be referred to as DQW Bloch oscillations. This similarity is particularly salient when the electric field of the DQW is weak, i.e., small with respect to its maximum value 2π. For a wide, i.e., spatially extended initial condition, one numerically observes semi-classical oscillations, i.e., oscillations of a localized particle, both for the electric TBH and the electric DQW. More precisely: The numerical simulations strongly suggest that the semi-classical DQW Bloch oscillations, correspond to two counter-propagating semi-classical TBH Bloch oscillations. In this work, this result is proven semi-analytically, that is: It is shown, under certain assumptions, that the solution of the electric DQW for a weak electric field and a wide initial condition is well approximated by the superposition of two continuous-time expressions, which are counter-propagating solutions of an electric TBH whose hopping amplitude is the cosine of the arbitrary coin-operator mixing angle. The result is semi-analytical in the sense that the quality of the approximated expressions is checked numerically.
That the coin-operator mixing angle can be chosen arbitrary is because we chose a wide initial condition; If one wishes the continuous-time approximation to hold for spatially localized initial conditions, one needs at least the DQW to be lazy, as suggested by numerical simulations and by the fact that this has been proven in the case of a vanishing electric field.