Discrete-time quantum walks on two-dimensional regular directed lattices

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Discrete time quantum walks (DTQW) constitute a prominent paradigm on the quest for obtaining complex quantum dynamics based on simple building blocks [1]. For a number of DTQW applications based on square lattices, e. g. search [2], topologically protected dynamics [3], the lowest dimension is two. While the conventional definition of discrete time quantum walks (DTQWs) on two-dimensional square lattices requires four coin dimensions [4], DTQWs with reduced coin dimensionalities have been widely addressed both theoretically and experimentally due to easier analysis and implementation [3, 5–7]. Our work is partly motivated by the utilization rank-2 directed lattices in network models of quantum transport. We demonstrate the equivalence of the $U(1)$ Chalker–Coddington model [8, 9] with the split-step walk on a 2D lattice [3], then define and analyse uniform DTQWs on the Manhattan lattice [10]. We show that the two DTQW processes are not equivalent, manifesting for example in different Anderson localization properties. Moreover, we show that certain symmetries of these two lattices allow us to reduce their description to that of well-studied undirected DTQWs. Our conclusion is that DTQWs on Manhattan lattices may constitute an interesting alternative to the traditionally studied split-step walks on two-dimensional square lattices.

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