Title:

Quantum walks in external gauge fields

Abstract:

Describing particles in external electromagnetic fields is a basic task
of quantum mechanics. The standard scheme for this is known as "minimal
coupling", and consists of replacing the momentum operators in the
Hamiltonian by modified ones with an added vector potential. In lattice
systems it is not so clear how to do this, because there are no momenta.
Moreover, when time is also discrete, as in quantum walk systems, there
is no Hamiltonian, only a unitary step operator. We present a unified
framework of gauge theory for such discrete systems, keeping a close
analogy to the continuum case. In particular, we show how to implement
minimal coupling in a way that automatically guarantees unitary
dynamics. The scheme works in any lattice dimension, for any number of
internal degree of freedom, for walks that allow jumps to a finite
neighborhood rather than to nearest neighbors, is naturally gauge
invariant, and prepares possible extensions to non-abelian gauge groups.
We then focus on the case of constant magnetic fields. Although the
magnetic field is constant, the translation system which realizes the
field is no longer translation invariant. If, however, the entries of
the field-strength tensor are rational, translation invariance can by
retrieved by regrouping the underlying lattice in an appropriate way,
resulting in a translation invariant translation system with respect to
a coarser sublattice. This renders the possibility of classifying the
resulting eigenbundles via Chern classes. As the possible regroupings
are highly non-unique, the question arises whether the characteristic
classes of the eigenbundles are invariant under the chosen regrouping.
We answer this question in the affirmative, by showing that any two
minimally regrouped translation system for a given rational magnetic
field are unitarily equivalent.

Reference:

C. Cedzich, T. Geib, A. H. Werner, and R. F. Werner. Quantum walks in
external gauge fields. Journal of Mathematical Physics, 60(1): 012107,
2019. arXiv:1808.10850.