

Continuum Limits of the 1D Discrete Time Quantum Walk

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Abstract

The discrete time quantum walk (DTQW) is a universal quantum computational model. Significant relationships between discrete and corresponding continuous quantum systems have been studied since the work of Pauli and Feynman. I continue the study of relationships between discrete quantum models and their ostensive continuum counterparts by developing a formal transition between discrete and continuous quantum systems through a formal framework for continuum limits of the DTQW. Under this framework, I have proven two constructive theorems concerning which internal discrete transitions (“coins”) admit nontrivial continuum limits. Additionally, I have shown that the continuous space limit of the continuous time limit of the DTQW only yields massless states which obey the Dirac equation. Finally, I show that for a general coin the continuous time limit of the DTQW can be identified with the canonical continuous time quantum walk (CTQW), provided the coin is allowed to transition through the continuous limit process.