Title:
Decoherent quantum walks and relativistic diffusions

Abstract:
Two models are first presented, of one-dimensional discrete-time quantum walk (DTQW) with temporal noise on the internal degree of freedom (i.e., the coin): (i) a model with both a coin-flip and a phase-flip channel, and (ii) a model with random coin unitaries. It is then shown that both these models admit a common limit in the spacetime continuum, namely, a Lindblad equation with Dirac-fermion Hamiltonian part and, as Lindblad jumps, a chirality flip and a chirality-dependent phase flip, which are two of the three standard error channels for a two-level quantum system. This, as one may call it, Dirac Lindblad equation, provides a model of quantum relativistic spatial diffusion, which is evidenced both analytically and numerically. This model of spatial diffusion has the intriguing specificity of making sense only with original unitary models which are relativistic in the sense that they have chirality, on which the noise is introduced: The diffusion arises via the by- construction (quantum) coupling of chirality to the position. For a particle with vanishing mass, the model of quantum relativistic diffusion introduced in the present work, reduces to the well-known telegraph equation, which yields propagation at short times, diffusion at long times, and exhibits no quantumness. Finally, the results are extended to temporal noises which depend smoothly on position.