Action rigidity for free products of hyperbolic manifold groups

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Joint work with Daniel Woodhouse.

Model geometry

Definition

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- π_1(closed hyperbolic n-manifold) $\curvearrowright \mathbb{H}^n$

Three related notions and rigidity

G and G'
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G and G' have a common model geometry	$\Longrightarrow$	G and G' are *abstractly commensurable* i.e. G and G' have isomorphic finite-index subgroups
Milnor-Schwarz $\Downarrow$	$\Longleftarrow$	
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- If G is QI rigid, then G is action rigid.
- Groups that are action rigid but **not** QI rigid yield examples of quasi-isometric groups with no common model geometry.

QI groups with no common model geometry

1. Virtually free groups:

$$\left\{ G_p = \mathbb{Z}/p\mathbb{Z} * \mathbb{Z}/p\mathbb{Z} \,\middle|\, p \geq 3 \text{ prime} \right\}$$

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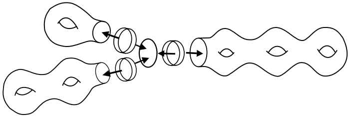

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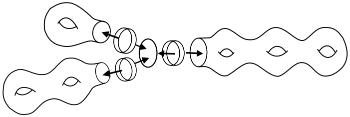

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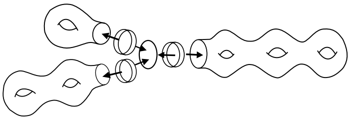

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Theorem (S.–Woodhouse)

Simple surface amalgams G and G' have a common model geometry if and only if G and G' are abstractly commensurable.

Proof strategy

Goal: To prove groups do not have a common model geometry.

Groups:

Step 1: Promote the common model geometry:

If $G, G' \curvearrowright X$, a proper geodesic metric space, then $G, G' \curvearrowright \ldots$

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Class of groups considered: Let

$$\mathcal{C} = \left\{ H_1 * H_2 * \ldots * H_k * F_n \right\},$$

where

- $k \geq 2$ and $n \geq 0$;
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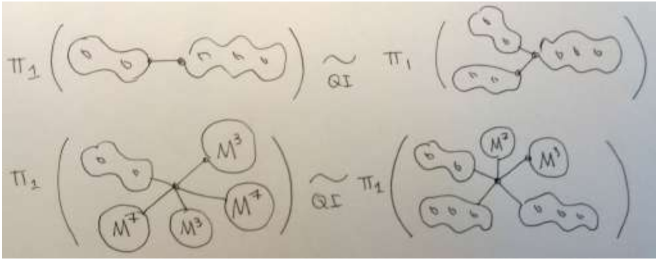
(Really, may take $\mathcal{C}$ to be the set of infinite-ended groups in which each 1-ended vertex group in the Stallings–Dunwoody decomposition is a uniform lattice in the isometry group of a rank-1 symmetric space.)

Theorem (Papasoglu–Whyte, 2002)

The groups $G, G' \in \mathcal{C}$ are quasi-isometric
if and only if
the quasi-isometry types of their one-ended factors agree,
ignoring multiplicity.



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- Within each quasi-isometry class in $\mathcal{C}$, there are infinitely many abstract commensurability classes.
- Thus, each group $G \in \mathcal{C}$ is **not** quasi-isometrically rigid.

Action rigidity theorem

Theorem (S.-Woodhouse)

Each group $G \in \mathcal{C}$ is action rigid.

That is, if G' is a group and G and G' act geometrically on the same proper geodesic metric space, then G and G' are abstractly commensurable.

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Corollary

There are quasi-isometric groups that do not **virtually** *have a common model geometry.*

Additional rigidity: two closed surfaces

Theorem (S.-Woodhouse)

Let $G = \pi_1(S_{g_1}) * \pi_1(S_{g_2})$, *and* $G' = \pi_1(S_{g'_1}) * \pi_1(S_{g'_2})$.
The groups G *and* G' *have a common model geometry if and only if* $G \cong G'$.

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There are torsion-free abstractly commensurable groups that do not have a common model geometry.

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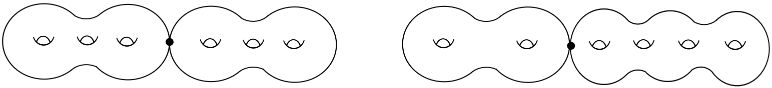
Example:

(Whyte) G and G' are abstractly commensurable if and only if $\chi(G) = \chi(G')$.



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2. If H and H' act geometrically on the same contractible simplicial complex, are H and H' abstractly commensurable?

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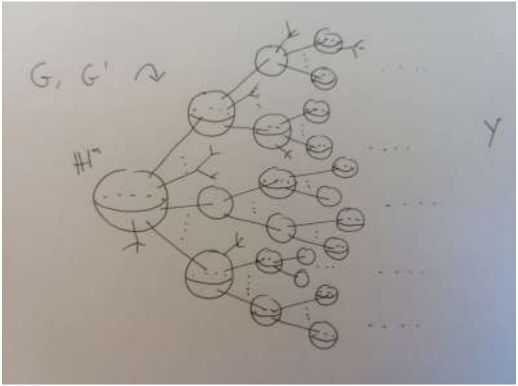
Proof Strategy Step 1: Promote the common model geometry

Let $G, G' \in \mathcal{C}$.
Suppose $G, G' \curvearrowright X$, a proper geodesic metric space geometrically.

We show G and G' act geometrically on an *ideal model geometry:*



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If G and G' act geometrically on a proper geodesic metric space,

then G and G' act geometrically on a simply connected simplicial

complex that has a tree of spaces decomposition with

Vertex spaces: *Either* 1*-ended or a point*
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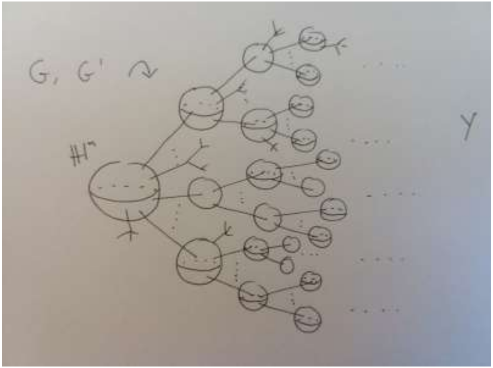
- Main tool: Structure of the visual boundary

IDEAL MODEL GEOMETRY



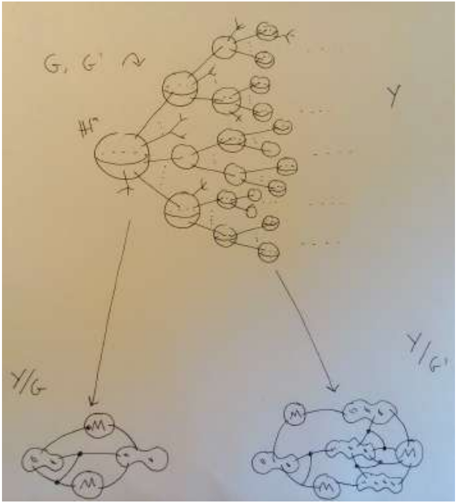
If $G, G' \in \mathcal{C}$, we apply work of

Tukia; Hinkkanen; Markovic; Chow; Pansu

to replace the 1-ended vertex spaces with copies of $\mathbb{H}^n_{\mathbb{F}}$.

$\longrightarrow$ Reformulate our goal topologically

Goal: Prove that G and G' are abstractly commensurable.



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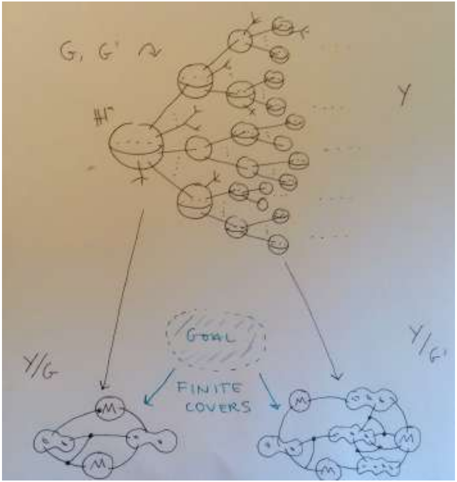

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- We want to exhibit homeomorphic finite-sheeted covering spaces

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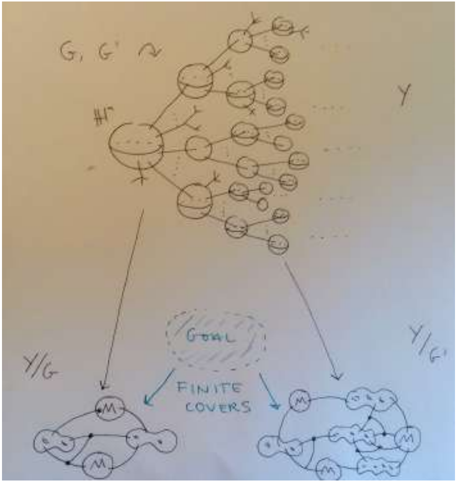

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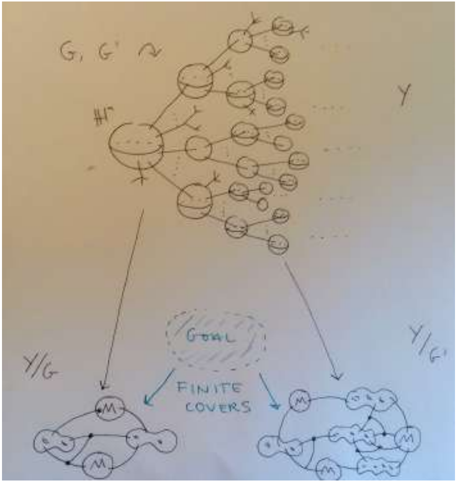

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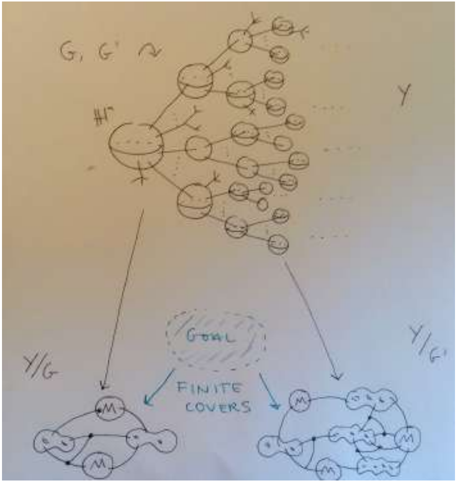
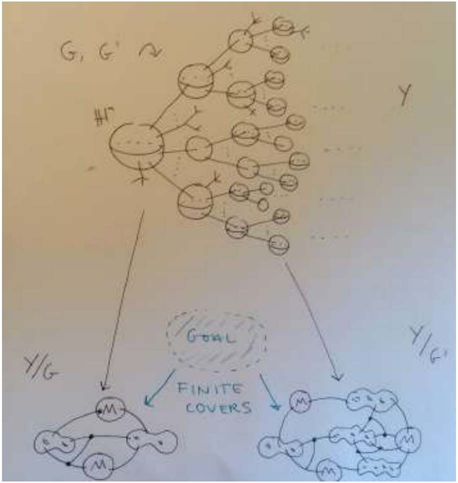


- If each manifold in Y/G and Y/G' was replaced with a point, then the existence of common finite covers follows from *Leighton's Theorem*.
- However, if Y/G and Y/G' were single hyperbolic n-manifolds with isometric universal covers, then the goal would be false in general.

Proof Strategy Part II: Symmetry-restricted Leighton's Theorem



Leighton's Theorem: If finite graphs Γ and Γ' have isomorphic universal covers, then Γ and Γ' have isomorphic finite-sheeted covers.

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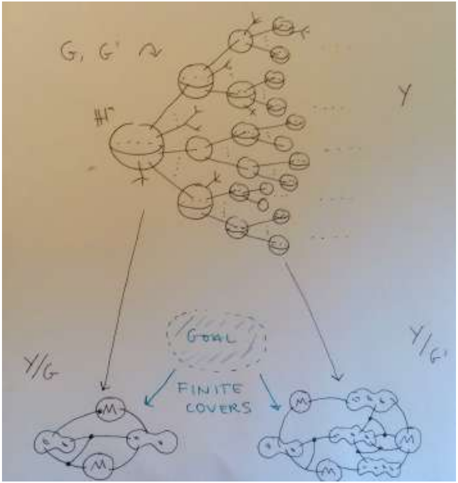

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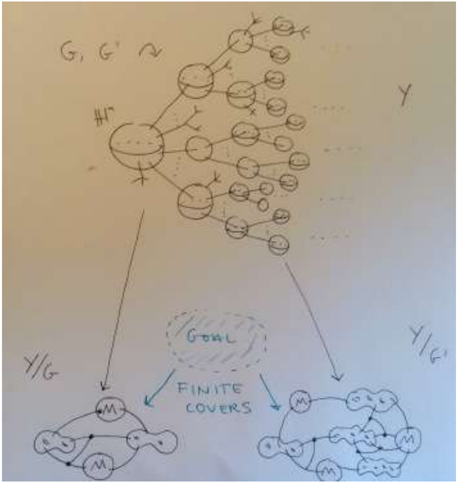

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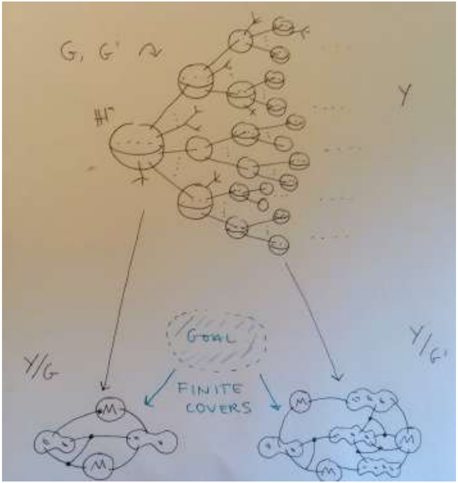

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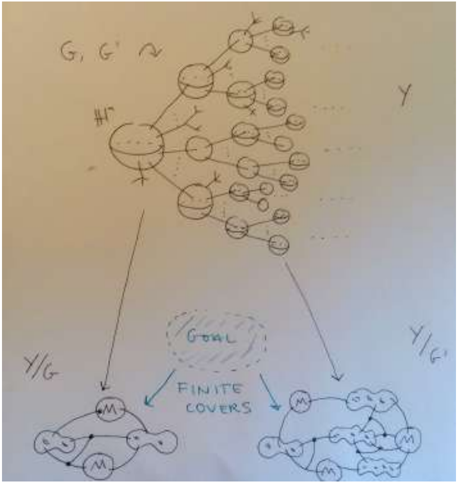

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Leighton's Theorem. Two free uniform lattices $F, F' \leq \mathrm{Aut}(T)$ are *weakly commensurable* in $\mathrm{Aut}(T)$.

(That is, $\exists g \in \mathrm{Aut}(T)$ so $gFg^{-1} \cap F'$ is finite index in gFg^{-1} and F'.)

Symmetry-restricted Leighton's Theorem.
(Gardam–Woodhouse; Shepherd)

Two free uniform lattices $F, F' \leq \mathrm{Aut}(T)$ contained in a *symmetry-restricted* $H \leq \mathrm{Aut}(T)$ are weakly commensurable in H.

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Def: A subgroup $H \leq \mathrm{Aut}(T)$ is *R-symmetry-restricted* if

$$H = \{\, g \in \mathrm{Aut}(T) \mid \forall v \in VT, \exists h \in H \text{ s.t. } g_v = h_v : B_R(v) \to B_R(gv) \,\}.$$

Example: If T is a colored tree, then the color-preserving automorphism group is symmetry-restricted.

Thank you!