A. Fan, "*μ-Dvoretzky random covering*"

Throw random intervals on the circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$. Assume that the centers are independent and follow a probability law μ and that the lengths are given positive numbes less than 1. The μ-Dvoretzy covering problem is to find conditions for the circle to be covered almost surely by the random intervals or for a given compact set to be covered almost surely. When μ is the Lebesgue measure, we find the classical Dvoretzky covering problem (1956), which was solved by L. Shepp (1972) for the circle, by J. P. Kahane for compact sets (1987). Many people were interested in and/or had contributed to this problem and related problems (P. Levy, P. Billard, S. Orey, P. Erd\"{o}s, B. Mandelbrot, J. Hawkes, A. H. Fan, J. Barral, J. Wu et al). Kahane's solution is based on B. Mandelbrot's Poisson model and S. Janson 's stopping time technique. Classical Dvoretzky problem of higher dimension remains unsolved (intervals are replaced by balls or cubes etc). With D. Karagulyan, we consider an absolutely continuous probability measure instead of Lebesgue measure. Under weak regularity condition on the density, we can solve the problem. However efforts are needed for singular probability measures. Let us mention that Fan, Schmeling and Troubetzkoy had considered the case that the centers are the orbits of the doubling dynamics according to a (singular) Gibbs measure (the indepedence is lost in this case). Only a weak result was obtained in this case. The multiplicative chaos is one of our useful tools and we base our discussions on Shepp's and Kahane's results by establishing some comparison principles. Open questions will be mentioned.