Estelle Basor (AIM) -

Title:t *Toeplitz determinants, Painlevé equations, and special functions. Part I: an operator approach*

Abstracts:

These lectures will focus on understanding properties of classical operators and their connections to other important areas of mathematics. Perhaps the simplest example is the asymptotics of determinants of finite Toepltiz matrices, which have constants along the diagonals. The determinants of these n by n size matrices, have (in appropriate cases) an asymptotic expression that is of the form G^n x E where both G and E are constants. This expansion is useful in describing many statistical quantities variables for certain random matrix models.

In other instances, where the above expression must be modified, the asymptotics correspond to critical temperature cases in the Ising Model, or to cases where the random variables are in some sense singular.

Generalizations of the above result to other settings, for example, convolution operators on the line, are also important. For example, for Wiener-Hopf operators, the analogue of the determinants of finite matrices is a Fredholm determinant. These determinants are especially prominent in random matrix theory where they describe many quantities including the distribution of the largest eigenvalue in the classic Gaussian Unitary Ensemble, and in turn connections to Painleve equations.

The lectures will use operator theory methods to first describe the simplest cases of the asymptotics of determinants for the convolution (both discrete and continuous) operators, then proceed to the more singular cases. Operator theory techniques will also be used to illustrate the links to the Painleve' equations.