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TILINGS OF A HEXAGON AND NON-HERMITIAN ORTHOGONALITY ON A CONTOUR

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Abstract

I will discuss polynomials P_N of degree N that satisfy non-Hermitian orthogonality conditions with respect to the weight $\frac{(z+1)^N(z+a)^N}{z^{2N}}$ on a contour in the complex plane going around 0. These polynomials reduce to Jacobi polynomials in case $a = 1$ and then their zeros cluster along an open arc on the unit circle as the degree tends to infinity.

For general a, the polynomials are analyzed by a Riemann-Hilbert problem. It follows that the zeros exhibit an interesting transition for the value of $a = 1/9$, when the open arc closes to form a closed curve with a density that vanishes quadratically. The transition is described by a Painlevé II transcendent.

The polynomials arise in a lozenge tiling problem of a hexagon with a periodic weighting. The transition in the behavior of zeros corresponds to a tacnode in the tiling problem.

This is joint work in progress with Christophe Charlier, Maurice Duits and Jonatan Lenells and we use ideas that were developed in [2] for matrix valued orthogonal polynomials in connection with a domino tiling problem for the Aztec diamond.

References

[1] **C. Charlier, M. Duits, A.B.J. Kuijlaars, and J. Lenells**, in preparation

[2] **M. Duits and A.B.J. Kuijlaars**, The two periodic Aztec diamond and matrix valued orthogonal polynomials, to appear in J. Eur. Math. Soc., preprint arXiv:1712.05636.

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