

CIRM 2019 Integrability workshop:

Title: Determinantal structure of eigenvector correlations in the complex Ginibre ensemble

Abstract: We study the expectation of the matrix of overlaps of left and right eigenvectors in the complex Ginibre ensemble, conditioned on a fixed number of  $k$  complex eigenvalues. The diagonal ( $k=1$ ) and off-diagonal overlap ( $k=2$ ) were introduced by Chalker and Mehlig. They provided exact expressions for finite matrix size  $N$ , in terms of a large determinant of size proportional to  $N$ . In the large- $N$  limit these overlaps were determined on the global scale and heuristic arguments for the local scaling at the origin were given. The topic has seen a rapid development in the recent past. Our contribution is to derive exact determinantal expressions of size  $k \times k$  in terms of a kernel, valid for finite  $N$  and arbitrary  $k$ . It can be expressed as an operator acting on the complex eigenvalue correlation functions and allows us to determine all local correlations in the bulk close to the origin, and at the spectral edge. The methods we use are bi-orthogonal polynomials in the complex plane and the analyticity of the diagonal overlap for general  $k$ .

This is joint work with Roger Tribe, Athanasios Tsareas, and Oleg Zaboronski as appeared in [arXiv:1903.09016](https://arxiv.org/abs/1903.09016) [math-ph]