A non arithmetic approach to the Coleman-Oort conjecture

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The Jacobian locus $\overline{J_g}$ is defined as the closure in the moduli space A_g of principally polarized abelian varieties of the image of the Torelli map $j : M_g \to A_g$, mapping any smooth projective curve to its Jacobian variety (modulo isomorphisms). The Coleman-Oort conjecture asserts that for g large enough there should be no Shimura varieties generically contained in $\overline{J_g}$. Even if the conjecture is of arithmetic kind, it can be detected by using complex methods after a paper of Mumford, classifying Shimura varieties as totally geodesic varieties with a CM-point. In the talk we address the problem in this way, by relating two things: unitary flat periods of a Jacobian and the Clifford index of the corresponding curve. Indeed, as by the Torelli theorem any Jacobian reconstructs a unique curve (up to isomorphism), complex Hodge theory in the Jacobian locus naturally reflects properties of the geometric Hodge theory in the moduli space of curves. The plan is to prove an obstruction to the existence of some deformations along geodesics and then to deduce from this a bound on the codimension of a totally geodesics variety generically contained in the Jacobian locus. This is a joint work with Alessandro Ghigi and Gian Pietro Pirola.