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Angela Ortega: Prym varieties and Prym maps.

Prym varieties are a particular type of polarized abelian varieties which arise from coverings between projective smooth curves. The main motivation behind this construction is that one can understand better abelian varieties (mostly with principal polarization but not only) via the geometry of the coverings that define them. By considering the associated Prym variety $P(f)$ of a covering $f : C' \to C$ over a genus g curve C, of a given degree and ramification pattern, one can define the corresponding Prym map $[f : C' \to C] \mapsto P(f)$ from the moduli space of such coverings to the moduli space of polarized abelian varietes $\mathcal{A}_d^D$ of dimension d and polarization type D, where d and D are determined by the invariants of the covering.

In the first talk we will recall the necessary preliminaries about abelian and Prym varieties, and define the Prym map. We will give some classical examples of Prym maps and related results.

In the second talk, we will explain the construction of the Prym-Tyurin map $PT : \mathcal{H}_{E_6} \to \mathcal{A}_6$, where $\mathcal{H}_{E_6}$ denotes the moduli space of coverings $C \to \mathbb{P}^1$ of degree 27 and ramified over 24 points with monodromy group the Weyl group $W(E_6)$. This map is dominant, so the general principally polarized abelian sixfold is a Prym-Tyurin variety associated to a covering in $\mathcal{H}_{E_6}$.

In the last talk we will discuss two other examples of Prym maps, which corresponding abelian varieties are no longer principally polarized. We will give the main ingredients to show that:

(1) The Prym map $\mathcal{R}_{2,6} \to \mathcal{A}_4^{(1,1,2,2)}$ is generically injective, where $\mathcal{R}_{2,6}$ denotes the moduli space of double coverings of genus 2 curves ramified in 6 points. In particular, its image defines a divisor in $\mathcal{A}_4^{(1,1,2,2)}$.

(2) The Prym map $\mathcal{R}_2(7) \to \mathcal{A}_6^{(1,1,1,1,1,7)}$ is generically finite of degree 10 onto its image, where $\mathcal{R}_2(7)$ denotes the moduli space of étale cyclic coverings over a genus 2 curve.