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Philip Boalch: Introduction to wild character varieties and wild surface groups.

As is well known the category of tame (or *regular singular*) connections on algebraic vector bundles on a smooth noncompact complex algebraic curve has a purely topological description as the category of local systems of complex vector spaces. Considering suitable moduli spaces one gets a noncompact moduli space with two distinct algebraic structures (that can be viewed as the De Rham and Betti realisation of the underlying nonabelian motive). As we vary the curve this change in algebraic structure can be used as a kind of nonlinear Fourier transform to solve nonlinear differential equation (such as Painleve 6). Somewhat less well-known is the natural extension of this story to the other (irregular/wild) connections, and how that solves even more nonlinear differential equations (such as Painleve 1-5). Amusingly the space of deformation parameters can now be much larger than just the moduli of the underlying punctured curve, leading to the notion of *wild Riemann surfaces*. If time permits I'll also discuss steps towards the classification of such nonabelian Hodge moduli spaces (global Lie theory), how a plethora of Nakajima quiver varieties occur as “Lie algebras” of such spaces, and also the third algebraic structure that such moduli spaces have (as meromorphic Higgs bundles) yielding the link to the algebraic integrable systems of Hitchin, and Garnier (the classical Gaudin model).