

Tordre les pattes des araignées

Interaction entre algèbres de Frobenius spéciales commutatives et structures compactes autoduales
dans les catégories monoidales symétriques

Titouan Carette

LORIA, équipe MOCQUA, Nancy

March 5, 2019

Connaître les ficelles

Nom

Symbole

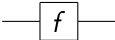
Diagramme

Exemple

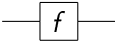
Connaître les ficelles

Nom	Symbole	Diagramme	Exemple
Transformation	f		



Connaître les ficelles

Nom	Symbole	Diagramme	Exemple
Transformation	f		Matrices

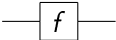

Connaître les ficelles

Nom	Symbole	Diagramme	Exemple
Transformation	f		Matrices
Composition	$g \circ f$		


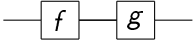

Connaître les ficelles

Nom	Symbole	Diagramme	Exemple
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Composition	$g \circ f$		Produit de matrices


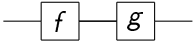

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Nom	Symbole	Diagramme	Exemple
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Composition	$g \circ f$		Produit de matrices
Identité	id		

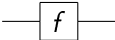


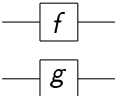
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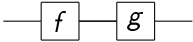

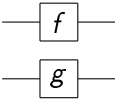
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Nom	Symbole	Diagramme	Exemple
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Tenseur	$f \otimes g$		

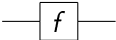


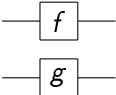

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Unité	I		

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Unité	1		Scalars

Connaître les ficelles

$$(g \otimes k) \circ (f \otimes h) = (g \circ f) \otimes (k \circ h)$$

Connaître les ficelles

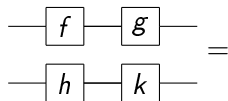
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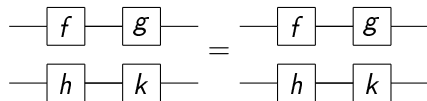
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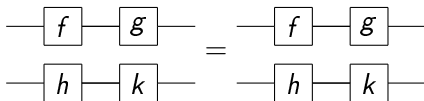
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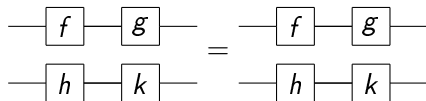


☹ Les diagrammes nous libèrent de la bureaucratie.

Connaître les ficelles

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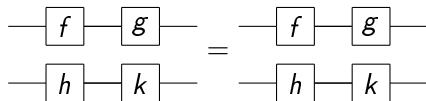


- ⊕ Les diagrammes nous libèrent de la bureaucratie.
- ⊕ On peut toujours revenir aux symboles si besoin.

Connaître les ficelles

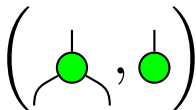
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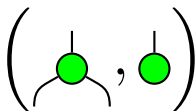


- ⊕ Les diagrammes nous libèrent de la bureaucratie.
- ⊕ On peut toujours revenir aux symboles si besoin.
- ⊕ On obtient un nouveau point de vue sur des structures bien connues.

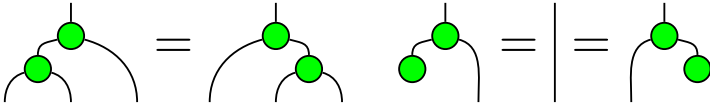
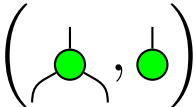
Le monoïde



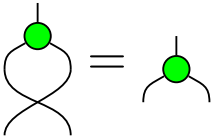
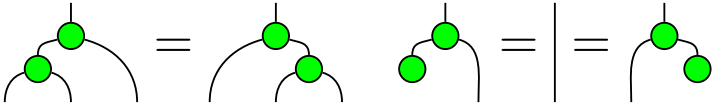
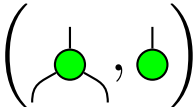
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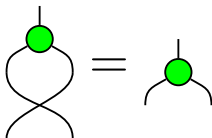
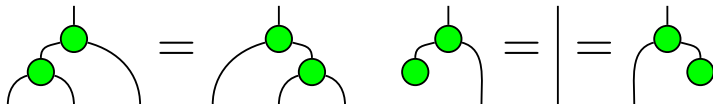
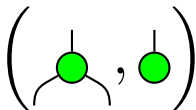
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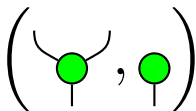


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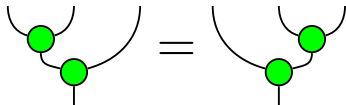
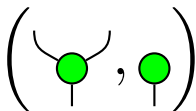


⊕ Exemples: concatenation et mot vide, addition et 0, produit d'Hadamard et matrice Atila, convolution et Dirac, etc...

Le comonoïde



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Le comonoïde

$$\left(\begin{array}{c} \text{---} \text{---} \\ \text{---} \bullet \\ \text{---} \end{array}, \begin{array}{c} \bullet \\ \text{---} \end{array} \right)$$

$$\begin{array}{c} \text{---} \text{---} \\ \text{---} \bullet \\ \text{---} \end{array} = \begin{array}{c} \text{---} \text{---} \\ \text{---} \bullet \\ \text{---} \end{array} \quad \begin{array}{c} \bullet \\ \text{---} \end{array} = \begin{array}{c} | \\ \text{---} \end{array} = \begin{array}{c} \text{---} \text{---} \\ \text{---} \bullet \\ \text{---} \end{array}$$

$$\begin{array}{c} \text{---} \text{---} \\ \text{---} \bullet \\ \text{---} \end{array} = \begin{array}{c} \text{---} \text{---} \\ \text{---} \bullet \\ \text{---} \end{array}$$

Le comonoïde

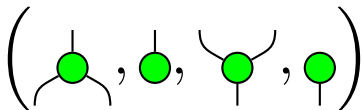
$$\left(\begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \bullet \\ \text{---} \end{array} , \begin{array}{c} \bullet \\ \text{---} \end{array} \right)$$

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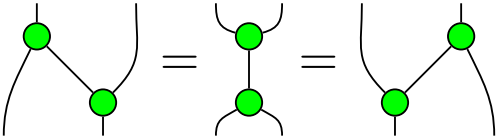
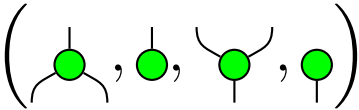
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⊕ Exemples: copie et effacement, une habile magouille, si vous en trouvez d'autres intéressants je suis preneur !

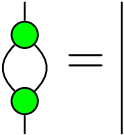
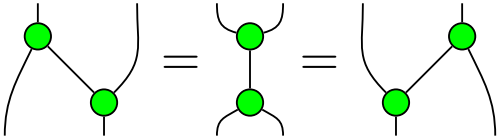
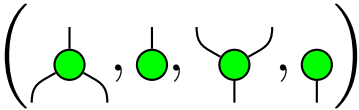
Les algèbres de Frobenius



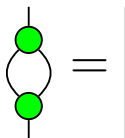
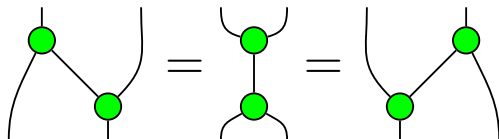
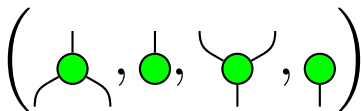
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
⊕ Exemples: relation d'égalité, autres idées ?

Basique, simple ?

- ⊖ On se donne une base e_j et les formes linéaires associées $\hat{e}_j : e_j \mapsto 1$.

Basique, simple ?

- ⊖ On se donne une base e_i et les formes linéaires associées $\hat{e}_i : e_i \mapsto 1$.



$: e_i \otimes e_j \mapsto \delta_{i,j} e_i$

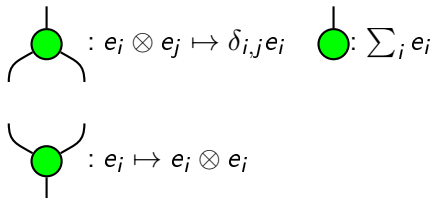
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$$\begin{array}{c} | \\ \bullet \\ / \quad \backslash \\ \text{---} \end{array} : e_i \otimes e_j \mapsto \delta_{i,j} e_i \quad \begin{array}{c} | \\ \bullet \\ \text{---} \end{array} : \sum_i e_i$$

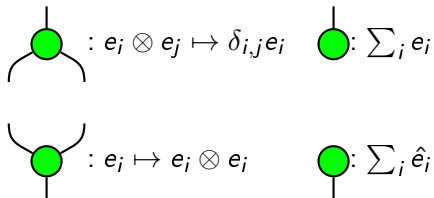
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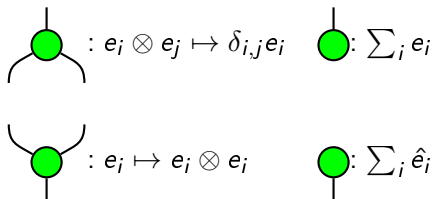
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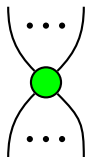
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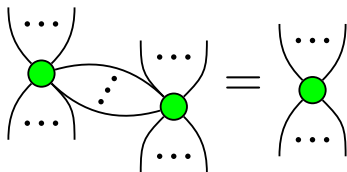
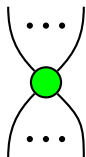


- ⊗ Il y a en fait une correspondance entre les bases d'un \mathbb{C} espace vectoriel et les algèbres de Frobenius !

Le théorème de l'araignée



Le théorème de l'araignée

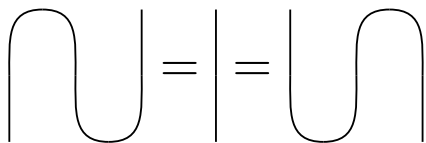


L'équation du serpent

$$(n, U)$$

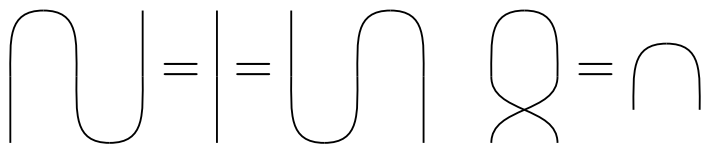
L'équation du serpent

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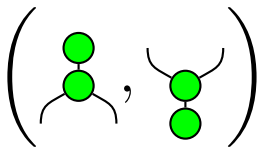
L'équation du serpent

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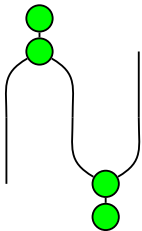
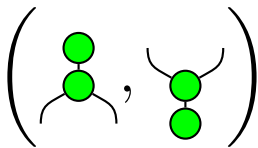
The diagram shows two equations. The first equation is the snake equation: a curve that starts as a vertical line, goes up, forms a hump, goes down, forms a U-shape, and ends as a vertical line, is equal to a vertical line, which is equal to a curve that starts as a vertical line, goes up, forms a U-shape, goes down, forms a hump, and ends as a vertical line. The second equation is the Reidemeister move: a figure-eight curve (two loops sharing a point) is equal to a single hump curve.

- ⊕ Exemples: produit scalaire, axiome et coupure, et bien sûr une autre subtile magouille...

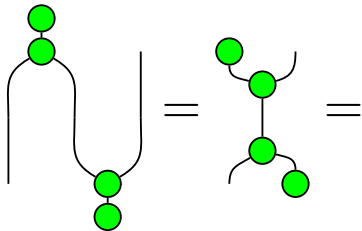
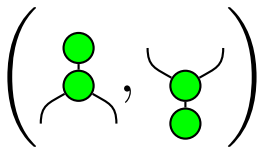
Et les araignées donneront naissance à des serpents...



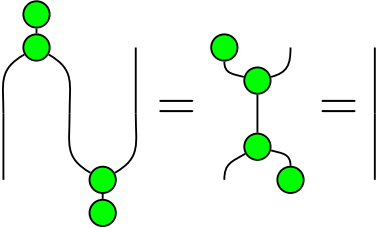
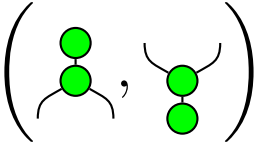
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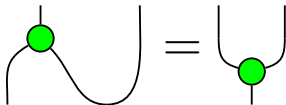
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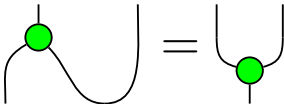
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“Only Adjacency Matters”

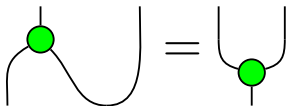


“Only Adjacency Matters”



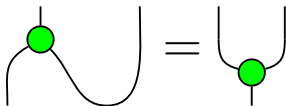
⊕ Les diagrammes sont des graphes !

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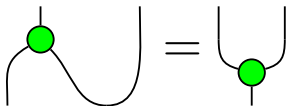
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“Only Adjacency Matters”



- ⊖ Les diagrammes sont des graphes !
- ⊖ Représentation mémoire.
- ⊖ Calcul par réécriture de graphes.

“Only Adjacency Matters”



- ⊗ Les diagrammes sont des graphes !
- ⊗ Représentation mémoire.
- ⊗ Calcul par réécriture de graphes.
- ⊗ Application de résultats de théorie des graphes en algèbre linéaire.

Pour aller plus loin:



Pour aller plus loin:

Le dodo

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Pour aller plus loin:

Le dodo

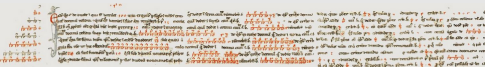
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1. Makélélé and Linear Algebra



linear algebra is the **Claude Makélélé** of science and mathematics. Makélélé is a well-known, retired football player, a French international. He played in the famous Real Madrid team of the early 2000s. That team was full of “galácticos” — the most famous and glamorous players of their generation. Players like Zidane, Figo, Ronaldo and Roberto Carlos. Makélélé was hardly

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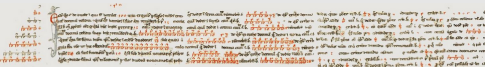
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C'est la fin !