Tordre les pattes des araignées

Interaction entre algèbres de Frobenius spéciales commutatives et structures compactes autoduales dans les categories monoidales symétriques

Titouan Carette

LORIA, équipe MOCQUA, Nancy

March 5, 2019

Connaître les ficelles

Nom	Symbole	Diagramme	Exemple

Connaître les ficelles

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Transformation	f		

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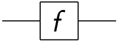
Transformation	f		Matrices

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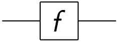
Transformation	f		Matrices
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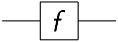
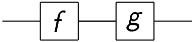
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices

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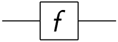
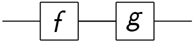
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices
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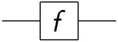
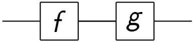
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices
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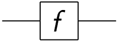
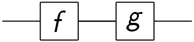
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices
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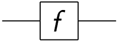
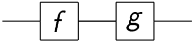
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices
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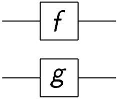
Tenseur	$f \otimes g$		Produit de Kronecker

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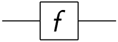
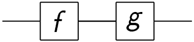
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices
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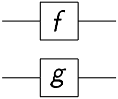
Tenseur	$f \otimes g$		Produit de Kronecker
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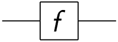
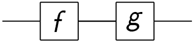
Transformation	f		Matrices
Composition	$g \circ f$		Produit de matrices
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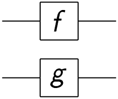
Tenseur	$f \otimes g$		Produit de Kronecker
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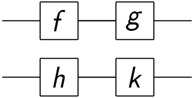

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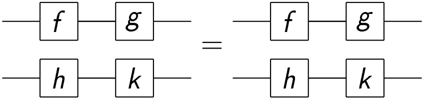

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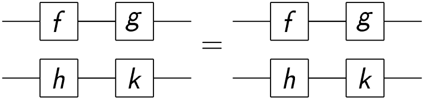

- Les diagrammes nous libèrent de la bureaucratie.

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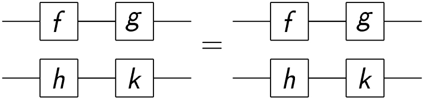

- ☺ Les diagrammes nous libèrent de la bureaucratie.
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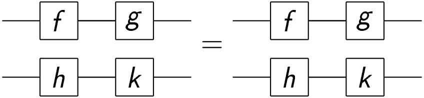
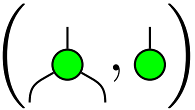


- Les diagrammes nous libèrent de la bureaucratie.
- On peut toujours revenir aux symboles si besoin.
- On obtient un nouveau point de vue sur des structures bien connues.

Le monoïde



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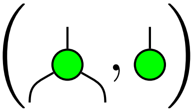

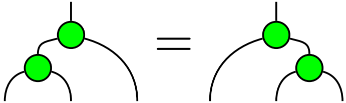

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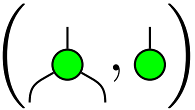

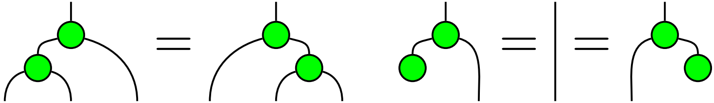

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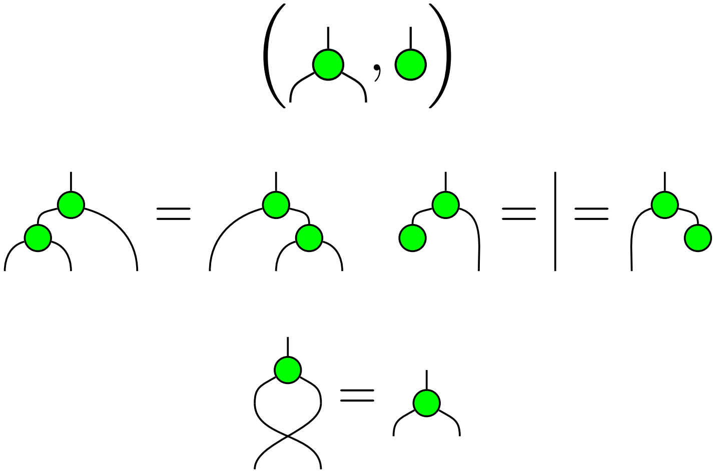

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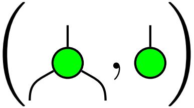
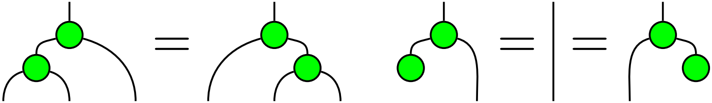
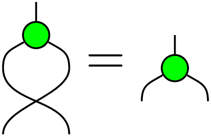
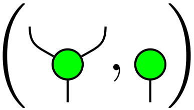






- Exemples: concatenation et mot vide, addition et 0, produit d'Hadamard et matrice Atilla, convolution et Dirac, etc...

Le comonoïde



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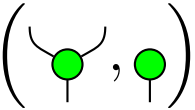

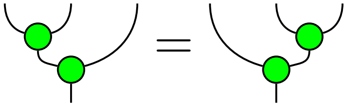

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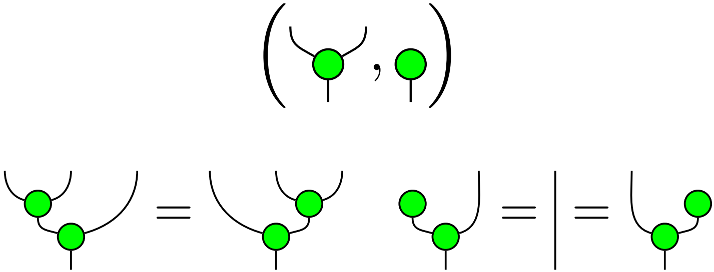

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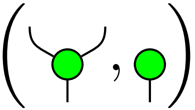

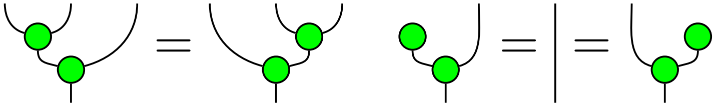

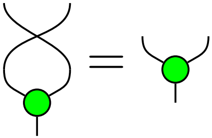

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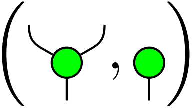
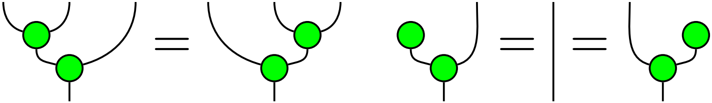
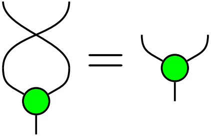
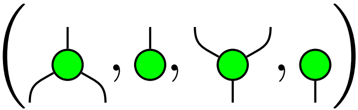






- Exemples: copie et effacement, une habile magouille, si vous en trouvez d'autres interressants je suis preneur !

Les algèbres de Frobenius



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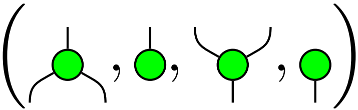

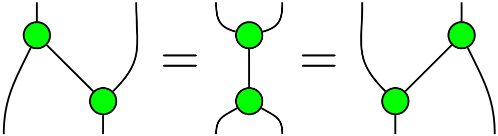

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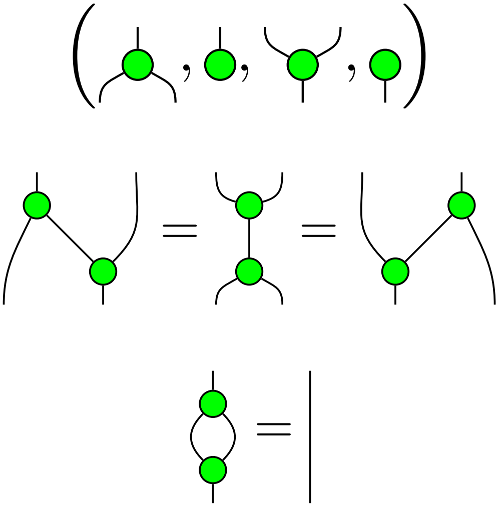

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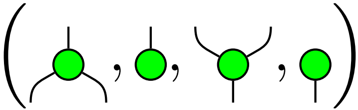

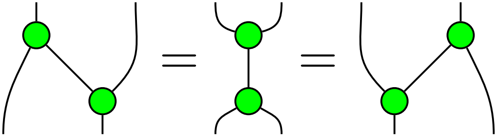

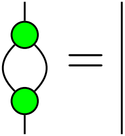

- Exemples: relation d'egalité, autres idées ?

Basique, simple ?

- On se donne une base e_i et les formes linéaires associées $\hat{e}_i : e_i \mapsto 1$.

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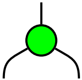

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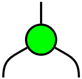
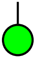
 $: e_i \otimes e_j \mapsto \delta_{i,j} e_i$  $: \sum_i e_i$

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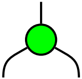

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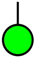

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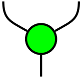

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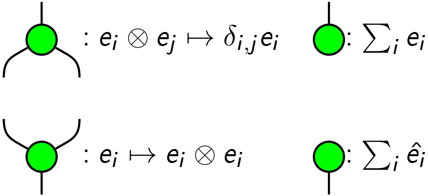

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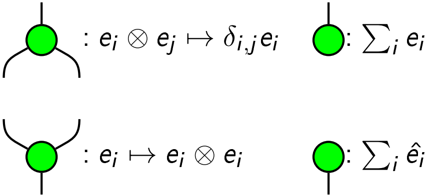

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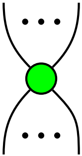
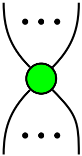
$: e_i \mapsto e_i \otimes e_i$ $\quad$ $: \sum_i \hat{e}_i$

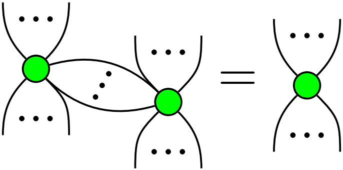
- Il y a en fait une correspondance entre les bases d'un $\mathbb{C}$ espace vectoriel et les algèbres de Frobenius !

Le théorème de l'araignée



Le théorème de l'araignée





L'équation du serpent

$$\left(\cap , \cup\right)$$

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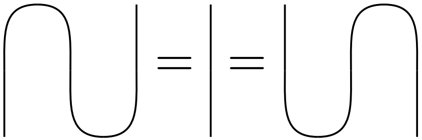
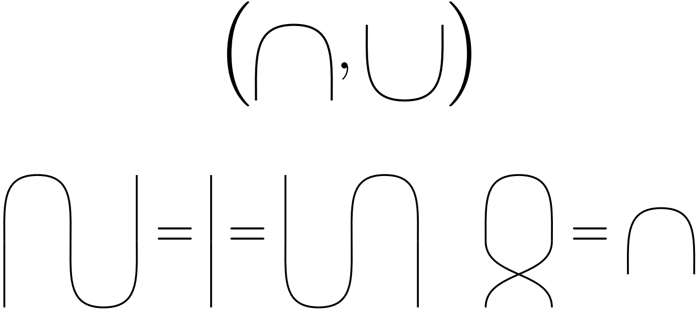
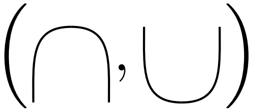
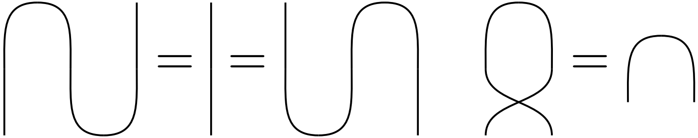


L'équation du serpent



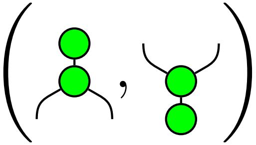
L'équation du serpent





- Exemples: produit scalaire, axiome et coupure, et bien sûr une autre subtile magouille...

Et les araignées donneront naissance à des serpents...



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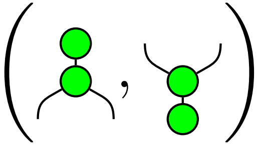

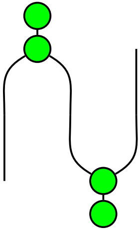

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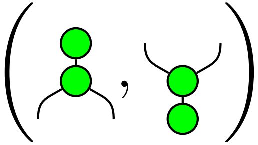

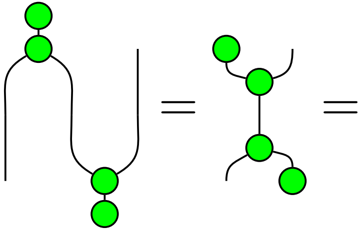

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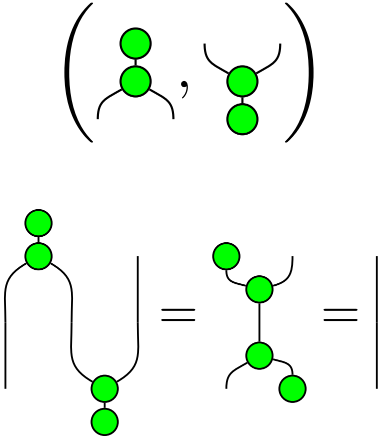
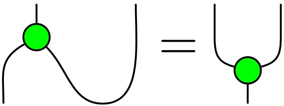


"Only Adjacency Matters"



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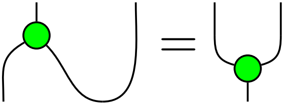

- Les diagrammes sont des graphes !

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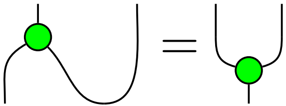
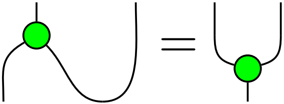


- Les diagrammes sont des graphes !
- Représentation mémoire.

“Only Adjacency Matters”



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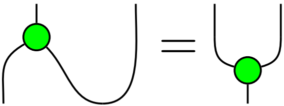
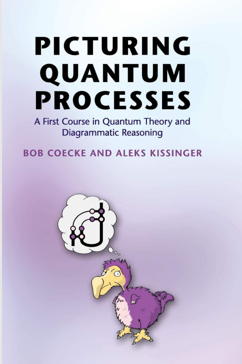


- Les diagrammes sont des graphes !
- Représentation mémoire.
- Calcul par réecriture de graphes.
- Application de résultats de théorie des graphes en algèbre linéaire.

Pour aller plus loin:

Pour aller plus loin:

Le dodo



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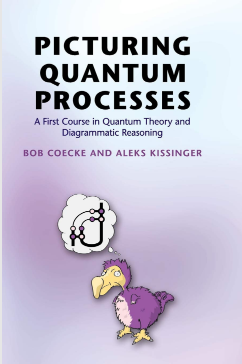

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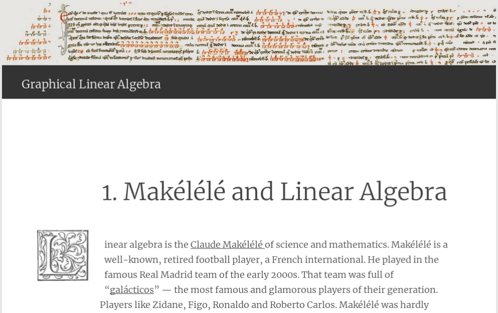

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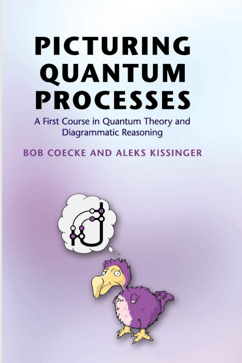

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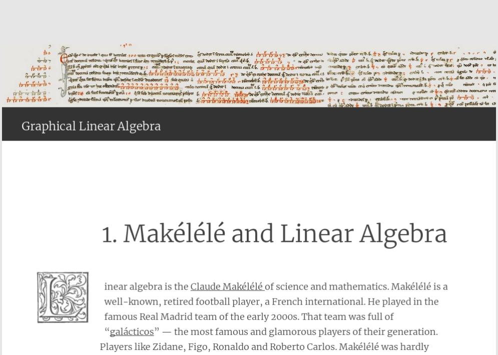

C'est la fin !