

# Dynamics in graph games

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Ecole Jeunes Chercheurs en Informatique Mathématique



## 1 General context

## 2 Definitions

## 3 Particular kind of games

- Sequential games
- Reachability games

## 4 Dynamics

## 5 Results

- Sequential games
- Reachability games

## 6 Computer Networking

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# Reactive systems

## Reactive system

Coffee machine

Plane's autopilot

## Environment interaction

Customer

Weather

## Specification

Give coffee, give change,  
does not explode, ...

Arrive safe

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## Possible solutions :

- Testing

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Give coffee, give change,  
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## Possible solutions :

- Testing
- Model-checking

# What is model-checking ?

**Real system**  
*plane,...*

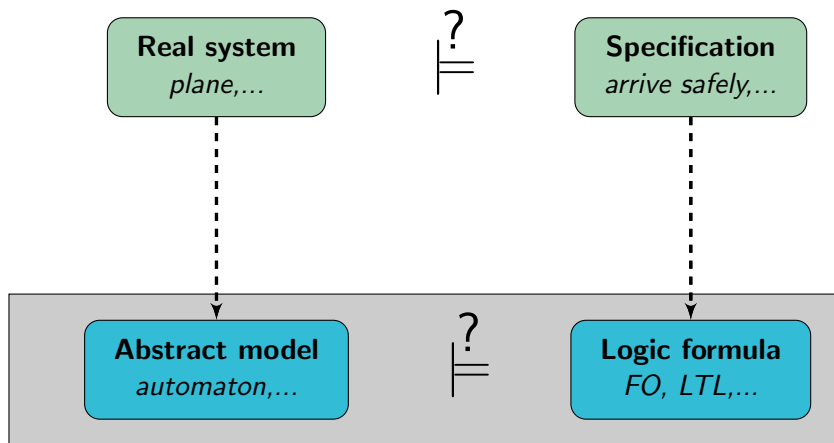
?

≡

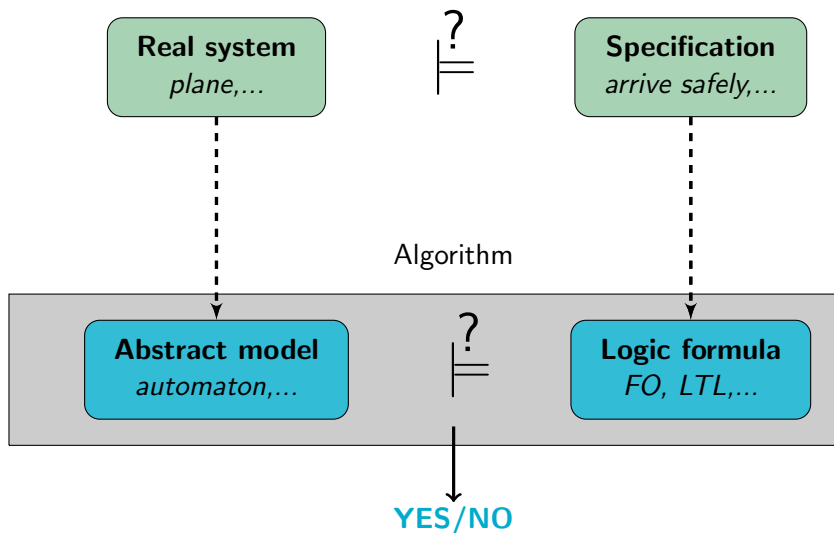
**Specification**  
*arrive safely,...*



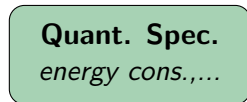
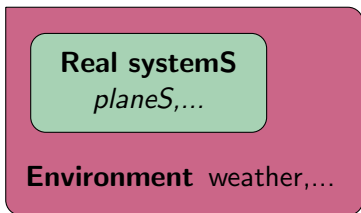
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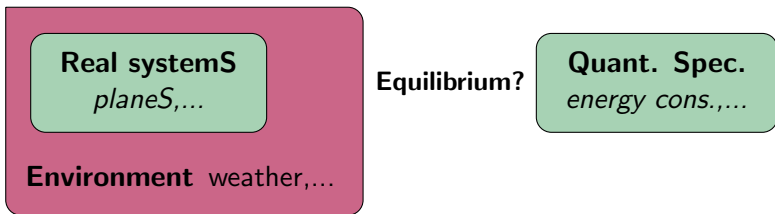
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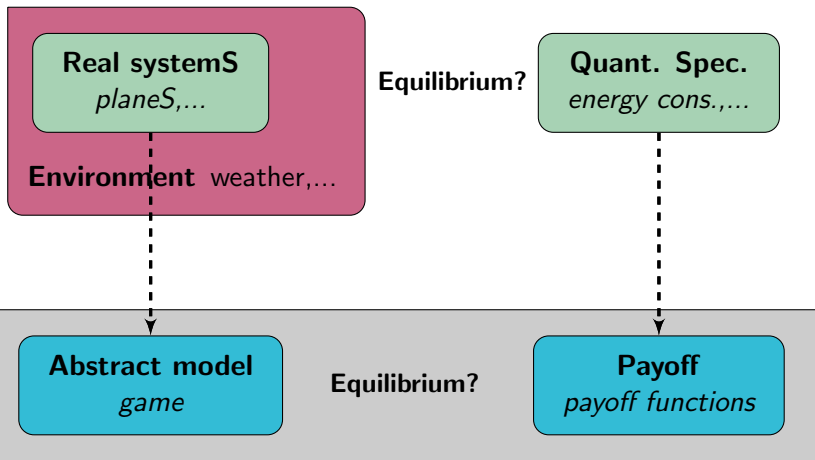
# From model-checking to (algorithmic) game theory



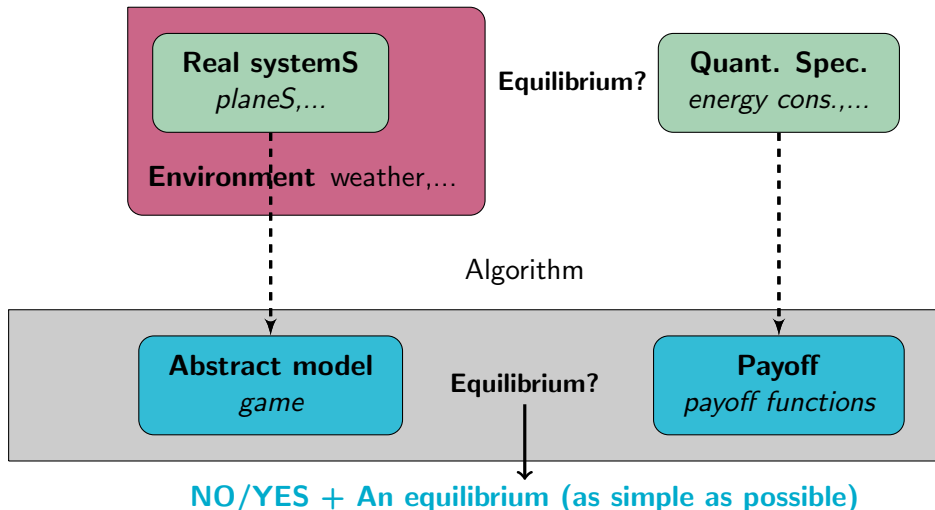
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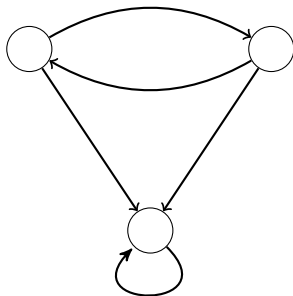
# Finite graph games

## ■ Nodes



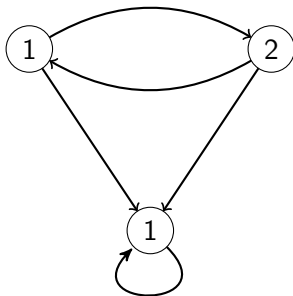


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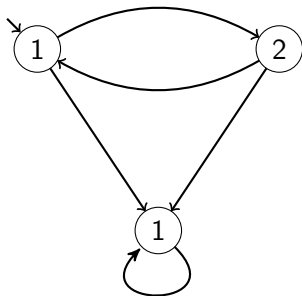
- Nodes
- Edges

# Finite graph games



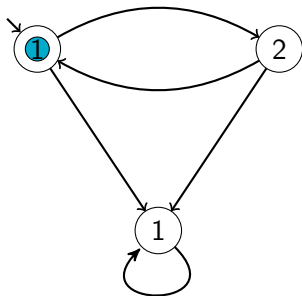
- Nodes
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- Players

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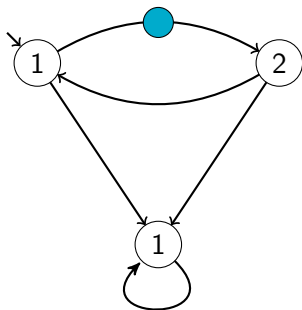
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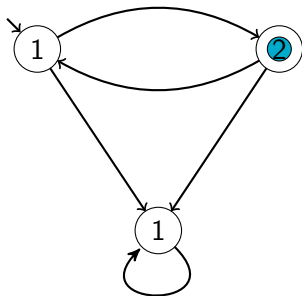
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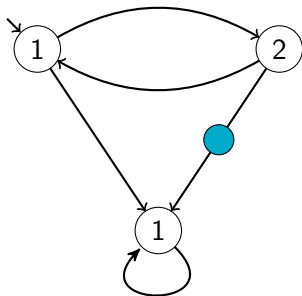
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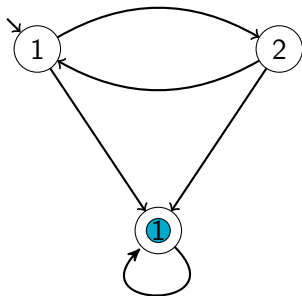
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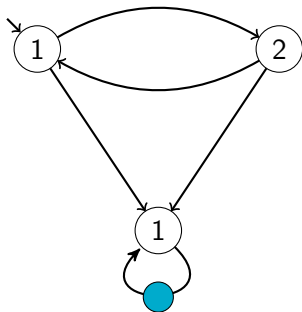
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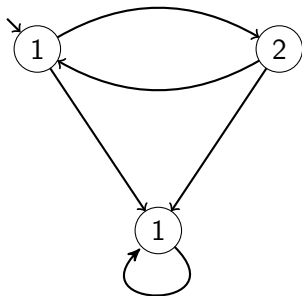


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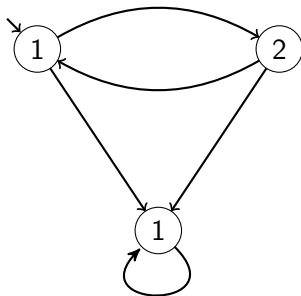
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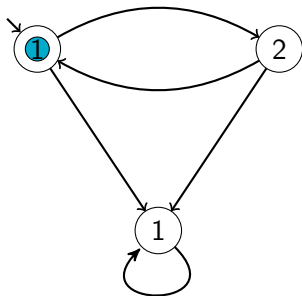
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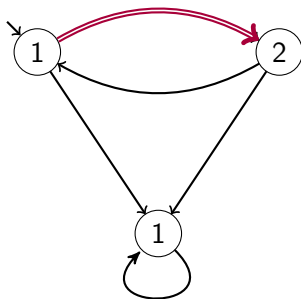
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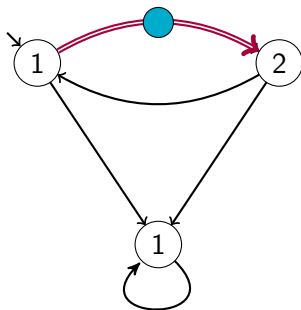
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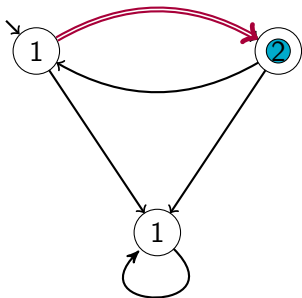
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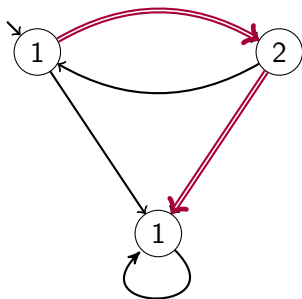
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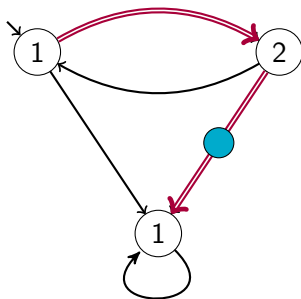
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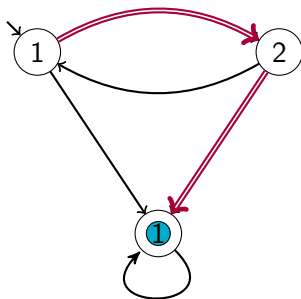


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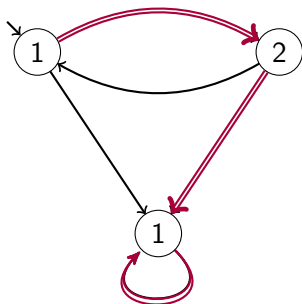
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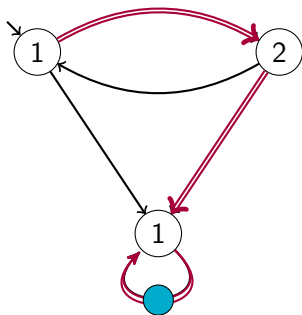
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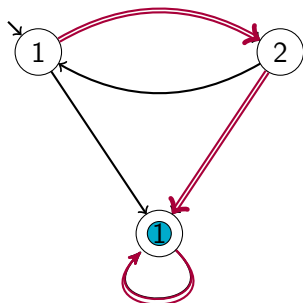
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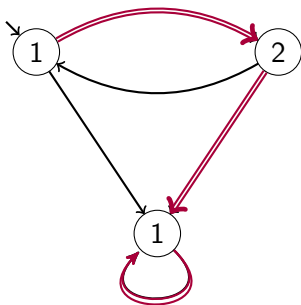
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We only deal with positional strategies (pure and memoryless)

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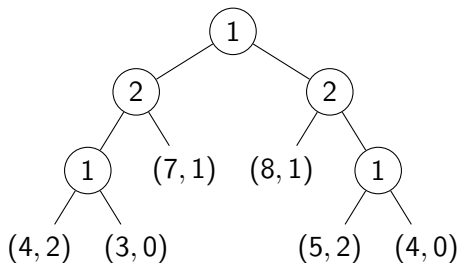
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## Finite sequential game or game played on tree

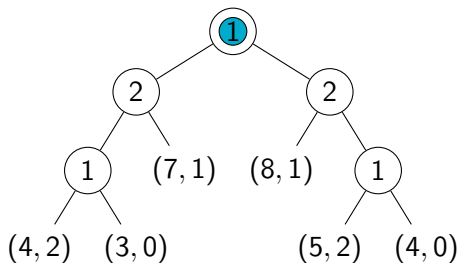


- $n$  players
- Finite play
- Quantitatif
- Selfish players who want maximise their payoff





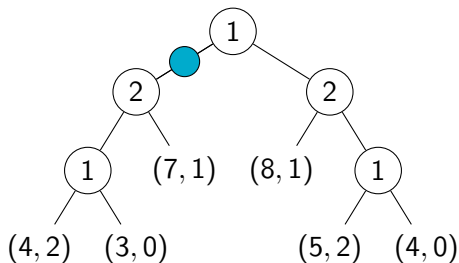
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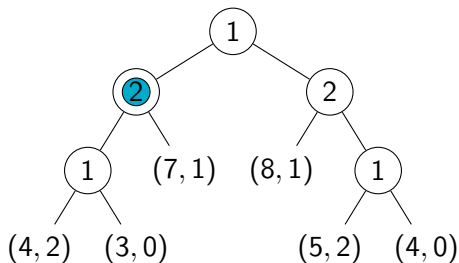
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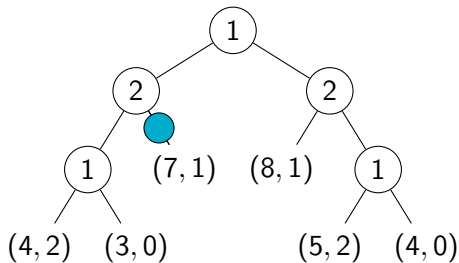
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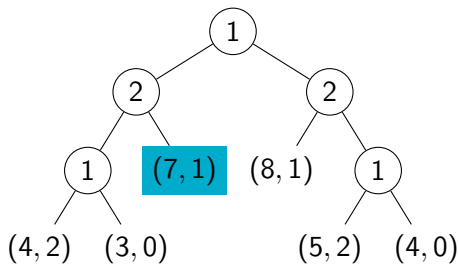
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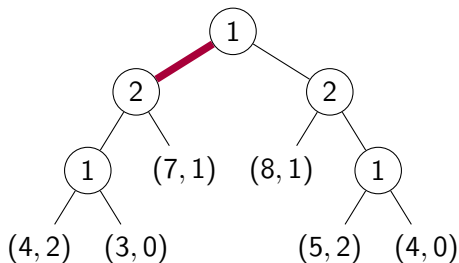
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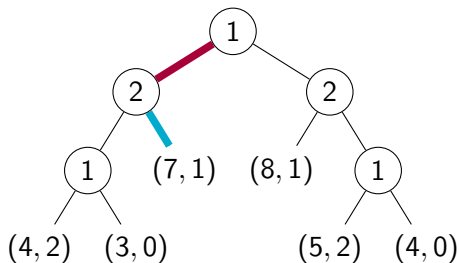
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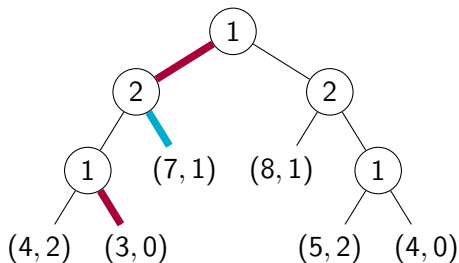
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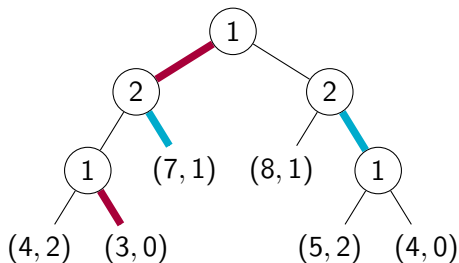


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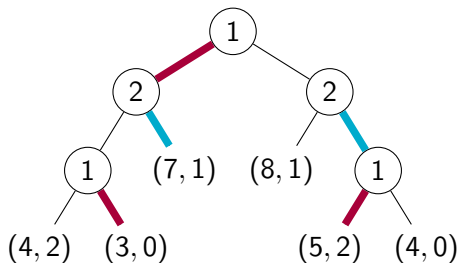
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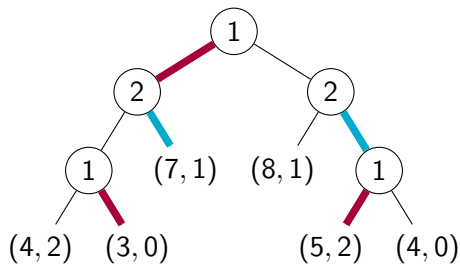
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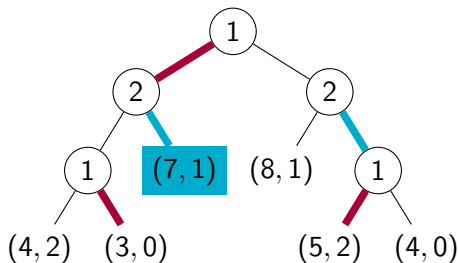


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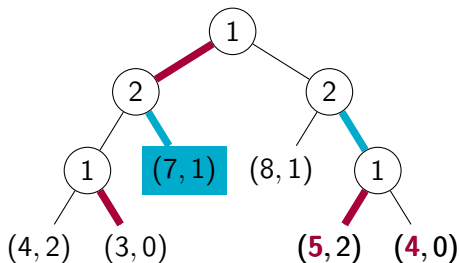
→ We will study *stable* strategy profiles

## Nash Equilibrium (NE)



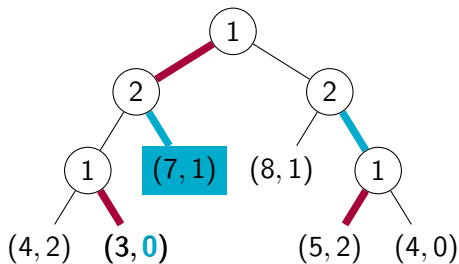
A strategy profile is a **Nash Equilibrium** (NE) if none of the players has a profitable deviation as long as the other players don't change their strategy.

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# Other notions of Equilibrium

## Subgame Perfect Equilibrium

A strategy profile is a **Subgame Perfect Equilibrium** (SPE) if this is a Nash Equilibrium in every subgame.

## Strong Nash Equilibrium

A strategy profile is a **Strong Nash Equilibrium** (SNE) if no coalition of players has a profitable deviation.



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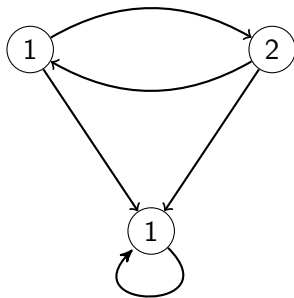
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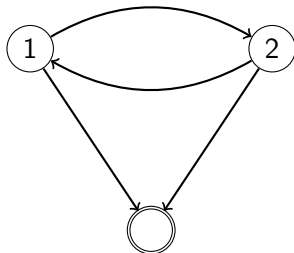


# Particular Reachability Game



■ **Not initialized**

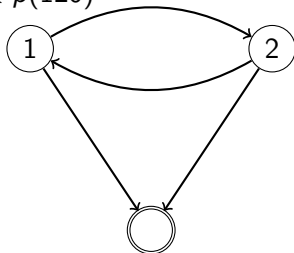
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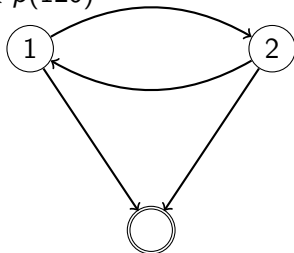
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# Particular Reachability Game

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- **Not** initialized
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- Players have preferences over the **paths**
  - Ex: shortest path

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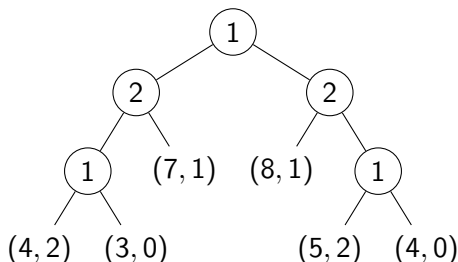
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## Static approach

- The game is played only once.
- Before playing, players decide which strategy they will play.
- If they decide to play a Nash Equilibrium, none of the player has interest to change his strategy.

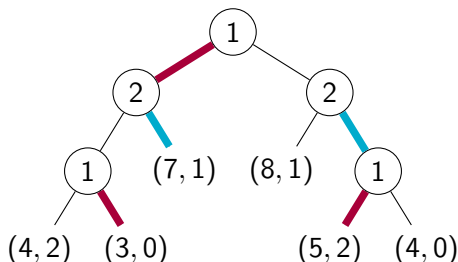
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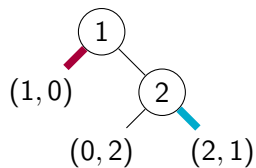
- Learning in games (e.g. fictitious play);
- Strategy improvement (e.g. in parity games);
- Evolutionary game theory (continuous time).

## Dynamics

Allowing players to reconsider their strategy and update it into a better one, observing some conditions.

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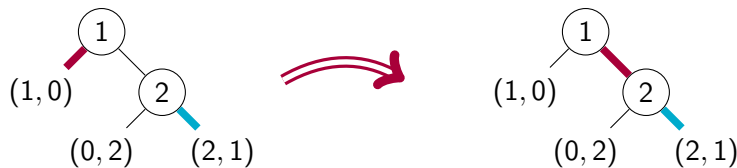
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## Questions

- What does *better one* means? What are the conditions over the updates?
- Does the dynamics always terminates?
- If so, what are the terminal profiles?

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# Sequential games

<i>I</i>	<i>SI</i>	<i>A</i>	<i>L</i>	<i>1P</i>	Games	Termination	Final Profiles
×	×	•	•	•		×	not appl.
•	✓	•	•	•	acyclic prefs	✓	$\supseteq$ SPEs
✓	•	✓	•	•	acyclic prefs	✓	$\supseteq$ NEs
✓	×	×	×	•		×	not appl.
					swo prefs	prefs can be layered (s.)	
✓	×	×	✓	×	swo prefs	prefs out of pattern (n.)	
					slo prefs	prefs out of pattern (n. & s.)	= SNEs
					swo prefs, 2 player	prefs out of pattern (n. & s.)	
✓	×	×	✓	✓	acyclic prefs	✓	= NEs

- *I* = Improvement
- *SI* = Subgame Improvement
- *A* = Atomicity
- *L* = Lazyness (along the play induced by the updated strategy)
- *1P* = One player

# Reachability games

	Lazy				Subgame			
	two players		n players		two players		n players	
	Termination	NE	Termination	NE	Termination	SPE	Termination	SPE
Qualitative reachability	✓	✓	???	???	×	✓	×	???
Quantitative reachability	???	???	×	???	×	×	×	×
Mean payoff	???	???	×	×	×	×	×	×

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- Sequential games
- Reachability games

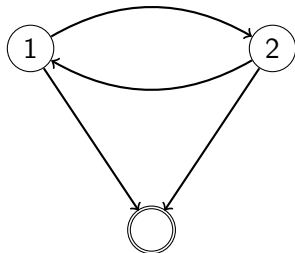
## 4 Dynamics

## 5 Results

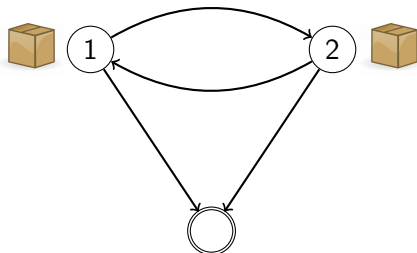
- Sequential games
- Reachability games

## 6 Computer Networking

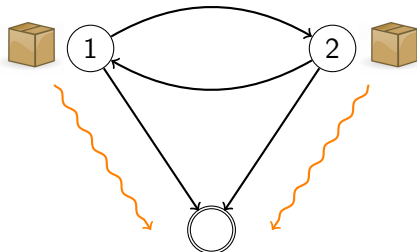
# Computer Networking



# Computer Networking

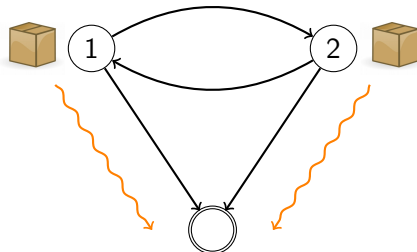


# Computer Networking



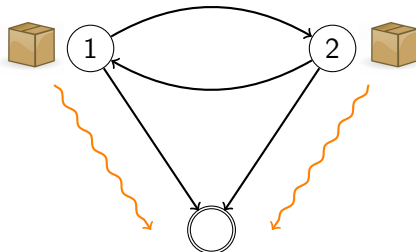


# Computer Networking



- Send a message to a specific destination = A unique target

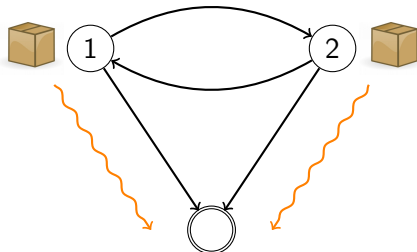
# Computer Networking



- Send a message to a specific destination = A unique target
- Only care about path starting at this node = Not initialized

# Computer Networking

$$p(10) < p(120) \quad p(20) < p(210)$$

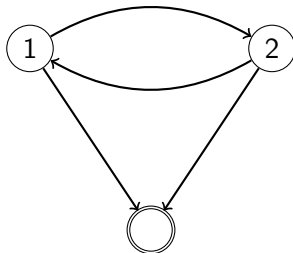


- Send a message to a specific destination = A unique target
- Only care about path starting at this node = Not initialized
- Has preference over the paths

# BGP $\equiv$ Concurrent dynamics

$$p(10) < p(120)$$

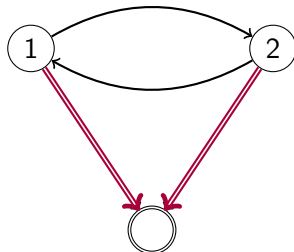
$$p(20) < p(210)$$



# BGP $\equiv$ Concurrent dynamics

$$p(10) < p(120)$$

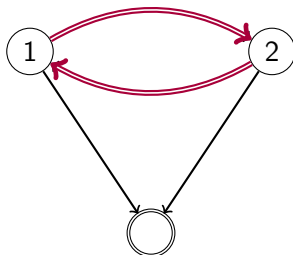
$$p(20) < p(210)$$



# BGP $\equiv$ Concurrent dynamics

$$p(10) < p(120)$$

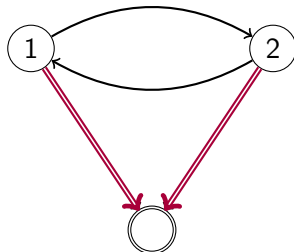
$$p(20) < p(210)$$



# BGP $\equiv$ Concurrent dynamics

$$p(10) < p(120)$$

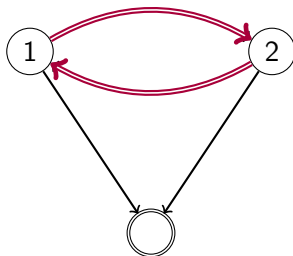
$$p(20) < p(210)$$



# BGP $\equiv$ Concurrent dynamics

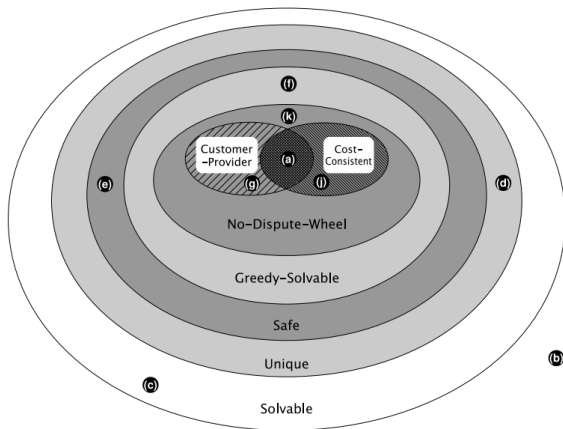
$$p(10) < p(120)$$

$$p(20) < p(210)$$





# Results



On the Stability of Interdomain Routing LUCA CITTADINI, GIUSEPPE DI BATTISTA, and MASSIMO RIMONDINI