

The Zimin word and words with Lyndon orbit

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This abstract is aimed to the organizers of the CIRM conference Spring School in Mathematical Computer Science, in Marseille, from the 4th to 8th March 2019, in order to present some results in the topic of combinatorics of words, obtained as a contribution in the Ph.D. thesis of the author, supervised by Boris Adamczewski and Luca Zamboni (Univ. Lyon 1).

Keywords : Zimin word, Lyndon words, 2-adic numbers, dynamical orbit of an infinite word, combinatorics on words.

Abstract :

We study the Zimin word, an infinite word defined over an infinite alphabet, through its factors and its dynamical orbit. We also introduce the notion of words with a Lyndon orbit, for which we prove some properties, and show that the Zimin word provides a non-trivial example of such a word.

Our presentation will be divided in three sections, the three of them presenting some results and perspective concerning an extensive study of the Zimin word, and the introduction of the class of words with a Lyndon orbit.

The first part of the presentation concerns the properties of the set of factors of the Zimin word. The Zimin word is defined as the limit $\lim Z_n$ where the finite word Z_n satisfy the recurrence $Z_1 = x_1$ and $Z_{n+1} = Z_n x_{n+1} Z_n$, and is defined over the infinite alphabet $\mathcal{A}_X = \{x_i, i \geq 1\}$. The Zimin word has been the subject of a number of studies, including, in some of which its algorithmic dimension comes into play. We will give a full description of the set of factors of the Zimin word, and explain how this characterisations may be used to prove, for example, that if uv and vw are two factors of the Zimin word, with u , v and w sufficiently large, then uvw is a factor of the Zimin word.

In a second part we will discuss the dynamical orbit of the Zimin word, defined as the set of infinite words sharing the same factors as the Zimin word. We will give a bijective description of its element, with the use of 2-adic numbers, and give a combinatorial interpretation of this result a a notion of duality between prefixes and suffixes in the case of the Zimin word. We may end this section by using these properties to run a similar study of the Doubling-period word, closely related to the Zimin word. This kind of study aims to understand the inherent differences between suffixes of a word and arbitrary elements of its dynamical orbit.

In a third part, we introduce the new notion of word with a Lyndon orbit, a more restrictive variant of the concept of Lyndon words, the latter being defined as words such that there exists a total order on the alphabet such that the considered word is lexicographically strictly smaller than all of its suffixes. We define a word with a Lyndon orbit if all its suffixes are Lyndon, and prove some properties about this idea, for example that such a word must be defined over an infinite alphabet. We end by showing that the Zimin word provides a non-trivial example of such a word.

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