Nonarchimedean Convex Programming and Its Relation to Mean-Payoff Games

Stéphane Gaubert
INRIA and CMAP, École polytechnique
Stephane.Gaubert@inria.fr

International Conference Dynamic, Games and Science
CIRM, June 2019

Linear programming, and more generally convex semialgebraic programming, makes sense over any ordered nonarchimedean field, like a field of real Puiseux series. Nonarchimedean instances encode classical parametric instances of convex programs with a suitably large parameter. Tropical geometry allows one to study such instances by combinatorial means. In particular, it reveals that, under a genericity condition, solving a nonarchimedean feasibility problem is equivalent to deciding who the winner is in a mean payoff game. Indeed, nonarchimedean linear programs correspond to deterministic mean payoff games, whereas nonarchimedean semidefinite programs correspond to perfect information stochastic mean payoff games. In this way, one can apply methods from convex programming to mean payoff games, and vice versa. This approach has led to several results: a counter example, with a family of linear programs, with large coefficients, for which log-barrier interior point methods have a non strongly polynomial behavior (they make a number of iterations exponential in the number of constraints and variables); a theorem transferring complexity results concerning pivoting methods in linear programming to mean payoff games; a notion of nonarchimedean condition number, governing the complexity of value iteration. We will present here the main ideas and tools from tropical geometry developed in this approach, emphasizing their relations with the "operator approach" of zero-sum games.

This is based on [1, 3, 2, 5, 6, 4].

References

[1] M. Akian, S. Gaubert, and A. Guterman. "Tropical polyhedra are equivalent to mean payoff games". In: *International Journal of Algebra and Computation* 22.1 (2012), p. 125001.

[2] X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. "Log-barrier interior point methods are not strongly polynomial". In: *SIAM Journal on Applied Algebra and Geometry* 2.1 (2018), pp. 140–178.

[3] X. Allamigeon, P. Benchimol, S. Gaubert, and M. Joswig. "Tropicalizing the Simplex Algorithm". In: *SIAM J. Disc. Math.* 29.2 (2015), pp. 751–795.

[4] X. Allamigeon, S. Gaubert, R. Katz, and M. Skomra. "Condition numbers of stochastic mean payoff games and what they say about nonarchimedean semidefinite programming". In: *Proceedings of the 23rd International Symposium on Mathematical Theory of Networks and Systems (MTNS2018)*. 2018. eprint: `arXiv:1802.07712`.

[5] X. Allamigeon, S. Gaubert, and M. Skomra. "Solving Generic Nonarchimedean Semidefinite Programs Using Stochastic Game Algorithms". In: *Journal of Symbolic Computation* 85 (2018), pp. 25–54.

[6] X. Allamigeon, S. Gaubert, and M. Skomra. "The tropical analogue of the Helton–Nie conjecture is true". In: *Journal of Symbolic Computation* 91 (2019). MEGA 2017, Effective Methods in Algebraic Geometry, Nice (France), June 12-16, 2017., pp. 129–148. ISSN: 0747-7171.