Minimality and stable ergodicity

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1 introduction

1 mechanisms that activate ergodicity

pugh - shub (1995)

a little hyperbolicity goes a long way toward guaranteeing stable ergodicity

1 conjecture

conjecture

generically in $\mathrm{Diff}^1_m(M)$

$$h_m(f) > 0$$

$$\Downarrow$$

stable ergodicity

1 original plan

conjecture

generically in $\mathrm{Diff}^1_m(M^3)$

$$h_m(f) > 0$$

$$\Downarrow$$

stable ergodicity

1 theorem 1

theorem 1 (G. Nuñez, JRH)

generically in $\mathrm{Diff}^1_m(M^3)$

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$\exists$ invariant expanding minimal foliation

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$\Downarrow$

stable Bernoulliness

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conjecture

generically in $\mathrm{Diff}^1_m(M^3)$

$$h_m(f) > 0$$

$$\Downarrow$$

$\exists$ a hyperbolic minimal invariant foliation

1 more modest conjecture

more modest conjecture

generically in $\mathrm{Diff}^1_m(M)$

$\exists$ invariant expanding minimal foliation
+ “periodic point”

$\Downarrow$

stable Bernoulliness

1 theorem 2

theorem 2 (G. Nuñez, JRH)

for a generic map $f \in \mathrm{Diff}^1_m(M)$

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- $\exists p \in \mathrm{Per}_H(f)$ with $u(p) = \dim W$

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- $\exists$ ergodic component $\mathrm{Phc}_g(q_g) \subset \mathrm{Nuh}(g)$
- $\overline{\mathrm{Phc}_g(q_g)}^{ess} = M$

① theorem 2

corollary (G. Nuñez, JRH)

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$\Rightarrow$ W is stably minimal
$\Rightarrow$ f stably topologically mixing in $\mathrm{Diff}^1_m(M)$

1 open question

open question

is the strongest foliation of an Anosov diffeomorphism minimal?

② elements

2 non-uniformly hyperbolic region

non-uniformly hyperbolic region

- $f \in \mathrm{Diff}^1_m(M)$

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- $f \in \mathrm{Diff}^1_m(M)$
-

$$\mathrm{Nuh}(f) = \left\{ x : \limsup_{n\to\infty} \frac{1}{n} \log \|Df^n(x)v\| \neq 0 \quad \forall v \in T_x M \setminus \{\vec{0}\} \right\}$$

2 Pesin stable manifold

Pesin stable manifold

$$W^{-}(x) = \left\{ y \in M : \limsup_{n \to \infty} \frac{1}{n} \log d(f^n(x), f^n(y)) < 0 \right\}$$

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$$W^{+}(x) = \left\{ y \in M : \limsup_{n\to\infty} \frac{1}{n} \log d(f^{-n}(x), f^{-n}(y)) < 0 \right\}$$

2 Pesin homoclinic class

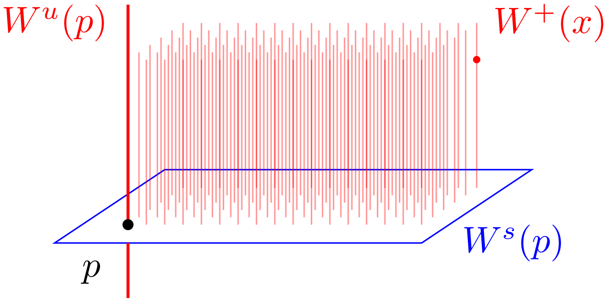
Pesin homoclinic class (HHTU11)

$p \in \mathrm{Per}_H(f)$

$$\mathrm{Phc}^+(p) = \{x : W^+(x) \pitchfork W^s(o(p)) \neq \emptyset\}$$

2 Pesin homoclinic class

$\mathrm{Phc}^+(p)$



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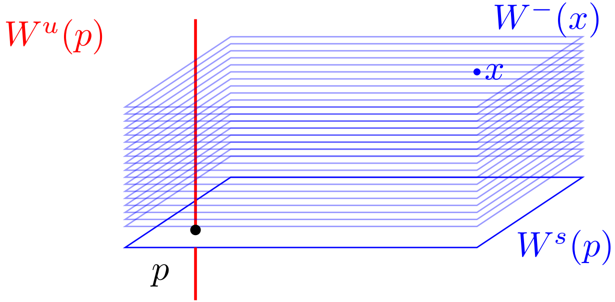
$\mathrm{Phc}^{-}(p)$



2 criterion of ergodicity

criterion of ergodicity (HHTU11)

- $f \in \mathrm{Diff}^2_m(M)$

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- $m(\mathrm{Phc}^-(p)) > 0$ and $m(\mathrm{Phc}^+(p)) > 0$

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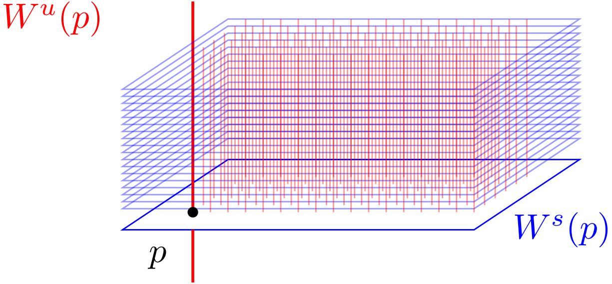
3. $\mathrm{Phc}(p) \subset \mathrm{Nuh}(f)$

2 Pesin homoclinic class

Phc(p)



2 small but useful remark 1

remark

- $p \in \mathrm{Per}(f)$ such that $W^u(f^k(p)) \pitchfork W^s(f^l(p)) \neq \emptyset\ \forall k, l$

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$$\mathrm{Phc}^+(p) = \{x : W^+(x) \pitchfork W^s(p) \neq \emptyset\}$$

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$$\mathrm{Phc}^+(p) = \{x : W^+(x) \pitchfork W^s(p) \neq \emptyset\}$$
- $\Rightarrow f|_{\mathrm{Phc}(p)}$ Bernoulli

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remark 2

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- $p \in \mathrm{Per}(f)$ such that $m(\mathrm{Phc}(p)) > 0$
- $f|_{\mathrm{Phc}(p)}$ Bernoulli
- $\Rightarrow W^u(p)$ dense in $\mathrm{Phc}(p)$

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generic property (ACW16)

- for a generic $f \in \mathrm{Diff}^1_m(M)$

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  2. $\exists q : \mathrm{Phc}(q) \overset{\circ}{=} \mathrm{Nuh}(f) \overset{\circ}{=} M$

2 generic property

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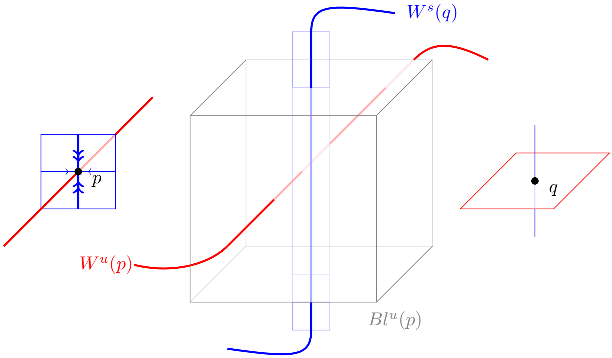
1. f ergodic,
2. $\exists q : \mathrm{Phc}(q) \overset{\circ}{=} \mathrm{Nuh}(f) \overset{\circ}{=} M$
3. the Oseledets splitting is globally dominated

2 blenders

blenders (BD)



2 blenders

blenders

- $p, q \in \mathrm{Per}_H(f)$

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- $T_pM = E^u \oplus (E^c_1 \oplus E^s)$

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- $u(q) = u(p) + 1$

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blenders

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$Bl^u(p)$ u-blender associated to p activated by q if

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- property is C^1-robust

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- every well-placed $(s+1)$-strip in $Bl^u(p) \pitchfork W^u(p)$
- property is C^1-robust
- $W^s(q)$ contains a well-placed s-strip in $Bl^u(p)$

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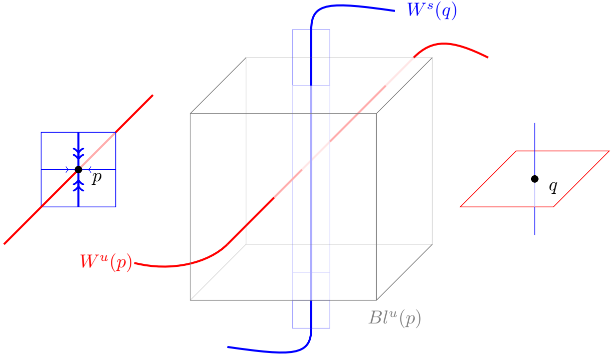
blenders (BD)



2 blenders

creation of blenders (HHTU)

generically in $\mathrm{Diff}^1_m(M)$

- $p, q \in \mathrm{Per}(f)$

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generically in $\mathrm{Diff}^1_m(M)$

- $p, q \in \mathrm{Per}(f)$
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$\exists Bl^s(p)$ *cs*-blender associated to p activated by q

2 blenders

property of blenders

- if ∃ blender associated to p activated by q

2 blenders

property of blenders

- if ∃ blender associated to p activated by q
- $u(q) = u(p) + 1$

② blenders

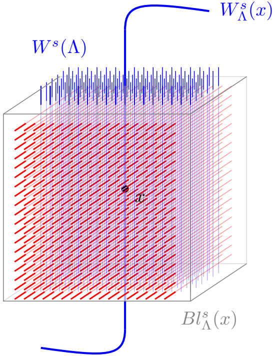
property of blenders

- if $\exists$ blender associated to p activated by q
- $u(q) = u(p) + 1$
- $\Rightarrow$

$$W(q) \subset \overline{W^u(p)}$$

2 superblenders

superblenders (Moreira-Silva 2012)



2 superbblenders

superblender

- Λ hyperbolic set

superblender

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- $T_\Lambda M = E_1^u \oplus \cdots \oplus E_k^u \oplus E^s$

2 superbleders

superblender

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2 superbenders

superblender

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$Bl_\Lambda^s(x)$ *s*-stable superblender associated to Λ if

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superblender

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$Bl_\Lambda^s(x)$ *s-stable superblender* associated to Λ if

- every well-placed u-strip in $Bl_\Lambda^s(x) \pitchfork_q W^s(y)$ for some $y \in \Lambda$, $\forall u = 1, \ldots, n-s$

2 superblenders

superblender

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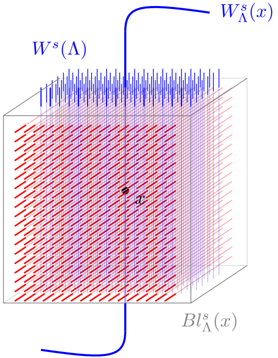

2 superblenders

creation of superblenders (ACW17)

generically in $\mathrm{Diff}^1_m(M)$ with $h_m(f) > 0$

- $\exists$ an s-superblender associated to Λ

2 superbleders

creation of superblenders (ACW17)

generically in $\mathrm{Diff}^1_m(M)$ with $h_m(f) > 0$

- $\exists$ an s-superblender associated to Λ
- where $s = u(q)$ such that $\mathrm{Phc}(q) \overset{\circ}{=} M$

3 proof

3 minimality criterion

minimality criterion

- W invariant expanding foliation

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3 minimality criterion

minimality criterion

- W invariant expanding foliation
- $\mathrm{Phc}^W(p) = M$
- $p \in \mathrm{Per}(f)$ hyperbolic with $u(p) = \dim W$
- $\overline{W^u(p)} = M$

3 minimality criterion

minimality criterion

- W invariant expanding foliation
- $\mathrm{Phc}^W(p) = M$
- $p \in \mathrm{Per}(f)$ hyperbolic with $u(p) = \dim W$
- $\overline{W^u(p)} = M$

$\Rightarrow$ W is minimal

3 SBM criterion

SBM criterion

W minimal invariant expanding foliation such that

1. $TM = F \oplus_< TW$ for some invariant bundle F

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SBM criterion

W minimal invariant expanding foliation such that

1. $TM = F \oplus_{<} TW$ for some invariant bundle F
2. $\mathrm{Phc}^W(p) = M$ with $u(p) = \dim W$
3. $\exists \mathcal{U}$: $m(\mathrm{Phc}^-(p_g)) > 0$ for all $g \in \mathcal{U} \cap \mathrm{Diff}^2_m(M)$

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W minimal invariant expanding foliation such that

1. $TM = F \oplus_< TW$ for some invariant bundle F
2. $\mathrm{Phc}^W(p) = M$ with $u(p) = \dim W$
3. $\exists \mathcal{U}$: $m(\mathrm{Phc}^-(p_g)) > 0$ for all $g \in \mathcal{U} \cap \mathrm{Diff}^2_m(M)$

$\Rightarrow$ f is stably Bernoulli and W is stably minimal

3 proof of SBM criterion

proof of SBM criterion

- $\Gamma(g) = M \setminus \mathrm{Phc}^W(p_g)$ compact u-saturated invariant

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proof of SBM criterion

- $\Gamma(g) = M \setminus \mathrm{Phc}^W(p_g)$ compact u-saturated invariant
- $g \mapsto \Gamma(g)$ varies u.s.c.
- then $\mathrm{Phc}^W(p) = M \Rightarrow \mathrm{Phc}^W(p_g) = M$ in some $\mathcal{U}$

③ proof of SBM criterion

proof of SBM criterion

- ② + ③ +[HHTU11] $\Rightarrow \mathrm{Phc}(p_g) \overset{\circ}{=} M \overset{\circ}{=} \mathrm{Nuh}(g)$

(3) proof of SBM criterion

proof of SBM criterion

- (2) + (3) +[HHTU11] $\Rightarrow \mathrm{Phc}(p_g) \overset{\circ}{=} M \overset{\circ}{=} \mathrm{Nuh}(g)$
- small remark 1 $\Rightarrow$ g Bernoulli

③ proof of SBM criterion

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- ② + ③ +[HHTU11] $\Rightarrow \mathrm{Phc}(p_g) \overset{\circ}{=} M \overset{\circ}{=} \mathrm{Nuh}(g)$
- small remark 1 $\Rightarrow g$ Bernoulli
- small remark 2 $\Rightarrow W^u(p_g)$ is dense in $\mathrm{Phc}^W(p_g)$

③ proof of SBM criterion

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- small remark 2 $\Rightarrow$ $W^u(p_g)$ is dense in $\mathrm{Phc}^W(p_g)$
- (minimality criterion) $\Rightarrow$ W_g minimal

③ proof of SBM criterion

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- ② + ③ +[HHTU11] $\Rightarrow \mathrm{Phc}(p_g) \overset{\circ}{=} M \overset{\circ}{=} \mathrm{Nuh}(g)$
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- small remark 2 $\Rightarrow$ $W^u(p_g)$ is dense in $\mathrm{Phc}^W(p_g)$
- (minimality criterion) $\Rightarrow$ W_g minimal $\square$

3 proof of theorem 1

proof of T1

cases, according to finest Oseledets splitting

- $TM = E_2^+ \oplus E_1^-$

3 proof of theorem 1

proof of T1

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3 proof of theorem 1

proof of T1

cases, according to finest Oseledets splitting

- $TM = E_2^+ \oplus E_1^-$ $\qquad \rightarrow TW = E_2^+$
- $TM = E_1^+ \oplus E_1^+ \oplus E_1^-$
- $TM = E_1^+ \oplus E_1^- \oplus E_1^-$
- $TM = E_1^+ \oplus E_2^-$

3 proof of theorem 1

proof of T1

cases, according to finest Oseledets splitting

- $TM = E_2^+ \oplus E_1^-$ $\to TW = E_2^+$
- $TM = E_1^+ \oplus E_1^+ \oplus E_1^-$ $\to$ PH
- $TM = E_1^+ \oplus E_1^- \oplus E_1^-$ $\to$ PH
- $TM = E_1^+ \oplus E_2^-$

3 proof of theorem 1

proof of T1

cases, according to finest Oseledets splitting

- $TM = E_2^+ \oplus E_1^-$ $\rightarrow TW = E_2^+$
- $TM = E_1^+ \oplus E_1^+ \oplus E_1^-$ $\rightarrow$ PH
- $TM = E_1^+ \oplus E_1^- \oplus E_1^-$ $\rightarrow$ PH
- $TM = E_1^+ \oplus E_2^-$ $\rightarrow$ SBM + [AB]

3 proof of theorem 1

proof of T1

cases, according to finest Oseledets splitting

- $TM = E_2^+ \oplus E_1^-$ $\rightarrow TW = E_2^+$
- $TM = E_1^+ \oplus E_1^+ \oplus E_1^-$ $\rightarrow$ PH
- $TM = E_1^+ \oplus E_1^- \oplus E_1^-$ $\rightarrow$ PH
- $TM = E_1^+ \oplus E_2^-$ $\rightarrow$ SBM + [AB]

□

3 proof of theorem 2

proof of T2

generically in our conditions

- $\exists\, q \in \mathrm{Per}_H(f)$ such that $\mathrm{Phc}(q) \overset{\circ}{=} M$

3 proof of theorem 2

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3 proof of theorem 2

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- $\exists\ s$-stable superblender $Bl^s_\Lambda(x)$ with $s = s(q)$
- all periodic points of the same index are homoclinically related [BC]

3 proof of theorem 2

proof of T2

exists a C^1-neighborhood $\mathcal{U}$ of f such that all g in $\mathcal{U}$ satisfy

- q_g homoclinically related to $r_g \in \Lambda_g$

3 proof of theorem 2

proof of T2

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3 proof of theorem 2

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- superblender property holds

3 proof of theorem 2

proof of T2

- $\mathrm{Phc}_g(p_g)$ invariant + positive measure [AB]

3 proof of theorem 2

proof of T2

- $\mathrm{Phc}_g(p_g)$ invariant + positive measure [AB]
- $\Rightarrow \overline{\mathrm{Phc}_g(p_g)}^{ess}$ essentially s- and u-saturated [HHTU]

3 proof of theorem 2

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- $r_g \sim q_g \Rightarrow \quad r_g \in \mathrm{Phc}_g(q_g) \quad \forall r \in \mathrm{Per}(g) \cap \Lambda_g$

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- all W_g-leaves intersect $W^s(\Lambda_g)$ (superblender)
- $\Rightarrow \overline{\mathrm{Phc}_g(q_g)}^{ess} = M$ (u-saturation)

3 proof of theorem 2

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- $u(q_{i+1}) = u(q_i) + 1$ for all $i = 0, \ldots, \ell - 1$ [BDPR, LSY]

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- $\to$ cascade of blenders associated to q_i activated by q_{i+1}

3 proof of theorem 2

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- $\Rightarrow \overline{W^u(q_g)} = \overline{\mathrm{Phc}_g(q_g)}^{ess} = M$

3 proof of theorem 2

proof of T2

- $\Rightarrow \overline{W^u(q_g)} = \overline{\mathrm{Phc}_g(q_g)}^{ess} = M$
- cascade of blenders

3 proof of theorem 2

proof of T2

- $\Rightarrow \overline{W^u(q_g)} = \overline{\mathrm{Phc}_g(q_g)}^{ess} = M$
- cascade of blenders
- $\Rightarrow C^1$-stably

$M = \overline{W^u(q)}$

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3 proof of theorem 2

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- $\Rightarrow W$ is stably minimal (minimality criterion)

3 proof of theorem 2

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- $\Rightarrow$ W is stably minimal (minimality criterion) $\square$

thank you!

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谢谢！