TRANSFER OPERATORS FOR SINAI BILLIARD MAPS

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1. Introduction

I will not have the possibility to provide any notes. Here are complements to the videos:

The main references are listed below, including [DZ1] and [BD], as well as the book [CM].

In lecture III, on May 17, I forgot to write that $h_{\mu_*} = h_*$ in Theorem 1.

The main idea to show that the entropy of μ_* is greater than or equal to h_* is the Brin–Katok local entropy theorem [BK], using [DWY] to forego continuity. (In the paper [BD], we prove ergodicity first but this is not needed.) In order to apply Brin–Katok, one has to prove an upper bound on the measure of Bowen balls. This is the easy "Gibbs bound." (There are confusing typos in v1 of arXiv:1807.02330: On page 45, the sentence before Corollary 7.17 should be "Now that we know that μ_* is ergodic, Proposition 7.12 will easily imply that $h_{\mu_*}(T) = h_*$." In footnote 33, same page, one should replace (7.25) by (7.24).)

We also show a (hard) "lower Gibbs bound" in [BD], using the consequence of the key lemma on the measure of neighbourhoods of singularity curves and Borel–Cantelli. Our lower bound allows us to find a necessary condition for $\mu_{SRB} = \mu_*$, but we are not able to get the exact e^{-nh_*} rate which would allow us to show uniqueness of the measure by [Bo2].

The key results we use (in §5.3 and §7.3 of [BD]) about Markov rectangles are Lemmas 7.87 and 7.90 in [CM]. Another ingredient to show absolute continuity of μ_* in §7.3 of [BD] is the control of smoothness of the Jacobian of holonomy maps obtained in Lemmas 6.6 and 6.8 of [BDL].

I did not have the time to define the norms of [BD] in the last lecture. The difference between these norms and the ones in [DZ1] presented in lecture II are:

(1) the weight $\cos W$ is removed everywhere (nonessential, due to the use of dx instead of $d\mu_{SRB}$ as reference);
(2) $|W|^{1/p}$ is replaced by $|\log |W||^{-\gamma}$, for $\gamma > 1$ so that $2^{s_0\gamma} < e^{h_*}$, in the strong stable norm (due to the new growth lemma);
(3) $\epsilon^{-\zeta}$ is replaced by $|\log \epsilon|^{\zeta}$ in the strong unstable norm (due to the new strong stable norm).

Finally, there is a "leafwise" interpretation of the distributions in our Banach spaces, given in §7.2 of [BD].

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Complements to the videos of the minicourse given in CRM, Luminy, on May 16 and 17, 2019.

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