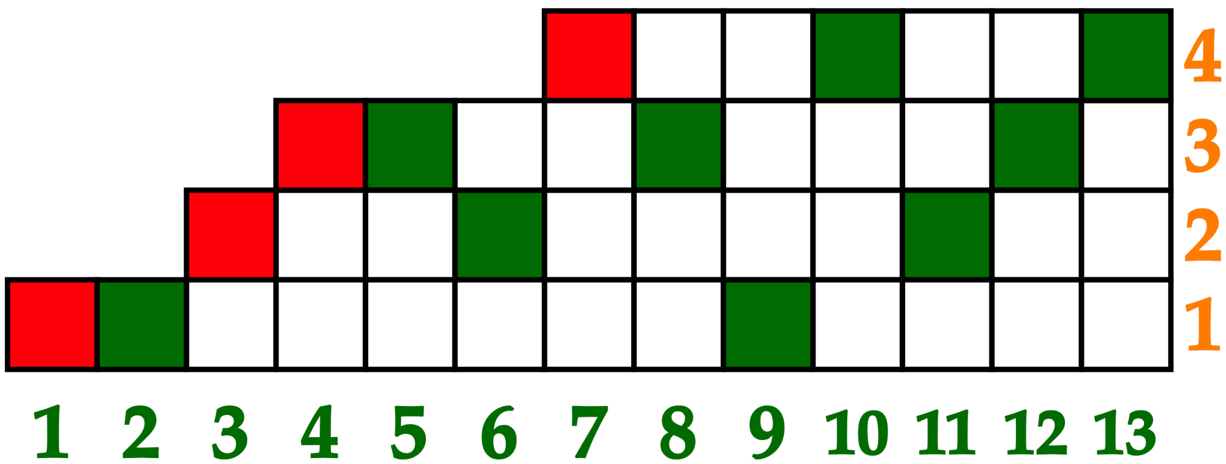
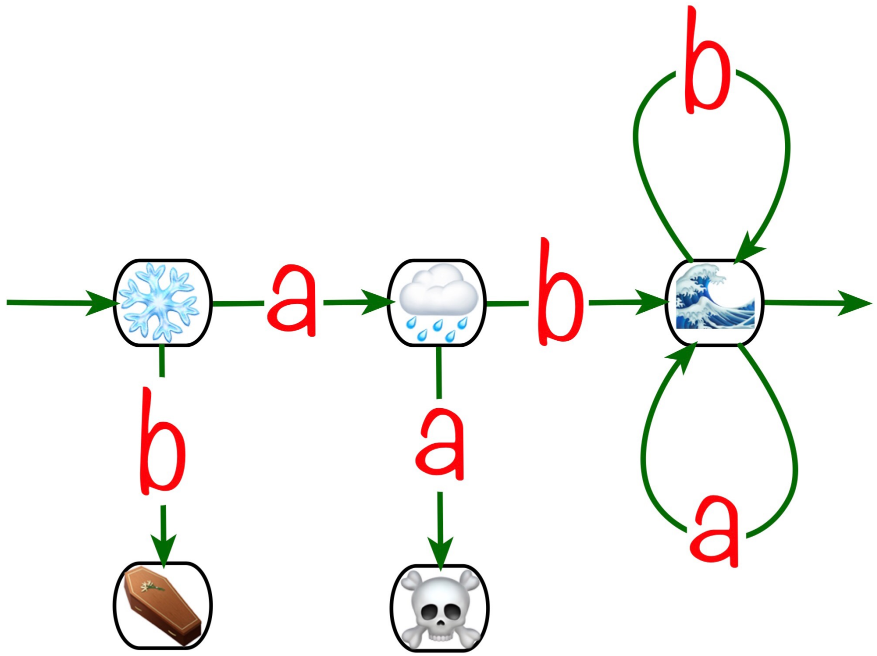
The impatient collector

Anis Amri, Philippe Chassaing





AOFA2019, 2019/06/24-28, CIRM Luminy

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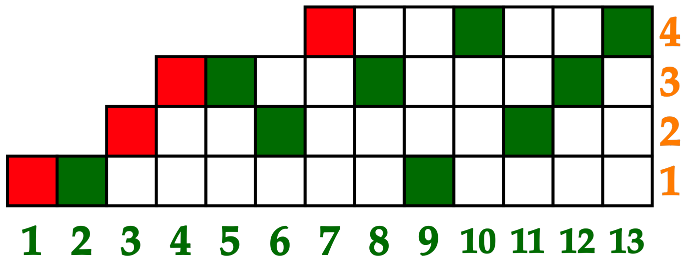

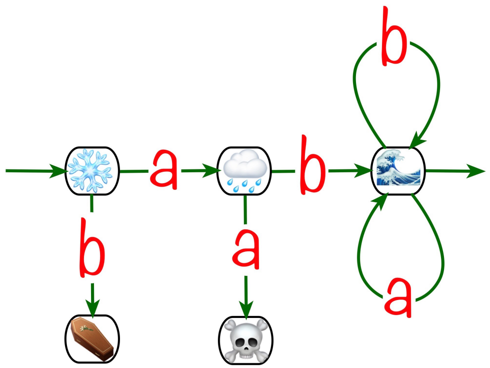

following papers by :

-Cyril Nicaud, Frédérique Bassino, Julien Clément, Andrea Sportiello, et al.

-Devroye and Cai

-Addario-Berry et al.

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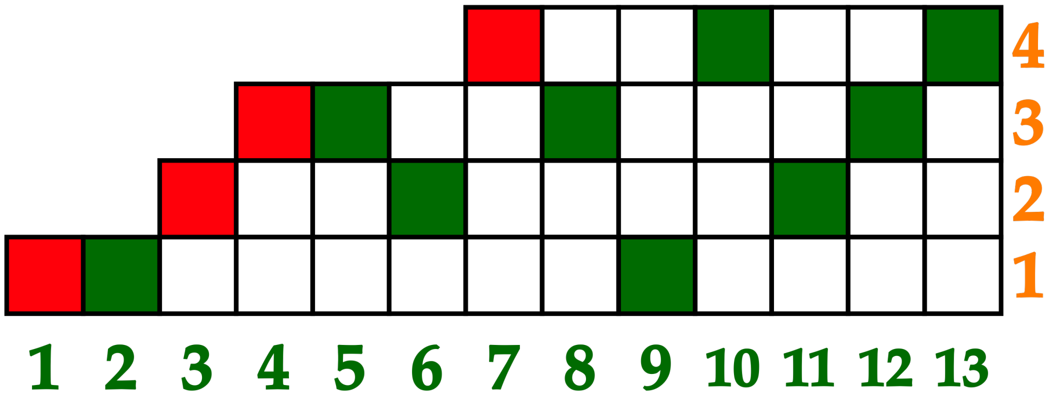

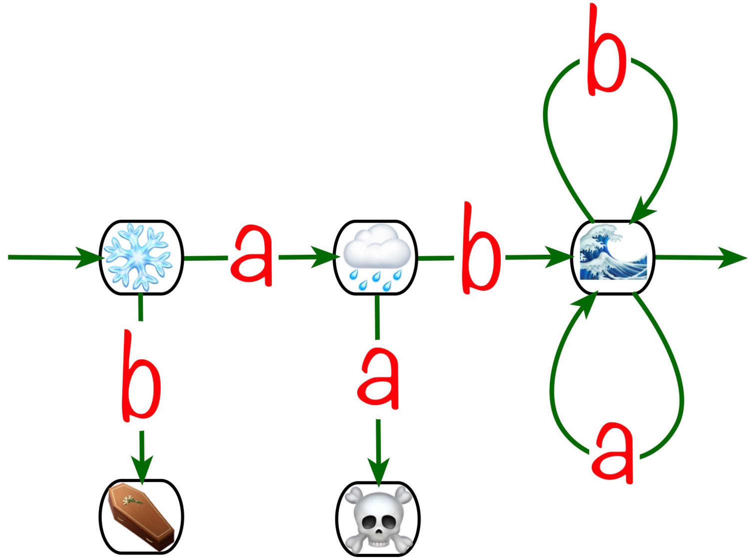

Byproduct 1: study of a variation of the coupon collector problem.

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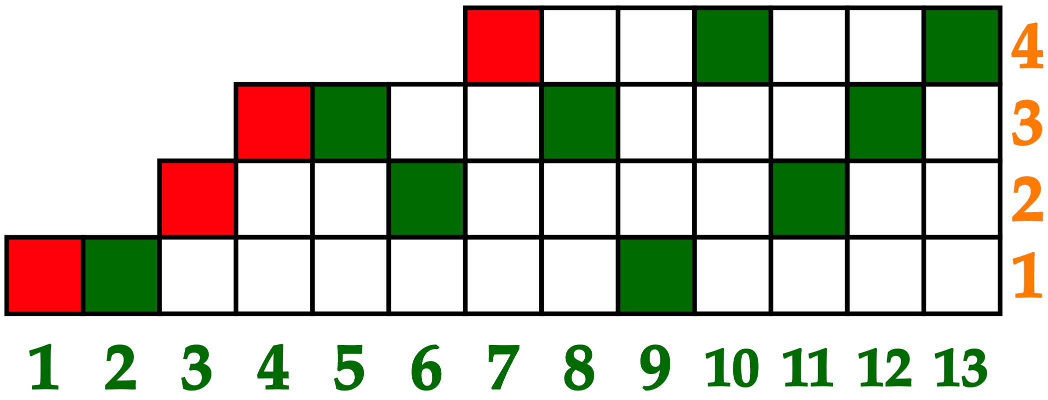

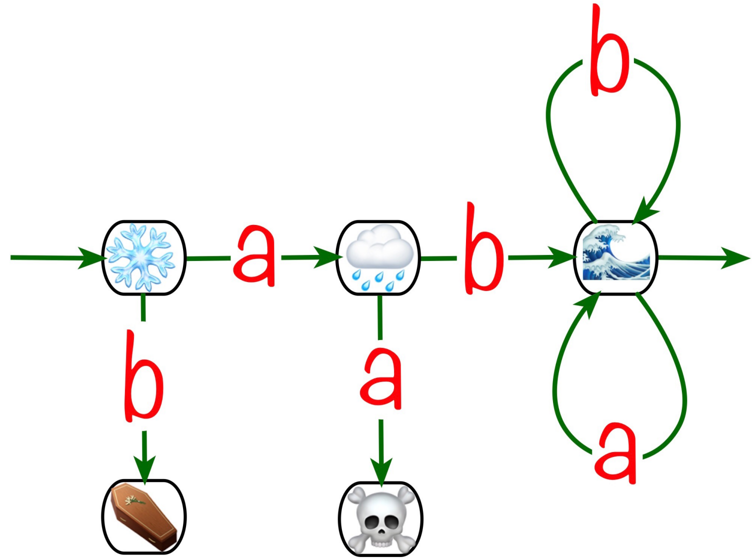

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Byproduct 2: refined asymptotic analysis of the Stirling numbers of the 2^{nd} kind.

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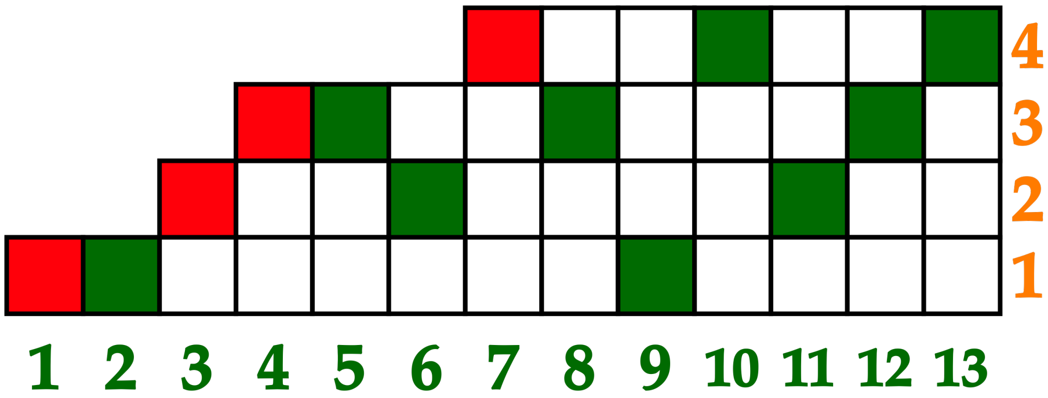
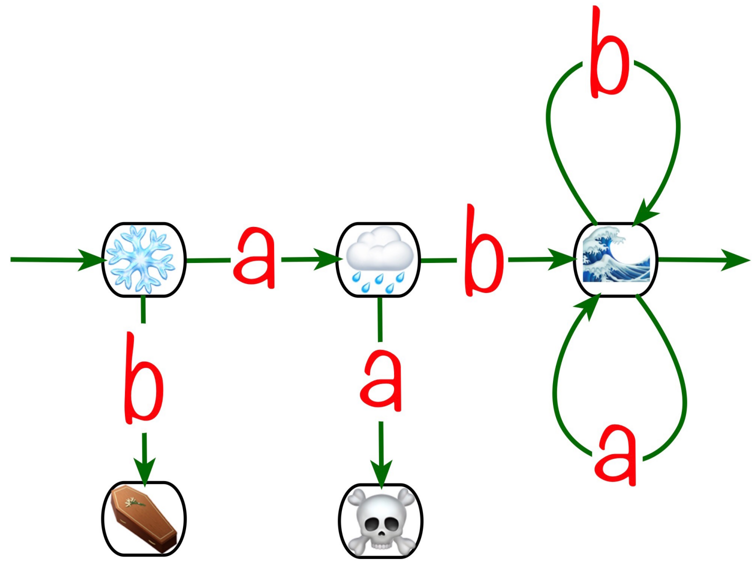


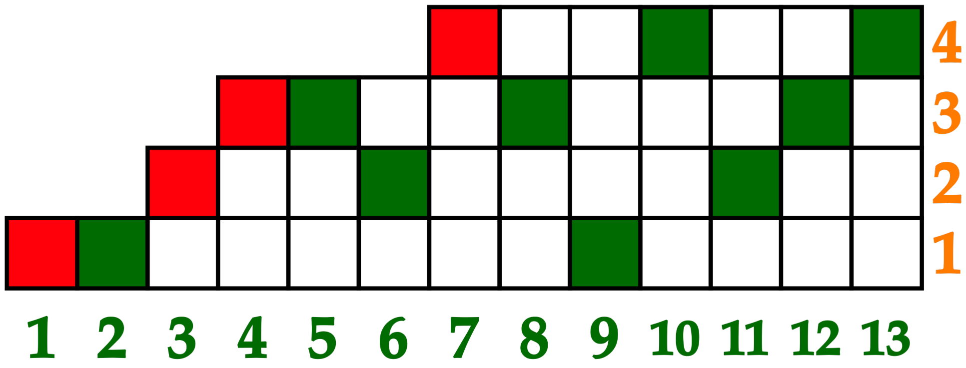


Byproduct 1: study of a variation of the coupon collector problem.

Byproduct 2: refined asymptotic analysis of the Stirling numbers of the 2^{nd} kind.

Byproduct 3: asymptotic analysis of large automata

The unhurried collector



$T_4 = 7$

n coupons (here $n = 4$)

$\omega = \omega_1 \omega_2 \omega_3 \cdots \in [\![1,n]\!]^{\mathbb{N}}$ ω_i's are i.i.d. uniform.

$\omega_{[a,b]} = \omega_a \omega_{a+1} \cdots \omega_b$ is a factor of ω.

$T_n(\omega)$: the smallest k s.t.

$\omega_{[1,k]}$ is a surjection onto $[\![1,n]\!]$.

$$\mathbb{E}[T_n] = n H_n \simeq n \ln(n).$$

Erdös Rényi 1961 If $T_n = n \ln n + n Z_n$, then:

$Z_n \xrightarrow{\text{dist}^n}$ Gumbel (distr^n function $e^{-e^{-x}}$).

ON A CLASSICAL PROBLEM OF PROBABILITY THEORY

by

P. ERDŐS and A. RÉNYI

We consider the following classical "urn-problem". Suppose that there are n urns given, and that balls are placed at random in these urns one after the other. Let us suppose that the urns are labelled with the numbers $1, 2, \ldots, n$ and let ξ_j be equal to k if the j-th ball is placed into the k-th urn. We suppose that the random variables $\xi_1, \xi_2, \ldots, \xi_N, \ldots$ are independent, and $\mathbf{P}(\xi_j = k) = \frac{1}{n}$ for $j = 1, 2, \ldots$ and $k = 1, 2, \ldots, n$. By other words each ball may be placed in any of the urns with the same probability and the choices of the urns for the different balls are independent. We continue this process so long till there are at least m balls in every urn $(m = 1, 2, \ldots)$. What can be said about the number of balls which are needed to achieve this goal?

We denote the number in question (which is of course a random variable) by $\nu_m(n)$. The "dixie cup"-problem considered in [1] is clearly equivalent with the above problem. In [1] the mean value $\mathbf{M}(\nu_m(n))$ of $\nu_m(n)$ has been evaluated (here and in what follows $\mathbf{M}(\ )$ denotes the mean value of the random variable in the brackets) and it has been shown that

$$\mathbf{M}(\nu_m(n)) = n \log n + (m-1)\, n \log\log n + n \cdot C_m + o(n) \tag{1}$$

where C_m is a constant, depending on m. (The value of C_m is not given in [1]).

In the present note we shall go a step further and determine asymptotically the probability distribution of $\nu_m(n)$; we shall prove that for every real x we have

$$\lim_{n \to +\infty} \mathbf{P}\left(\frac{\nu_m(n)}{n} < \log n + (m-1) \log\log n + x\right) = \exp\left(-\frac{e^{-x}}{(m-1)!}\right). \tag{2}$$

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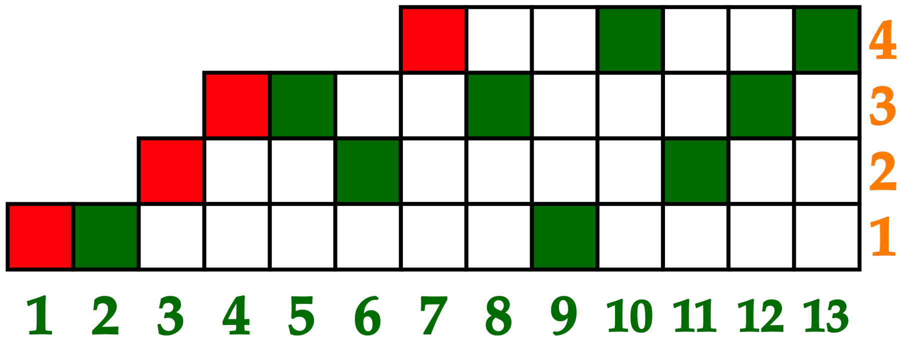

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on aura donc à très-peu près pour l'expression du nombre i de tirages, après lesquels la probabilité que tous les numéros seront sortis est $\frac{1}{k}$,

$$i = (\log n - \log\log k).(n - \tfrac{1}{2} + \tfrac{1}{2}\log k) + \tfrac{1}{2}\log k;$$

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cites sources such as Bernstein or Renyi (1940's)

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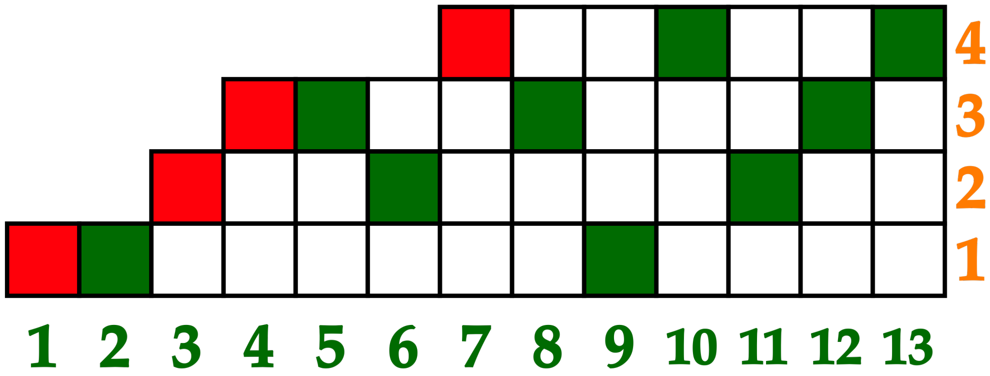

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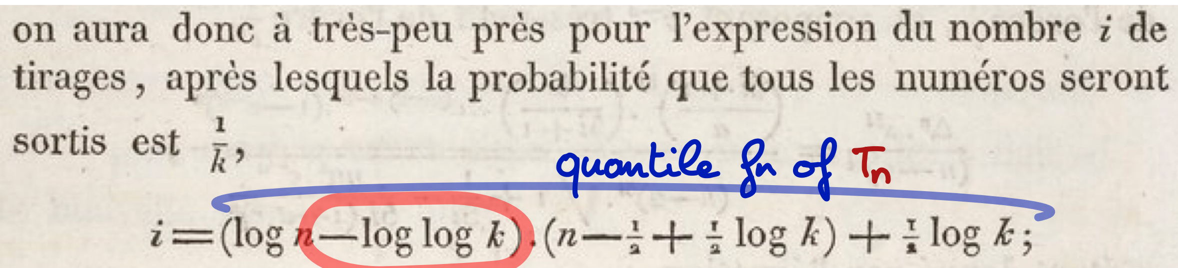

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quantile fn of T_n

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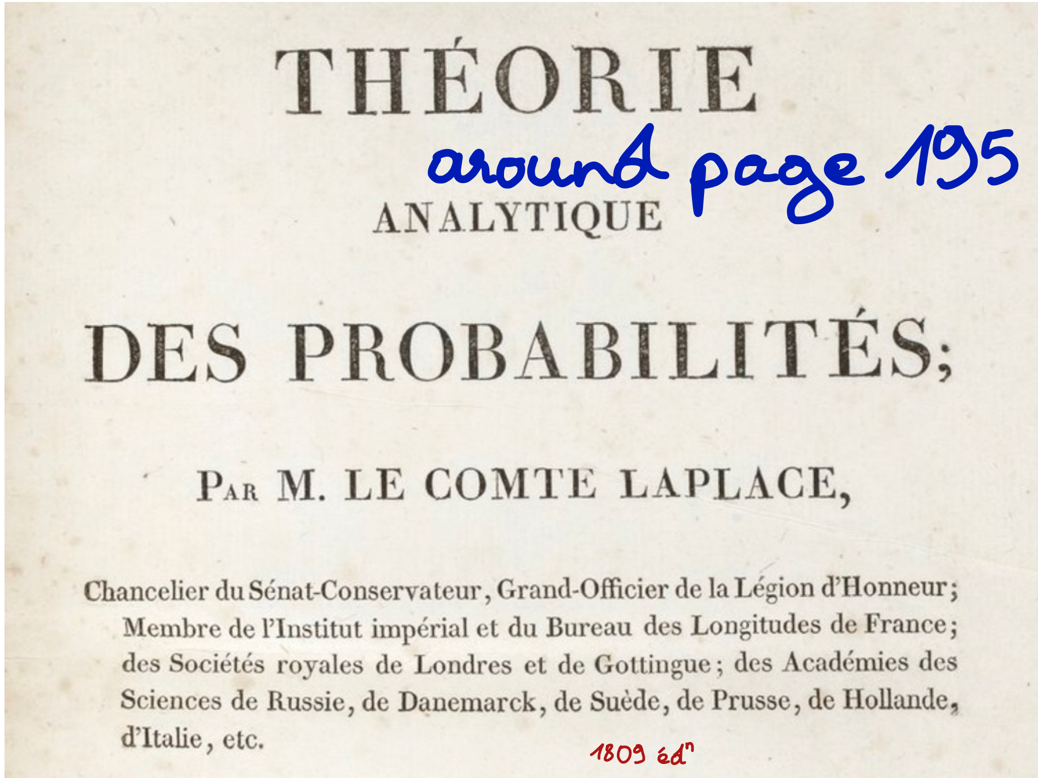
quantile fn of Gumbel



THÉORIE

ANALYTIQUE

around page 195

DES PROBABILITÉS;

PAR M. LE COMTE LAPLACE,

Chancelier du Sénat-Conservateur, Grand-Officier de la Légion d'Honneur; Membre de l'Institut impérial et du Bureau des Longitudes de France; des Sociétés royales de Londres et de Gottingue; des Académies des Sciences de Russie, de Danemarck, de Suède, de Prusse, de Hollande, d'Italie, etc.

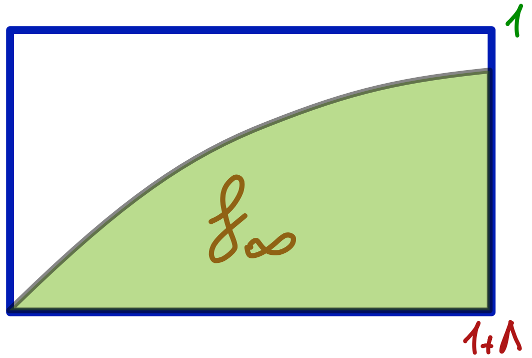
1809 ed^n

The completion curve

$\omega = \omega_1 \omega_2 \omega_3 \cdots$ is "uniform" on $[\![1,n]\!]^{\mathbb{N}}$

$\zeta_n(t,\omega) = \frac{1}{n} \#\{\omega_k \,,\; 1 \le k \le tn\}$

is the completion curve.



Proposition. (unhurried case)

$$\lim_n \mathbb{E}\left[\zeta_n(t)\right] = 1 - e^{-t}$$

$=: \zeta_\infty(t).$

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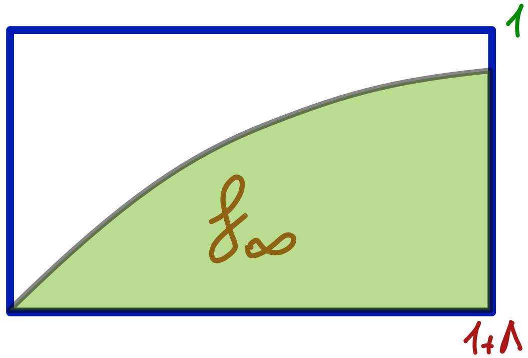
Proposition. (unhurried case)

$$\lim_n \mathbb{E}[\zeta_n(t)] = 1 - e^{-t} =: \zeta_\infty(t).$$

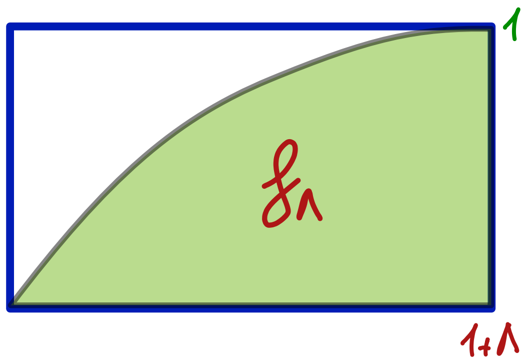
$\longleftarrow \quad \mathbb{E}[\ ] = 1 - (1 - \frac{1}{n})^{\lfloor tn \rfloor}$



Question (impatient case)

Given that $T_n \leq (1+\Lambda)n$, $(\Lambda > 0)$

asymptotic behaviour of $\zeta_n(t)$??



I.e. description of

$$\zeta(\Lambda, t) = \lim_n \mathbb{E}[\zeta_n(t) \mid T_n \leq (1+\Lambda)n] \ ??$$

Nota: $\mathbb{P}(T_n \leq (1+\Lambda)n) \approx C(\Lambda)\, e^{-n J(\Lambda)}$ in which $J(\Lambda) \searrow_{+\infty} 0$.

The completion curve

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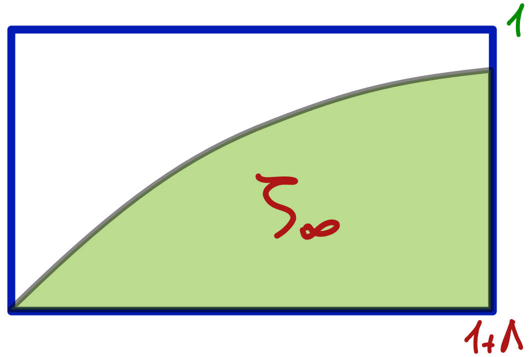
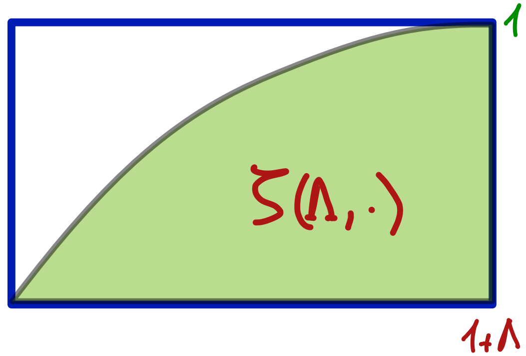


Question: asymptotic behaviour of $\zeta_n(t)$??



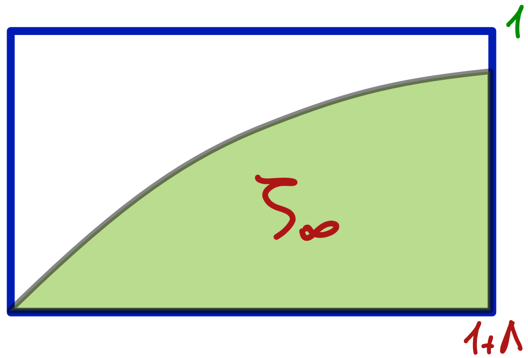
$$\zeta(\Lambda,t) = \lim_n \mathbb{E}\left[\zeta_n(t) \mid T_n < (1+\Lambda)n\right]$$

✱ exists,

The completion curve

Proposition. (unhurried case)

$$\lim_n \mathbb{E}[\zeta_n(t)] = 1 - e^{-t} =: \zeta_\infty(t).$$



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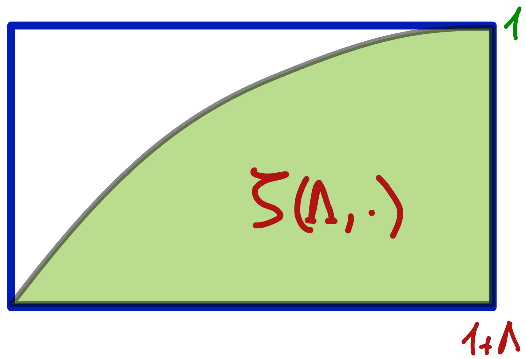

$$\zeta(\Lambda,t) = \lim_n \mathbb{E}[\zeta_n(t) \mid T_n < (1+\Lambda)n]$$

- exists,
- is solution of an ODE,

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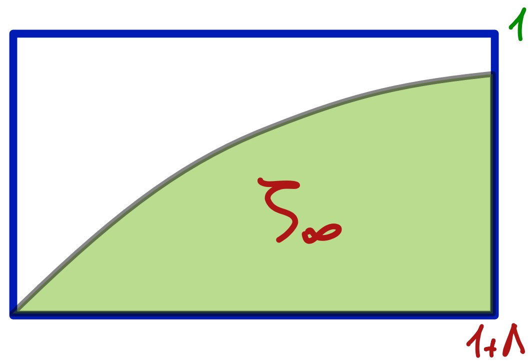
$$\lim_n \mathbb{E}\left[\zeta_n(t)\right] = \underbrace{1-e^{-t}}_{=\zeta_\infty(t)}.$$



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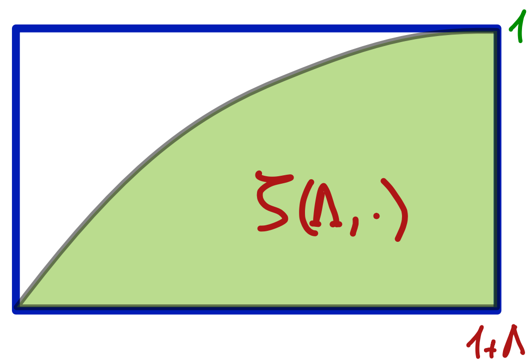


$$\zeta(\Lambda,t) = \lim_n \mathbb{E}\left[\zeta_n(t) \mid T_n < (1+\Lambda)n\right]$$

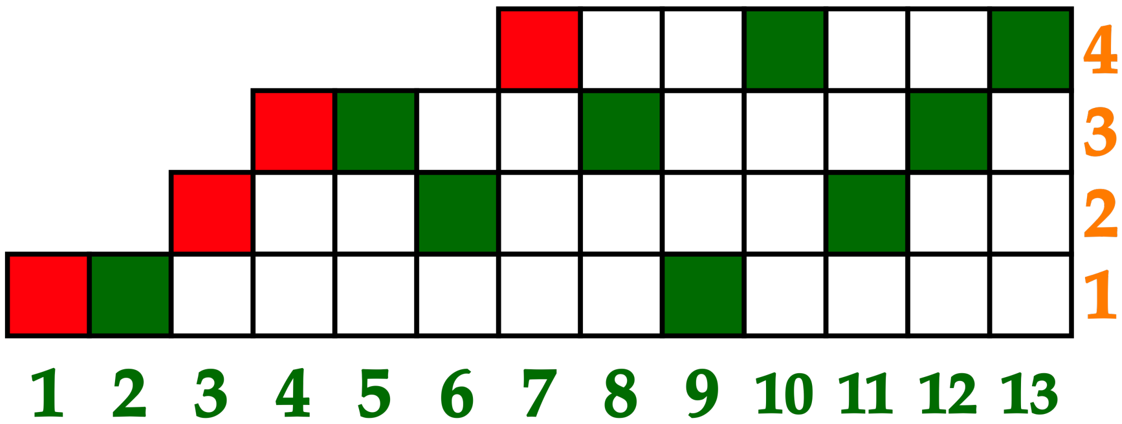
- exists,
- is solution of an ODE,
- satisfies $\zeta(\Lambda,t) = \lim_n \zeta_n(t)$ in probability ...

Stirling numbers of the 2nd kind $\left\{ {m \atop \ell} \right\}$

$\left\{ {m \atop \ell} \right\}$: number of partitions of $[\![1,m]\!]$ in ℓ parts, $\ell \leq m$.

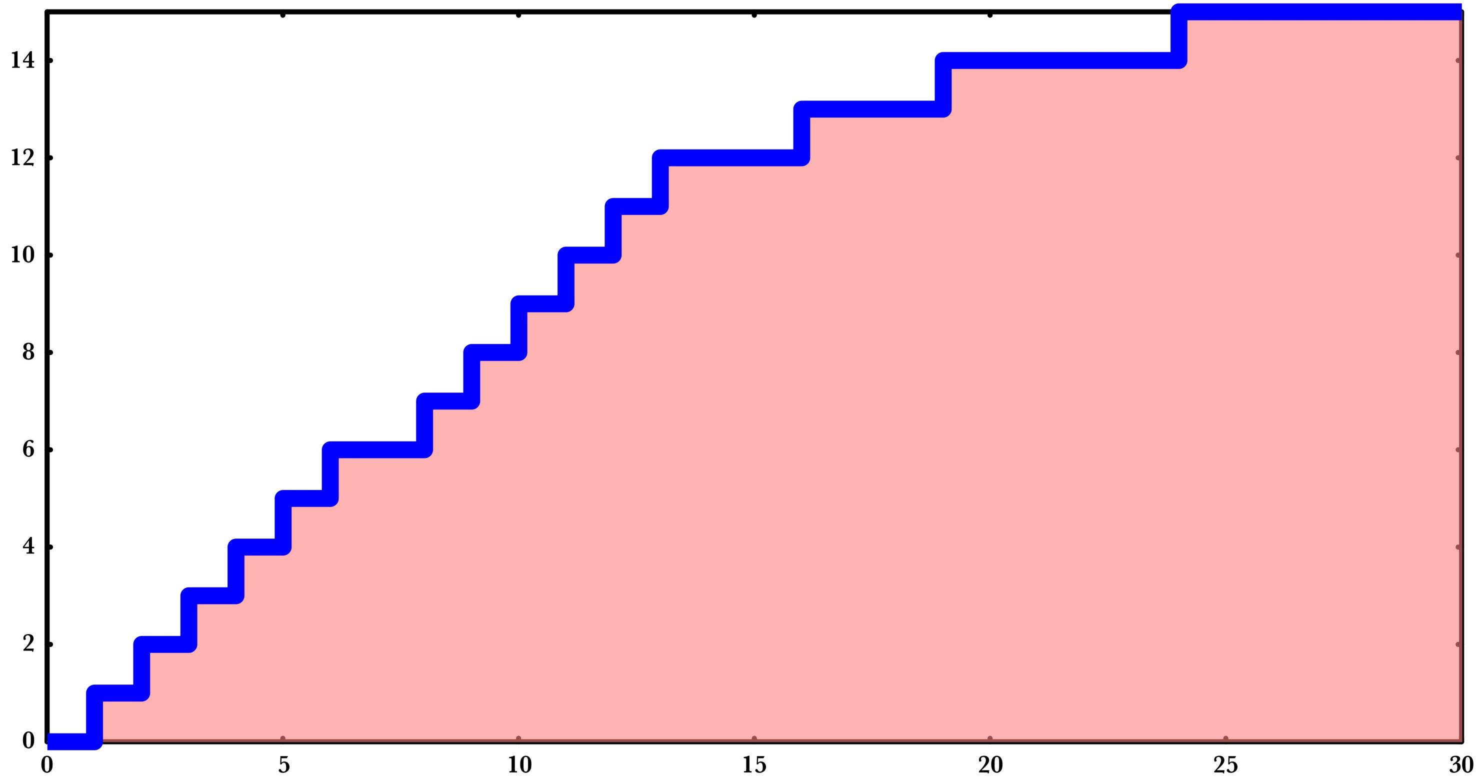
$\left\{ {m \atop \ell} \right\} \times (\ell!)$: number of surjections from $[\![1,m]\!]$ to $[\![1,\ell]\!]$.

$$\left\{ {m \atop \ell} \right\} \times (\ell!) \times \ell^{-m} = \mathbb{P}(T_\ell \leq m).$$



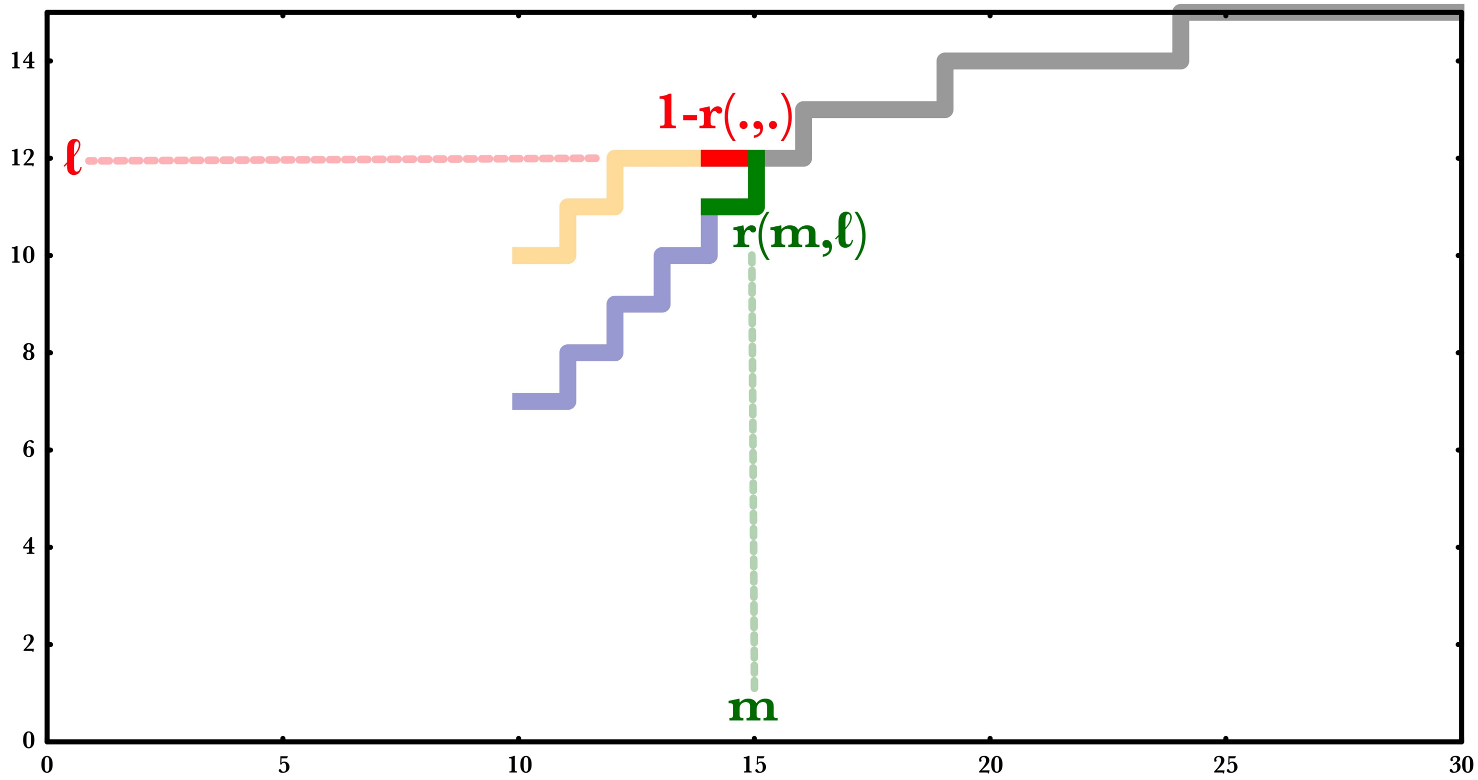
$\left\{ {13 \atop 4} \right\}$

A Markov chain



$\lambda = 1$

A Markov chain



$$r(m,\ell) = \frac{\left\{ {m-1 \atop \ell-1} \right\}}{\left\{ {m \atop \ell} \right\}}$$ is the probability of

$$1 - r(m,\ell) = \frac{\left\{ {m-1 \atop \ell} \right\} \ell}{\left\{ {m \atop \ell} \right\}}$$ is the probability of

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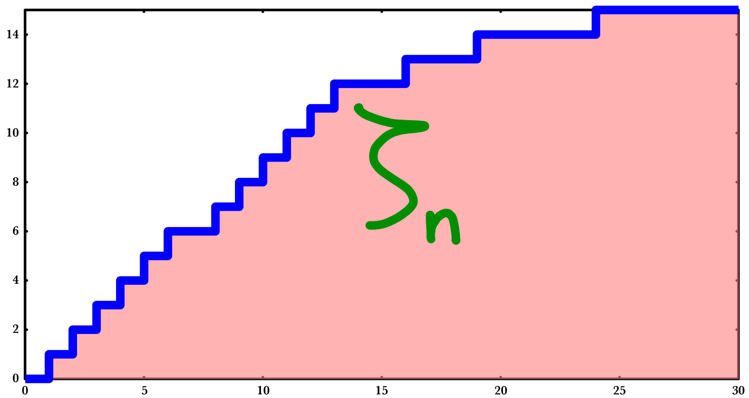

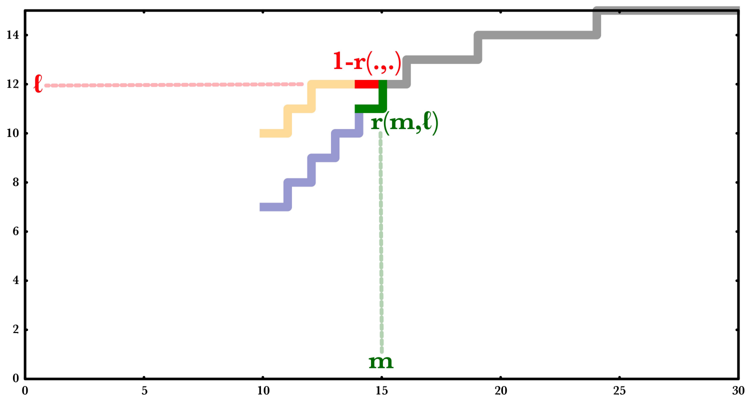

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The asymptotic shape $\zeta(1,.)$ is given by

the asymptotic behaviour of $\left\{ {m \atop \ell} \right\}$

Asymptotics for $\left\{ {m \atop \ell} \right\}$

$$\left\{ {m \atop \ell} \right\} \sim \psi(m,\ell) = \frac{m!(e^{\xi}-1)^{\ell}}{\ell!\xi^m \sqrt{2\pi m\left(1-\frac{m}{\ell}e^{-\xi}\right)}}.$$

Good 1961

$$\lambda(m,\ell) = \lambda = \frac{m-\ell}{\ell}$$

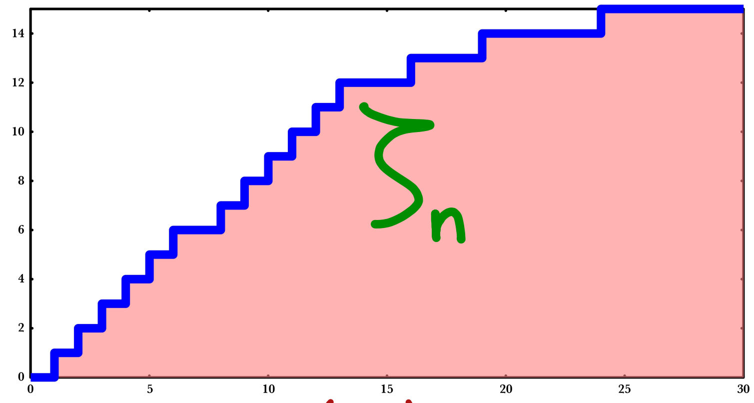
$$\xi(m,\ell) = \xi = 1+\lambda+W_0\left(-(1+\lambda)e^{-1-\lambda}\right)$$

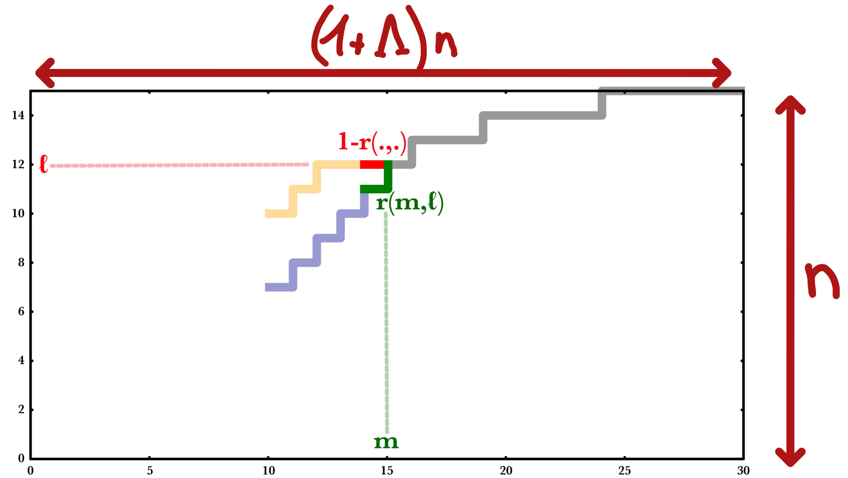
$$v = \frac{(\lambda+1)(\xi-\lambda)}{2}$$

W_0 : Lambert function

$\xi = (1-e^{-\xi})(1+\lambda)$

A Markov chain





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The asymptotic shape $\xi(\Lambda, .)$ is given by the asymptotic behaviour of $\left\{ {m \atop \ell} \right\}$

$$\left\{ {m \atop \ell} \right\} \sim \psi(m,\ell) = \frac{m!(e^{\xi}-1)^{\ell}}{\ell!\xi^m \sqrt{2\pi m\left(1-\frac{m}{\ell}e^{-\xi}\right)}}.$$

Good 1961

$$\left.\begin{array}{l} \lambda = \frac{m}{\ell} - 1 \\ \xi = (1-e^{-\xi})(1+\lambda) \end{array}\right\} \to r(m,\ell) \simeq \exp(-\xi(\lambda)) = f(\lambda)$$

$\to \xi(\Lambda, \cdot)$ is sol^n of $\left\{ \begin{array}{l} y' = f\left(\frac{y}{x} - 1\right) \\ y(1+\Lambda) = 1 \end{array} \right.$

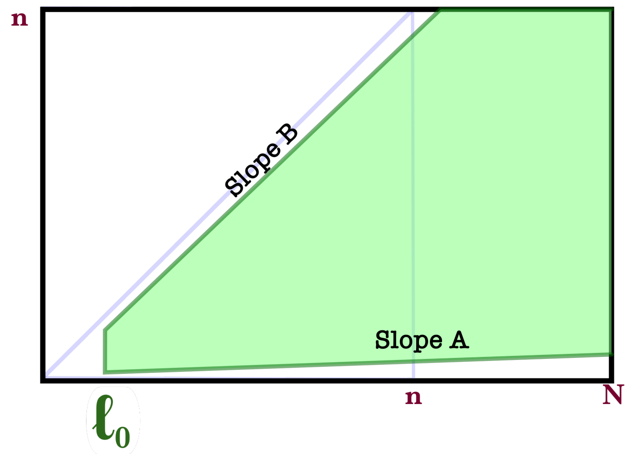
Proof: Euler scheme + Azuma inequality

$$\left| \frac{\left\{ {m \atop \ell} \right\} - \psi(m,\ell)}{\psi(m,\ell)} \right| \leq \frac{C_2(\ell_0, \delta)}{\ell}$$

Slope B $= \frac{1}{1+\delta}$

Slope A $= \frac{\delta}{1+\delta}$

for (m,ℓ) in



$$|r\,(m,\ell) - \rho(\lambda)| \leq \frac{C_1(\ell_0, \delta)}{\ell}$$

Theorem. *For any $a > 0$, and for n_0 large enough, there exists $C = C(n_0, a) > 0$ such that, for $n \geq n_0$,*

$$\mathbb{P}_{N,n}\left(\sup_{[a, 1+\Lambda(N,n)]} |\zeta_n - \zeta(\Lambda(N,n), .)| \geq C n^{-1/3} \right) \leq n^{1/3} e^{-\ln^2 n/2}.$$

Proof: Euler scheme + Azuma inequality & saddle point methods

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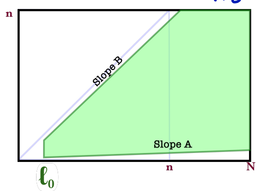
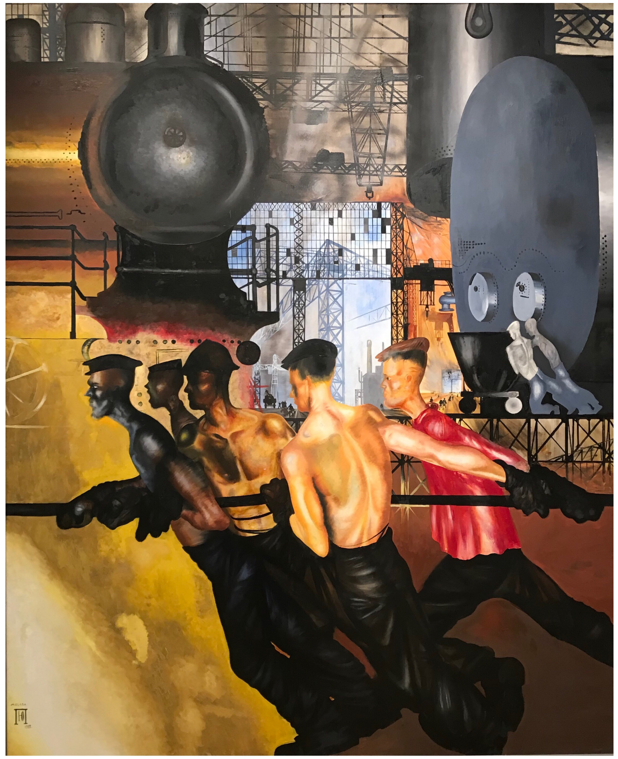




Theorem. *For any $a > 0$, and for n_0 large enough, there exists $C = C(n_0, a) > 0$ such that, for $n \geq n_0$,*

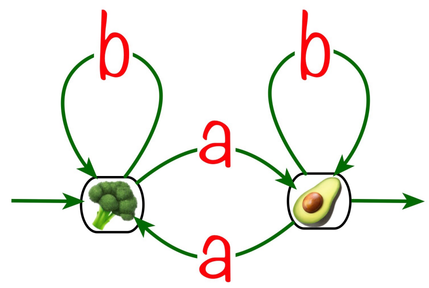
$$\mathbb{P}_{N,n}\left(\sup_{[a,1+\Lambda(N,n)]} |\zeta_n - \zeta(\Lambda(N,n),.)| \geq Cn^{-1/3}\right) \leq n^{1/3} e^{-\ln^2 n/2}.$$

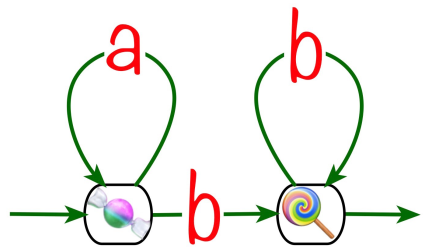
Automata : some terminology

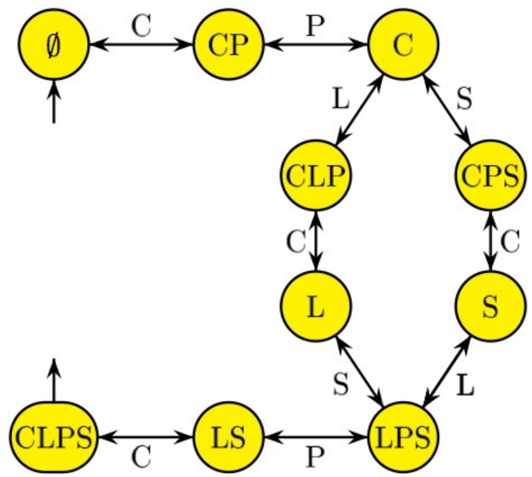
* state : vertex

* alphabet (A) : set of edges' labels.

* starting state, final states







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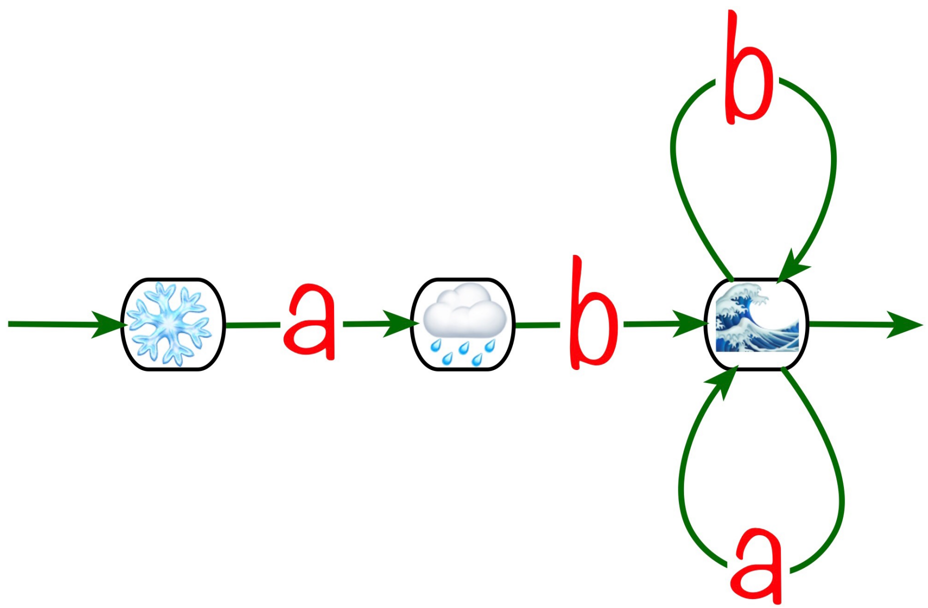

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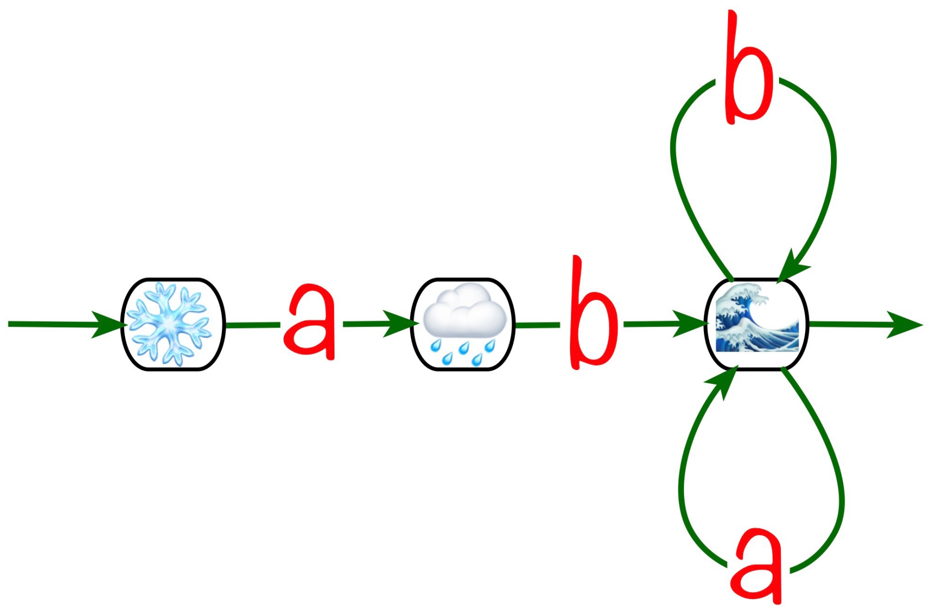

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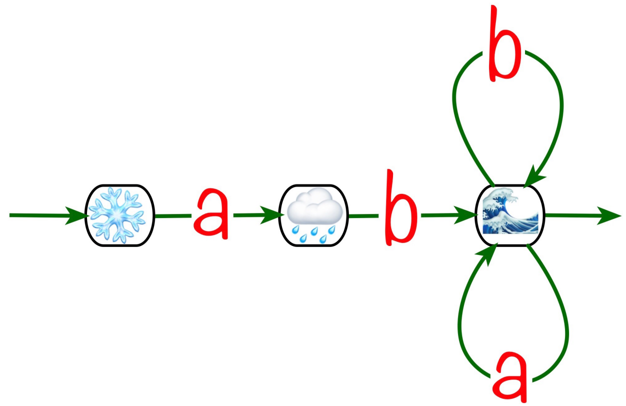

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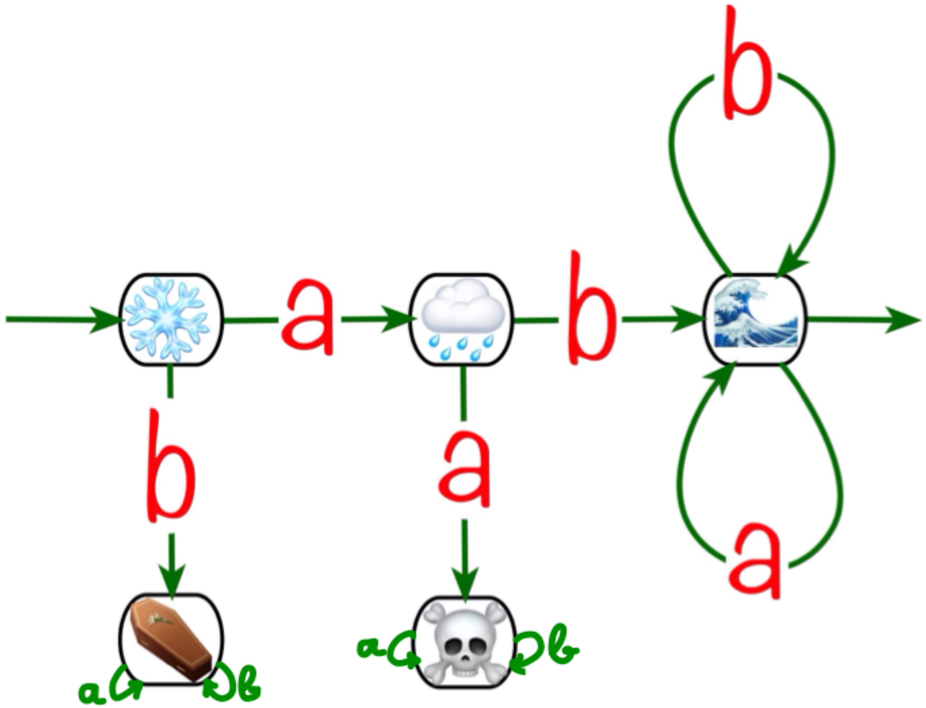
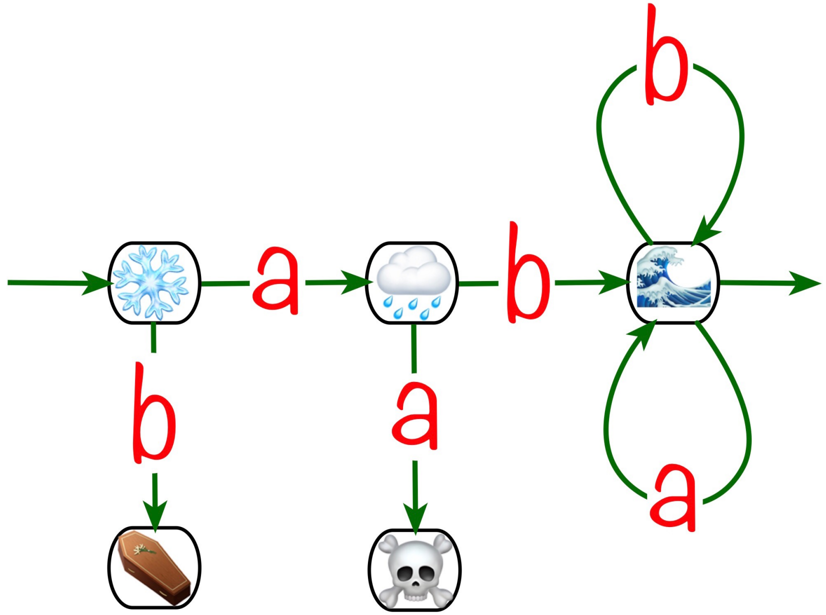


* accessible automata : each state can be
  reached, from the starting state.

Relation automata - coupon collector

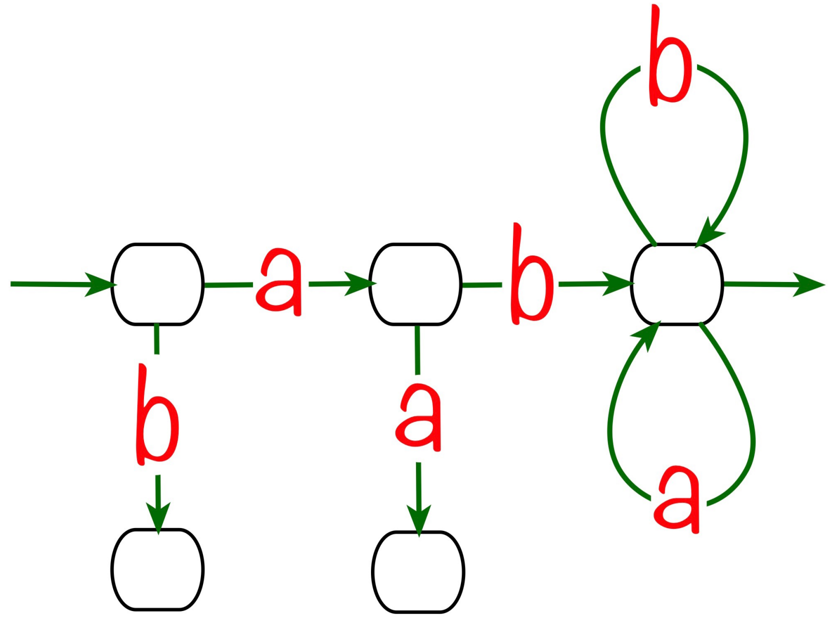
← input for Moore's algorithm

$\mathcal{A}_{n,k}$: set of the deterministic complete accessible automata (DCAA) with n vertices and a k-letters alphabet $\mathcal{A}$.



Koršunov: $\#\mathcal{A}_{n,k} \simeq \left\{ {kn+1 \atop n} \right\} n! \times C(k,n)$
1978

in which $\lim_n C(k,n) = C(k) \in]0,1[$.



Provided an alphabetic order on $\mathcal{A}$, breadth-first search provides an ordering of the vertices (if accessibility).

Relation automata - coupon collector

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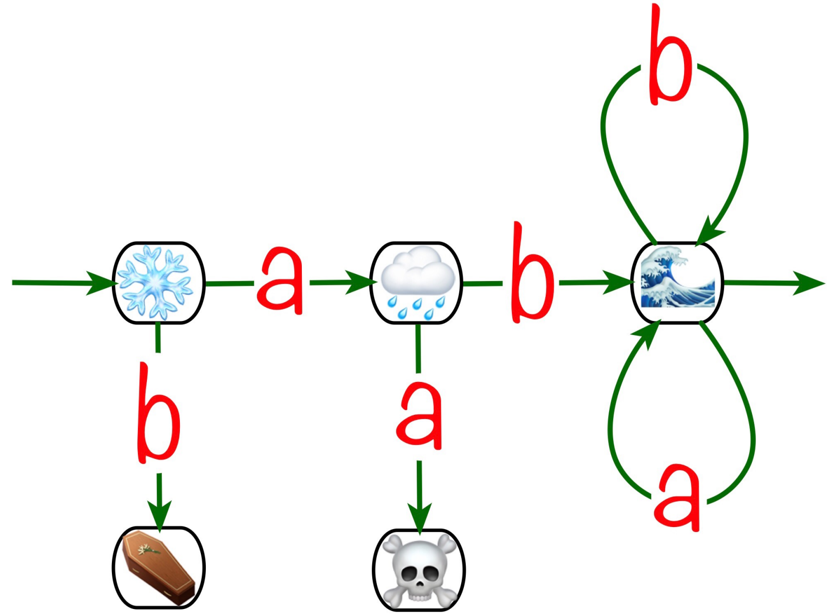

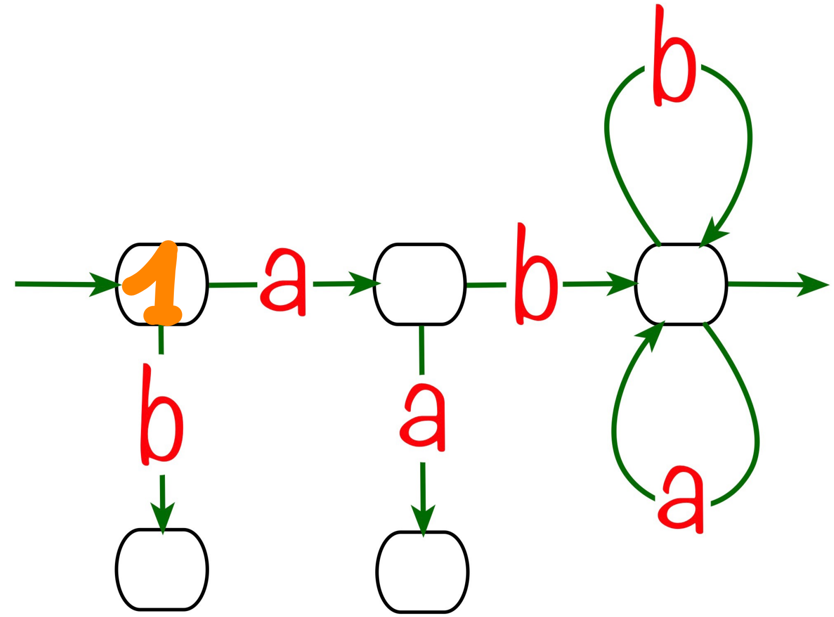

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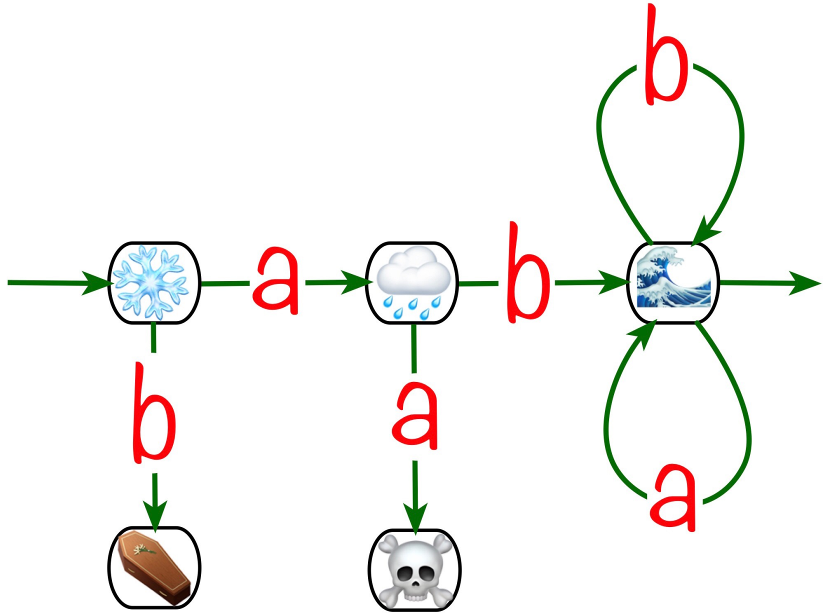

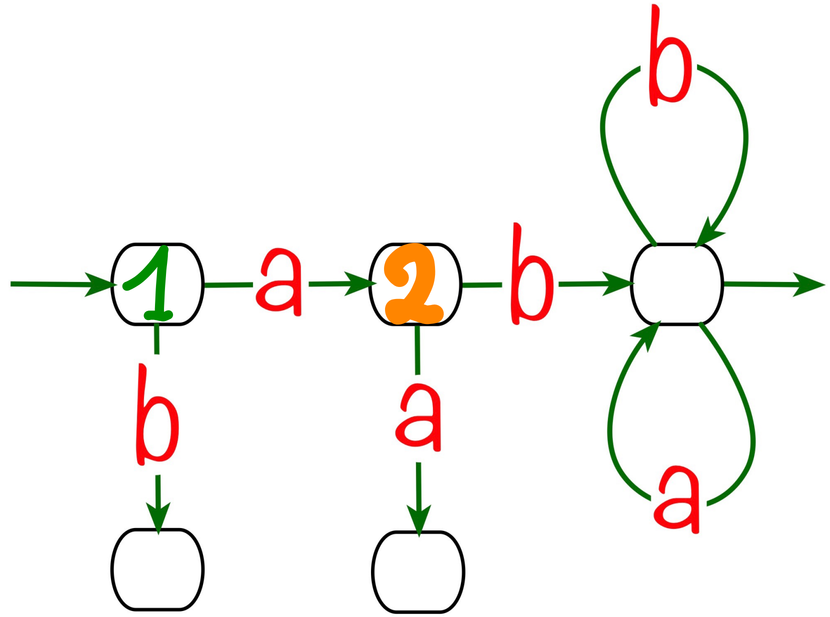

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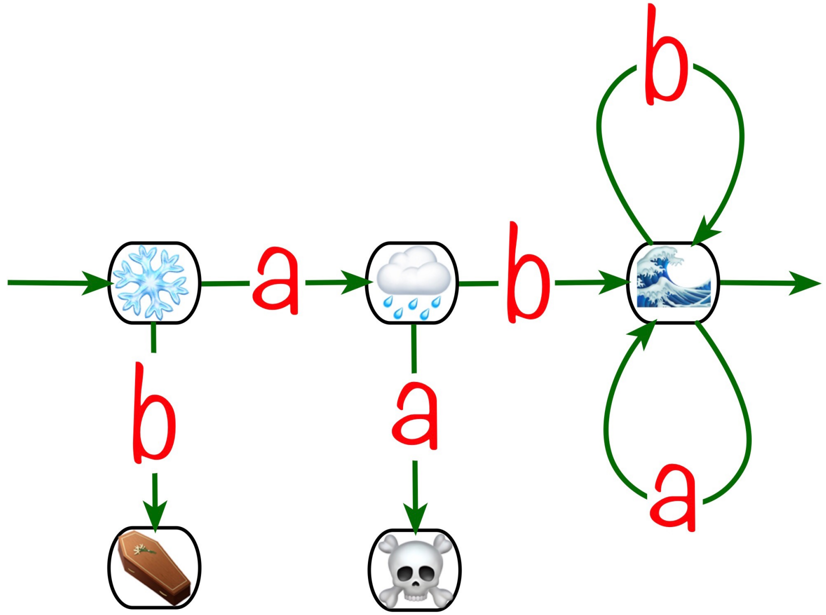

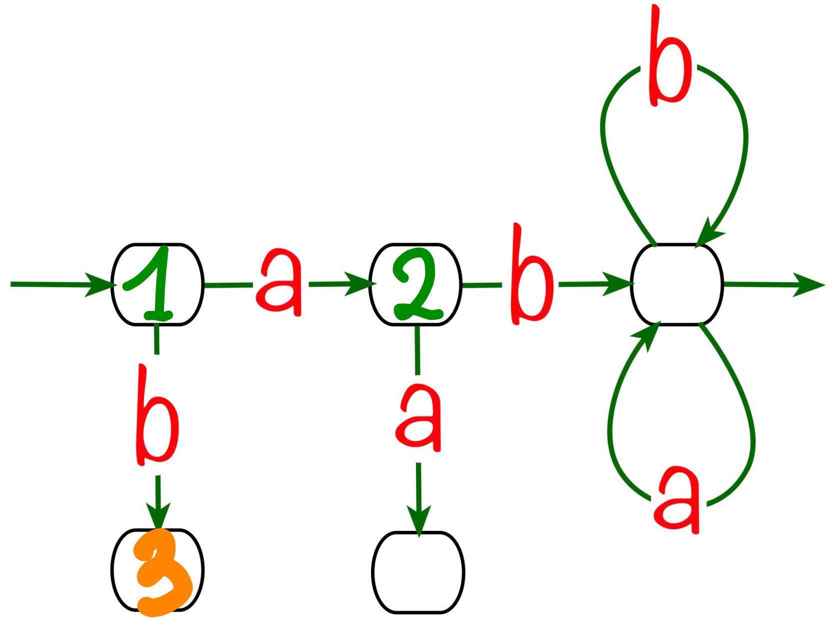

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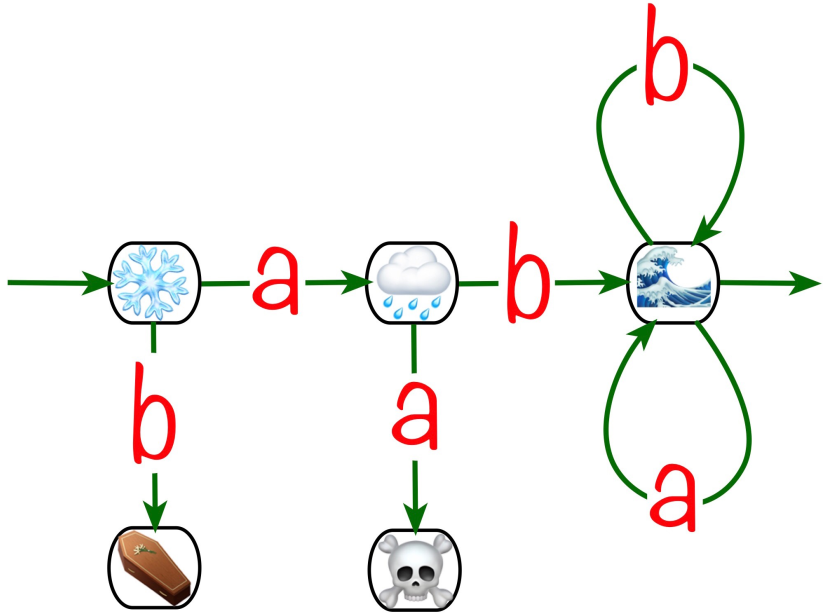

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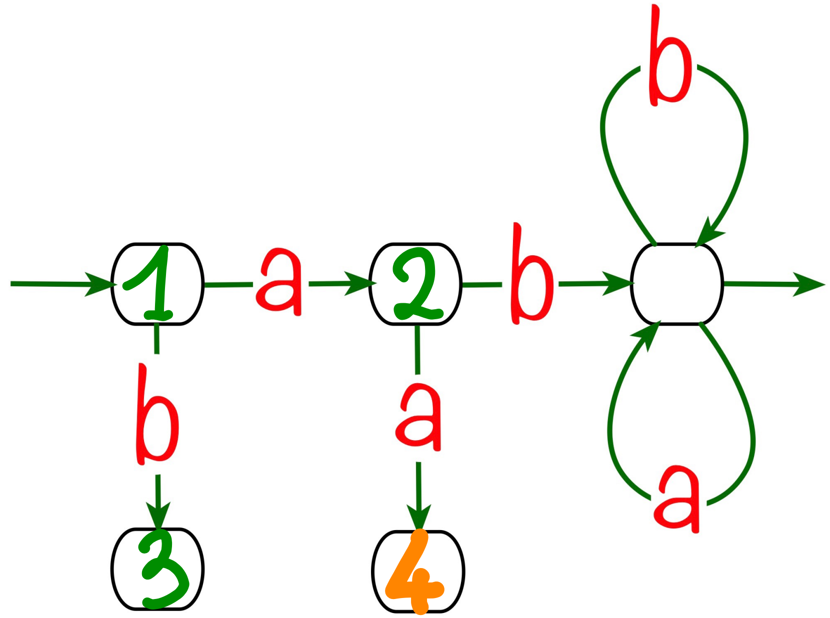

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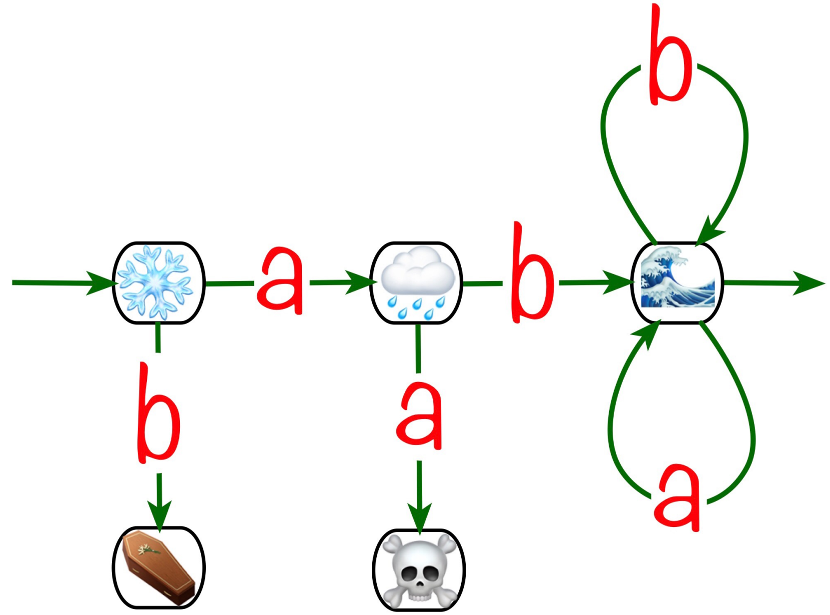

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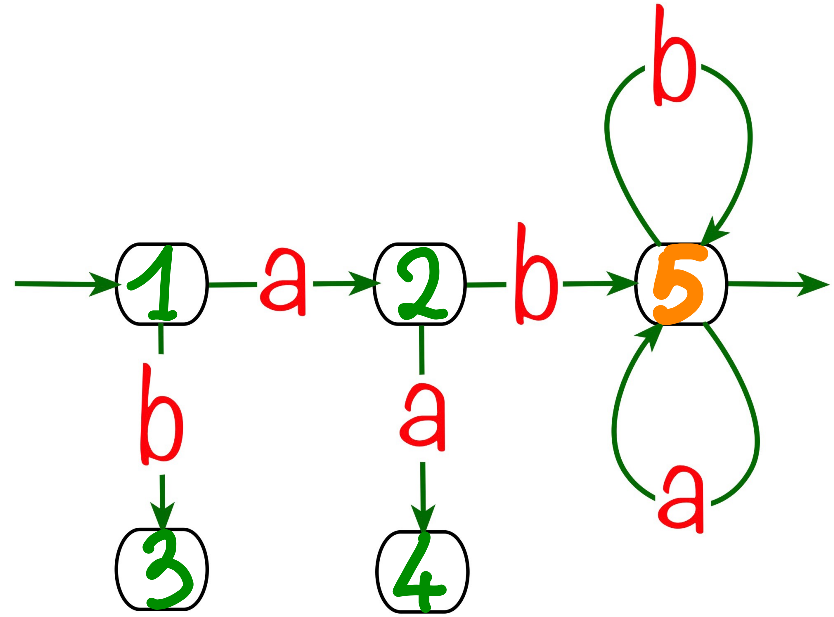

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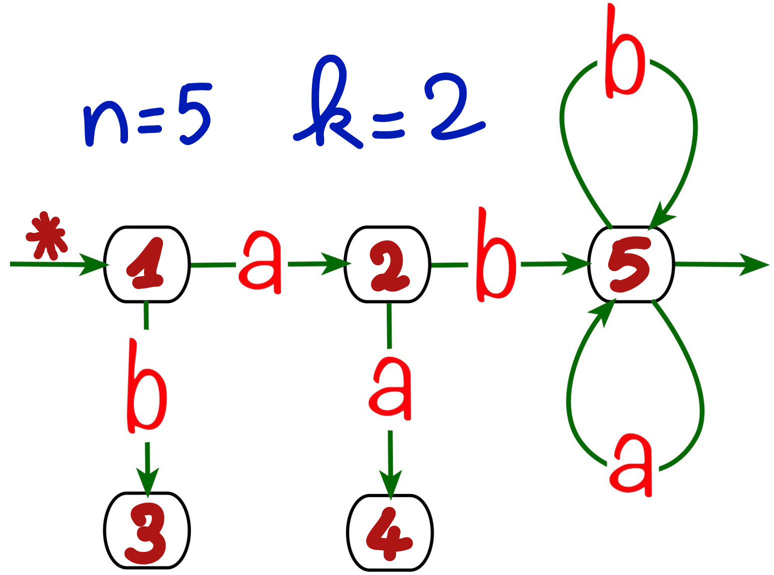
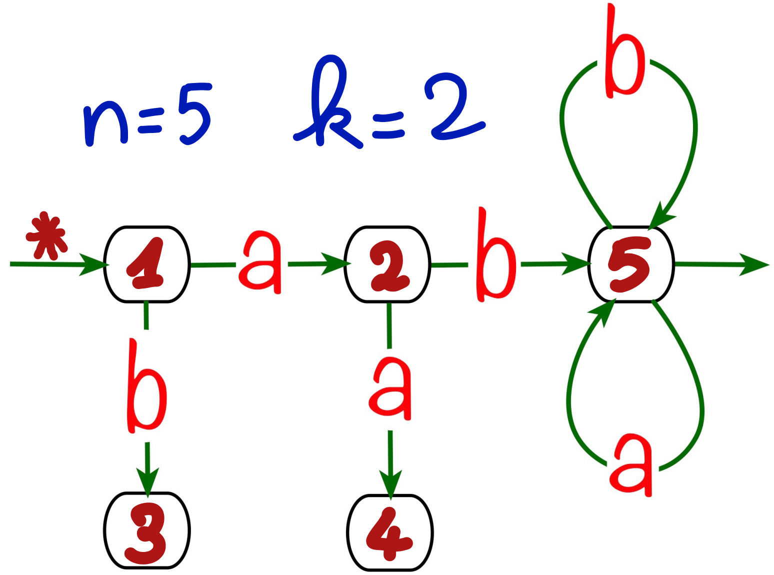
Relation automata - coupon collector

$n=5 \quad k=2$



$* \to$	1a	1b	2a	2b	3a	3b	4a	4b	5a	5b
1	2	3	4	5	3	3	4	4	5	5

Relation automata - coupon collector



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automata in $\mathcal{A}_{k,n}$

$\updownarrow$

map from $[\![1, 1+kn]\!]$ to $[\![1,n]\!]$.

Relation automata - coupon collector

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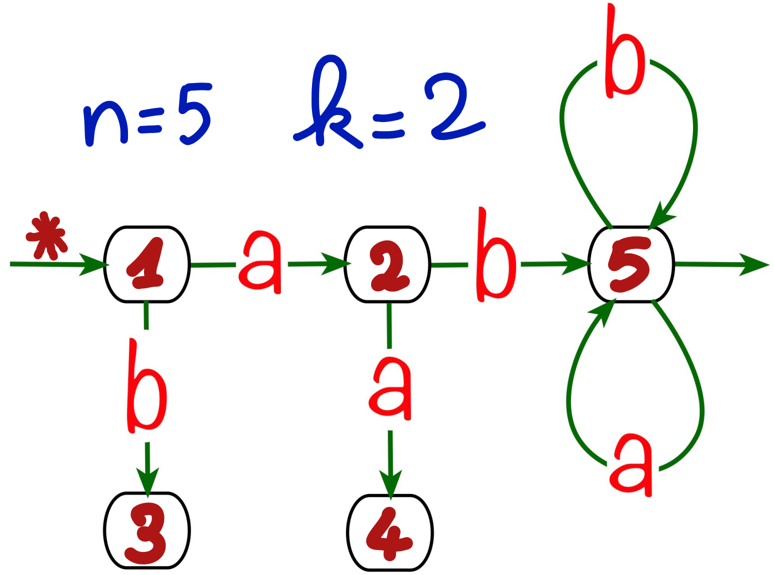

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accessible

$\Downarrow$

surjection !

Relation automata - coupon collector

automata

$\overset{?}{=}$

impatient collector

$\lambda = k$

$$\# \mathcal{A}_{k,n} \overset{?}{=} \left\{ {1+kn \atop n} \right\} \times n!$$

$\Uparrow$

accessible

$\Downarrow$

surjection !

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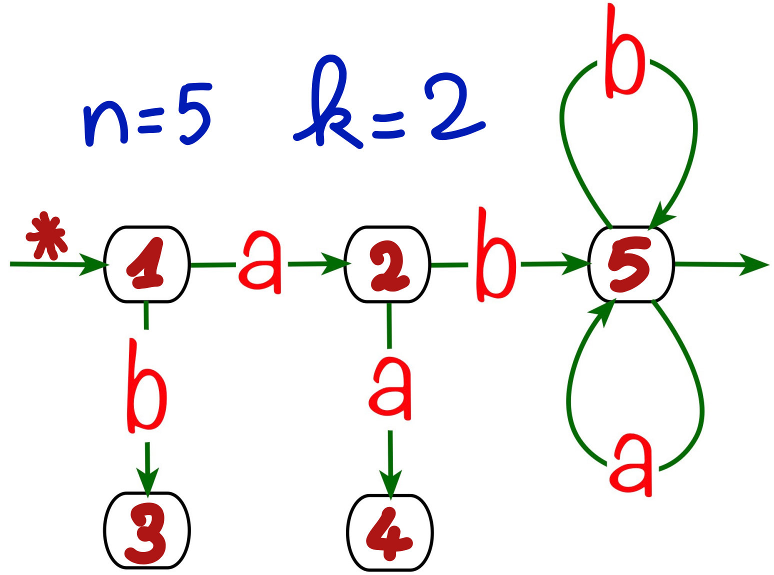

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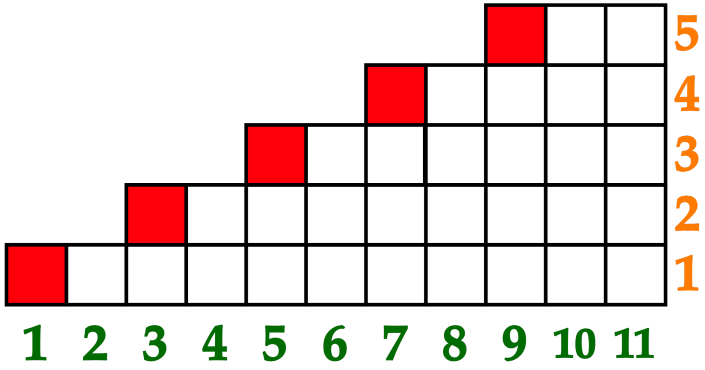

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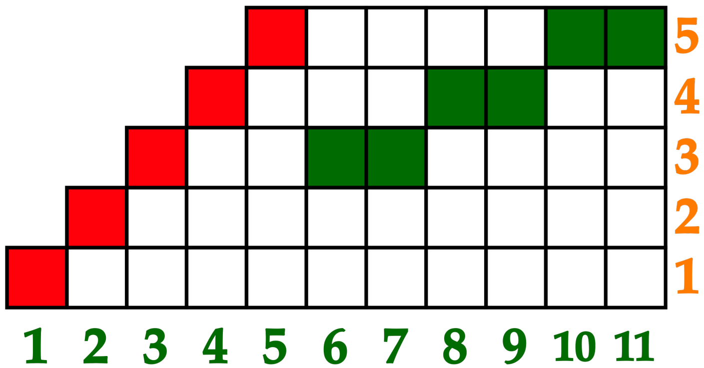

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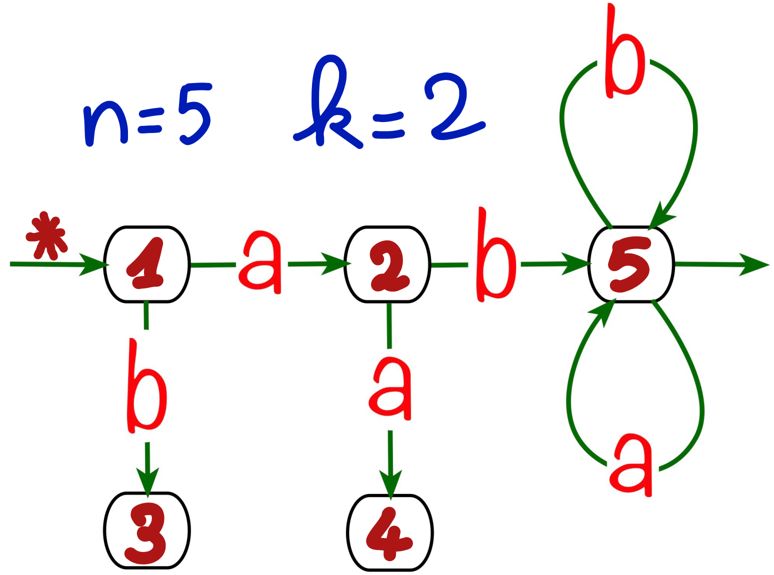

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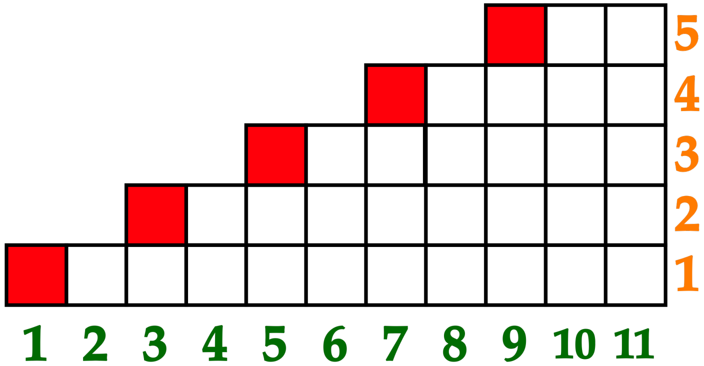
map from $[\![1, 1+kn]\!]$ to $[\![1,n]\!]$.

Relation automata - coupon collector



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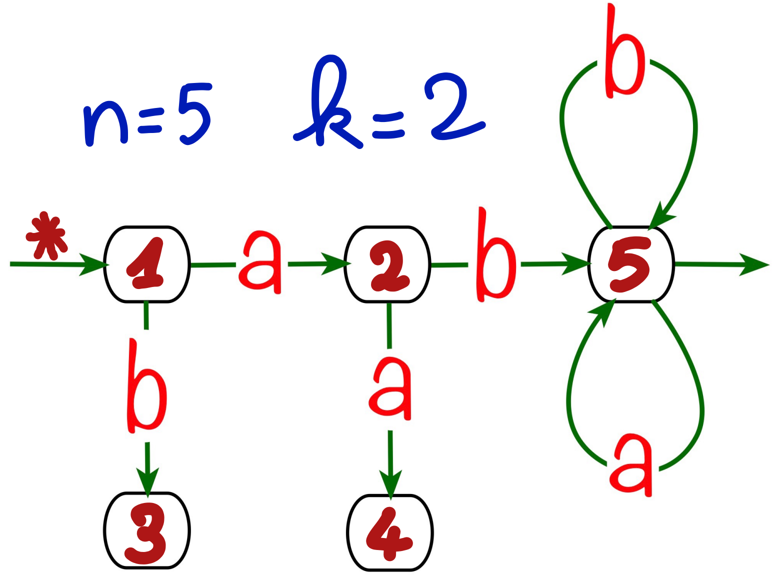


$\#\mathcal{A}_{k,n} = \left\{ {1+kn \atop n} \right\} \times n! \times C(k,n) \in]0,1[$

automata

$\stackrel{?}{=}$

impatient collector

$1 \le k$

$\#\mathcal{O}_{k,n} \stackrel{?}{=} \left\{ {1+kn \atop n} \right\} \times n!$

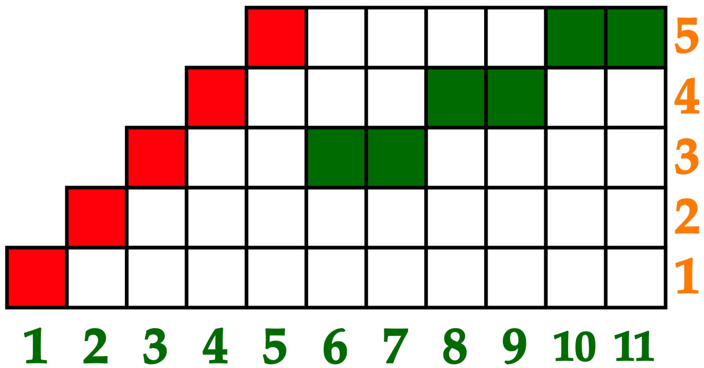
$\Uparrow$

$\xrightarrow{*}$	1a	1b	2a	2b	3a	3b	4a	4b	5a	5b
1	2	3	4	5	3	3	4	4	5	5

automata in $\mathcal{A}_{k,n}$

$\updownarrow$

map from $[\![1, 1+kn]\!]$ to $[\![1,n]\!]$.

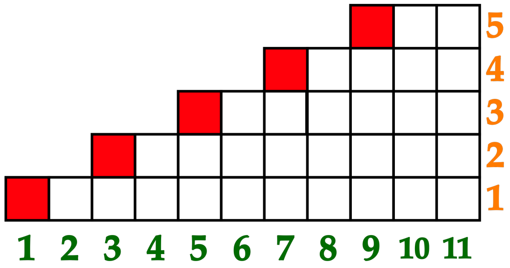


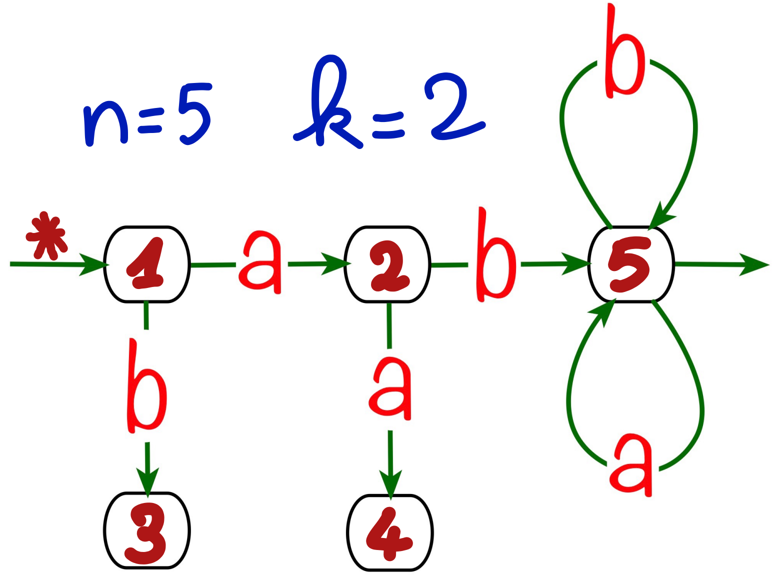
accessible

$\Downarrow \quad \cancel{\Uparrow}$

surjection !

Relation automata - coupon collector





$$\#\mathcal{A}_{k,n} = \left\{ {1+kn \atop n} \right\} \times n! \times \underbrace{C(k,n)}_{\text{some probability in the impatient collector model}} \in \,]0,1[$$

* →	1a	1b	2a	2b	3a	3b	4a	4b	5a	5b
1	2	3	4	5	3	3	4	4	5	5

~~automata $\Updownarrow$? impatient collector $1 \le k$~~

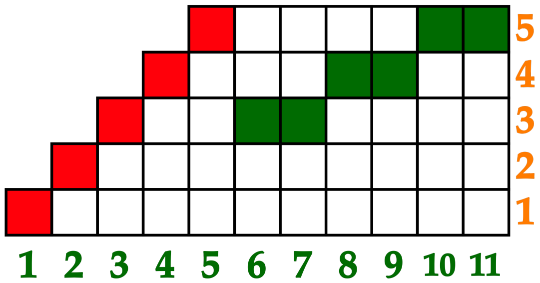
~~$\#\mathcal{A}_{k,n} \overset{?}{=} \left\{ {1+kn \atop n} \right\} \times n!$~~

~~$\Uparrow$~~

automata in $\mathcal{A}_{k,n}$

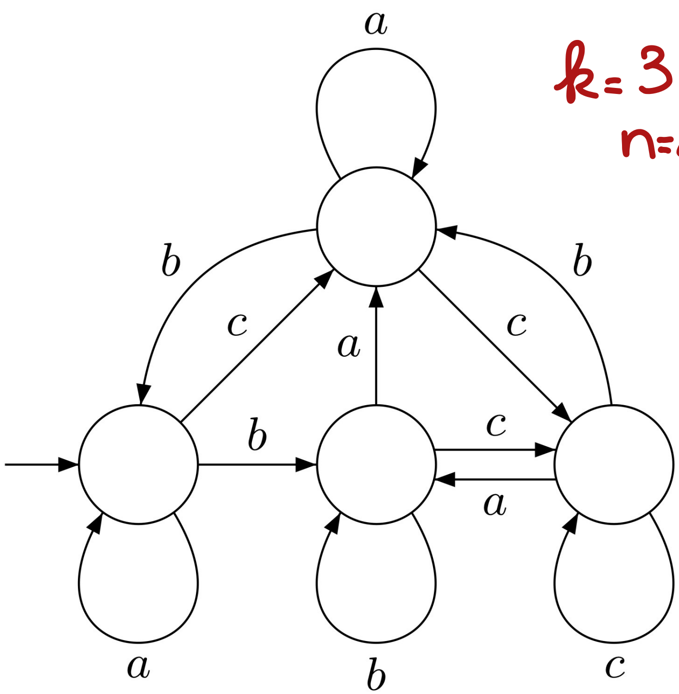
$\updownarrow$

accessible $\Downarrow$ $\cancel{\Uparrow}$ surjection ! map from $[\![1, 1+kn]\!]$ to $[\![1, n]\!]$.

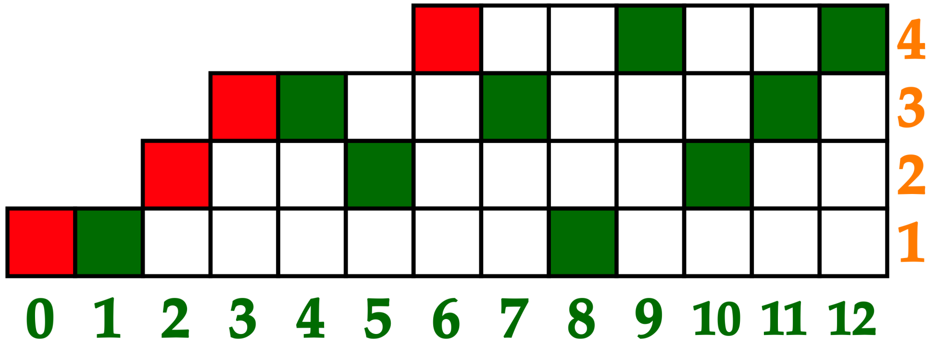


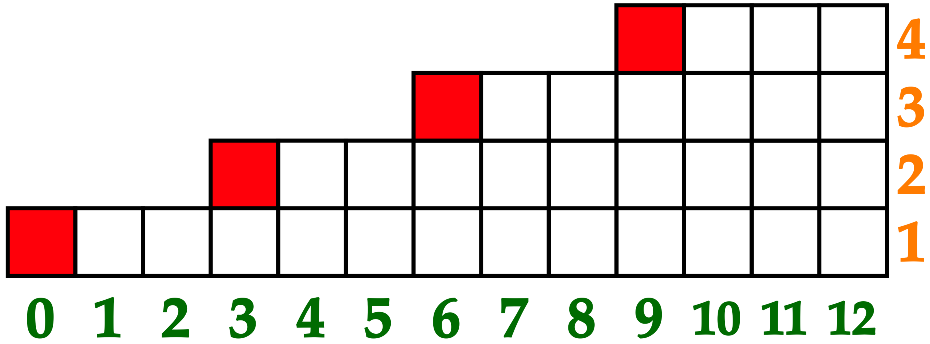
Koršunov 1978

$$\lim_n \frac{\#\mathcal{A}_{k,n}}{\left\{ {1+kn \atop n} \right\} \times n!} = C(k) \in]0,1[$$

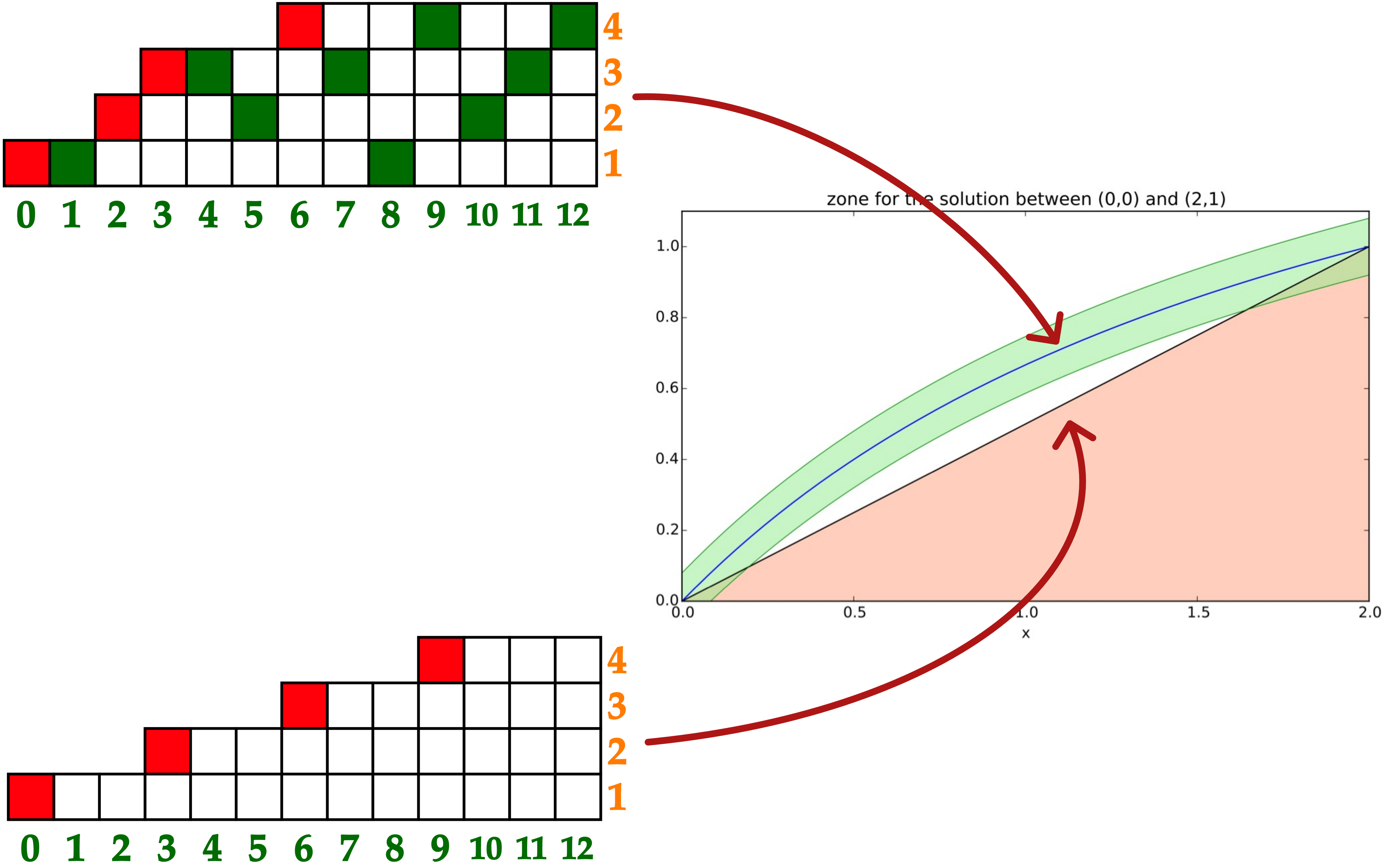


$k=3$
$n=4$





Limit profile & forbidden zone



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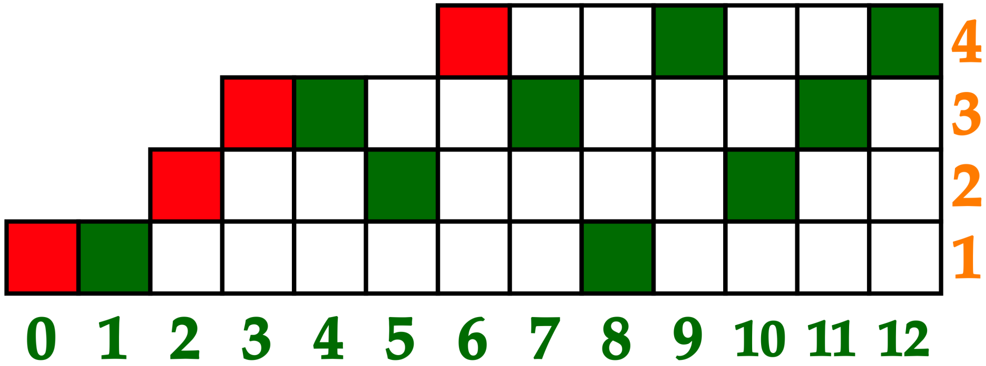
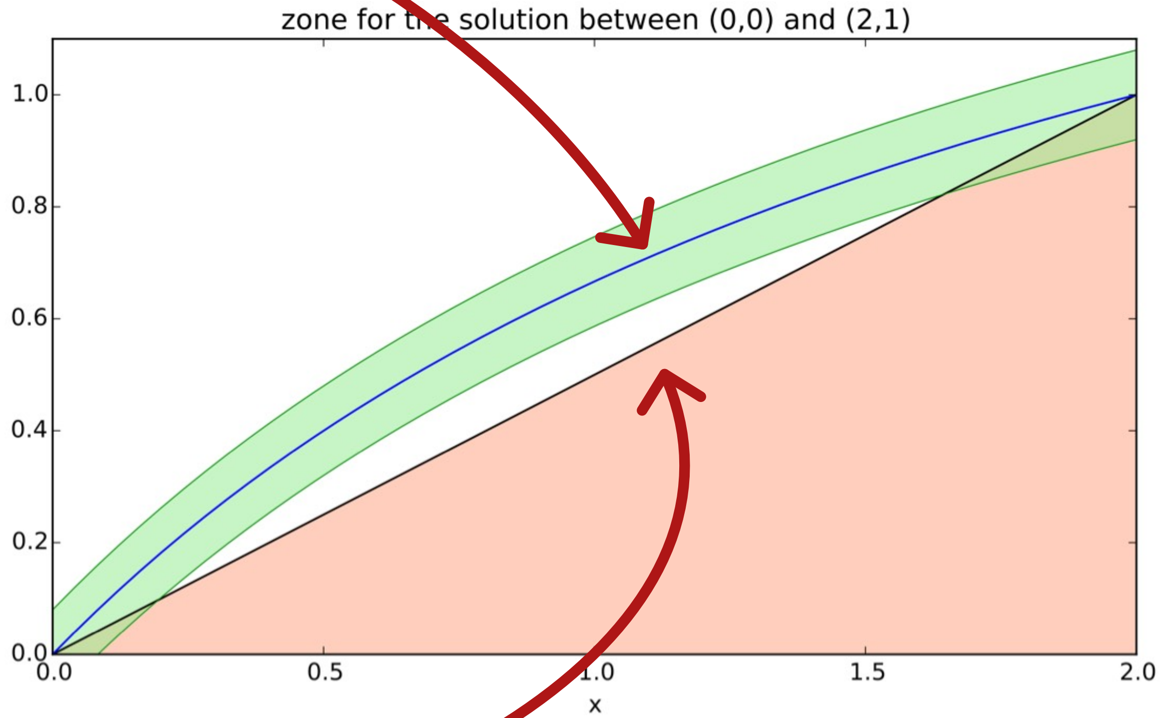
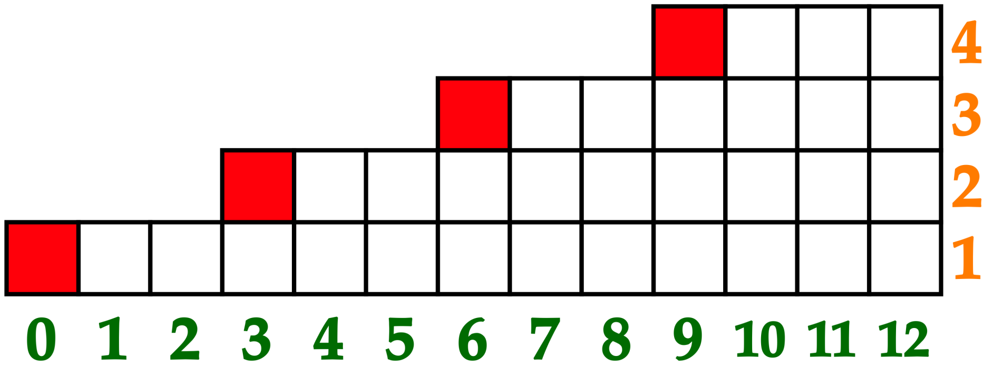


$$\mathbb{P}_{N,n}\left(\sup_{[a,1+\Lambda]}|\zeta_n-\zeta(\Lambda,.)|\geq Cn^{-1/3}\right)\leq n^{1/3}e^{-\ln^2 n/2}$$





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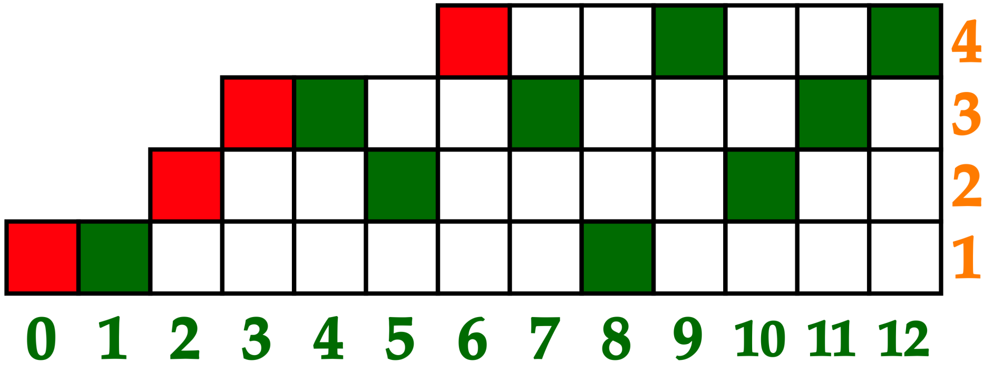
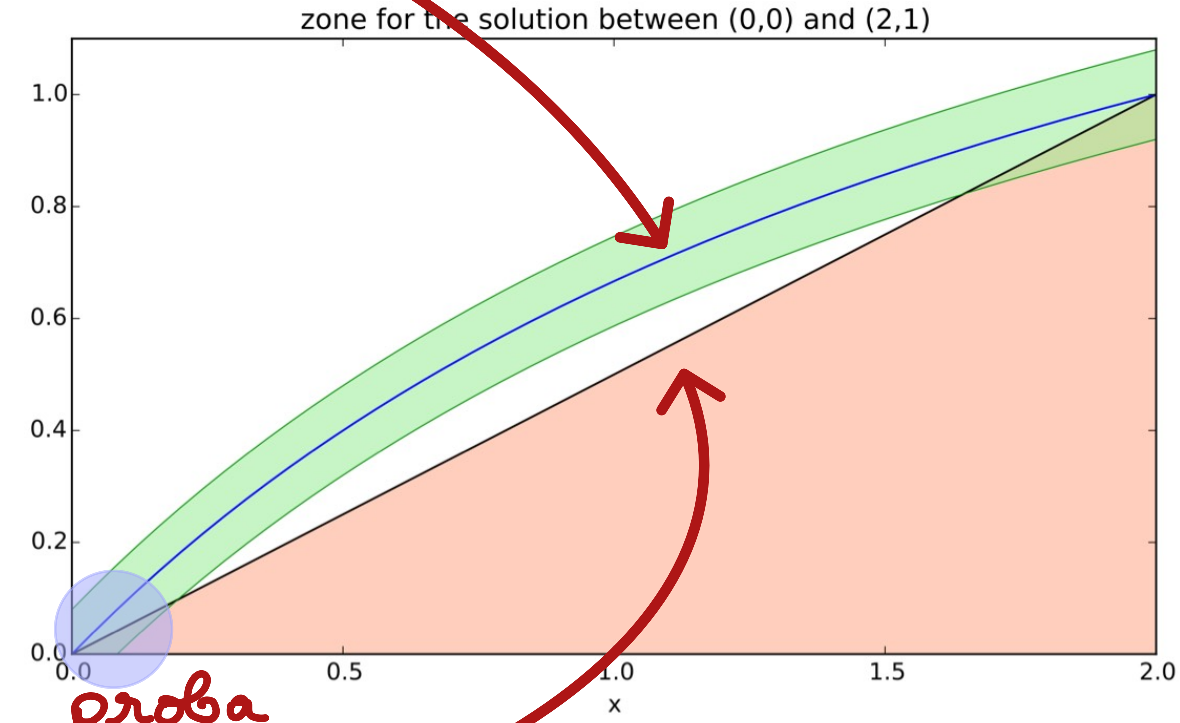
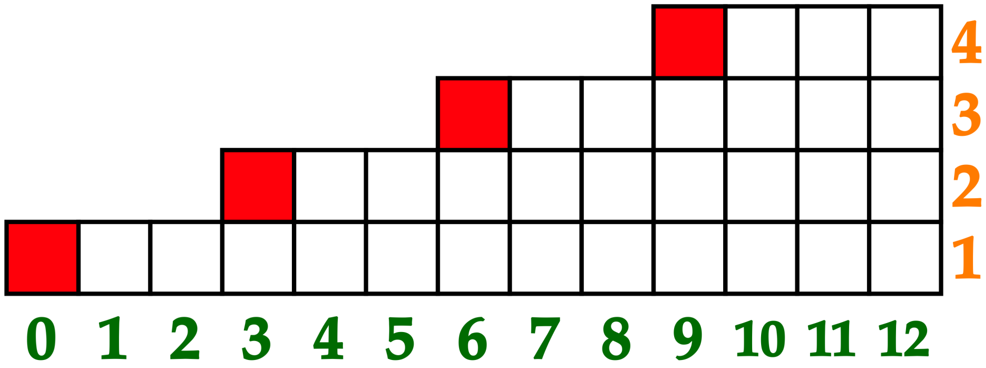




proba goes to 0



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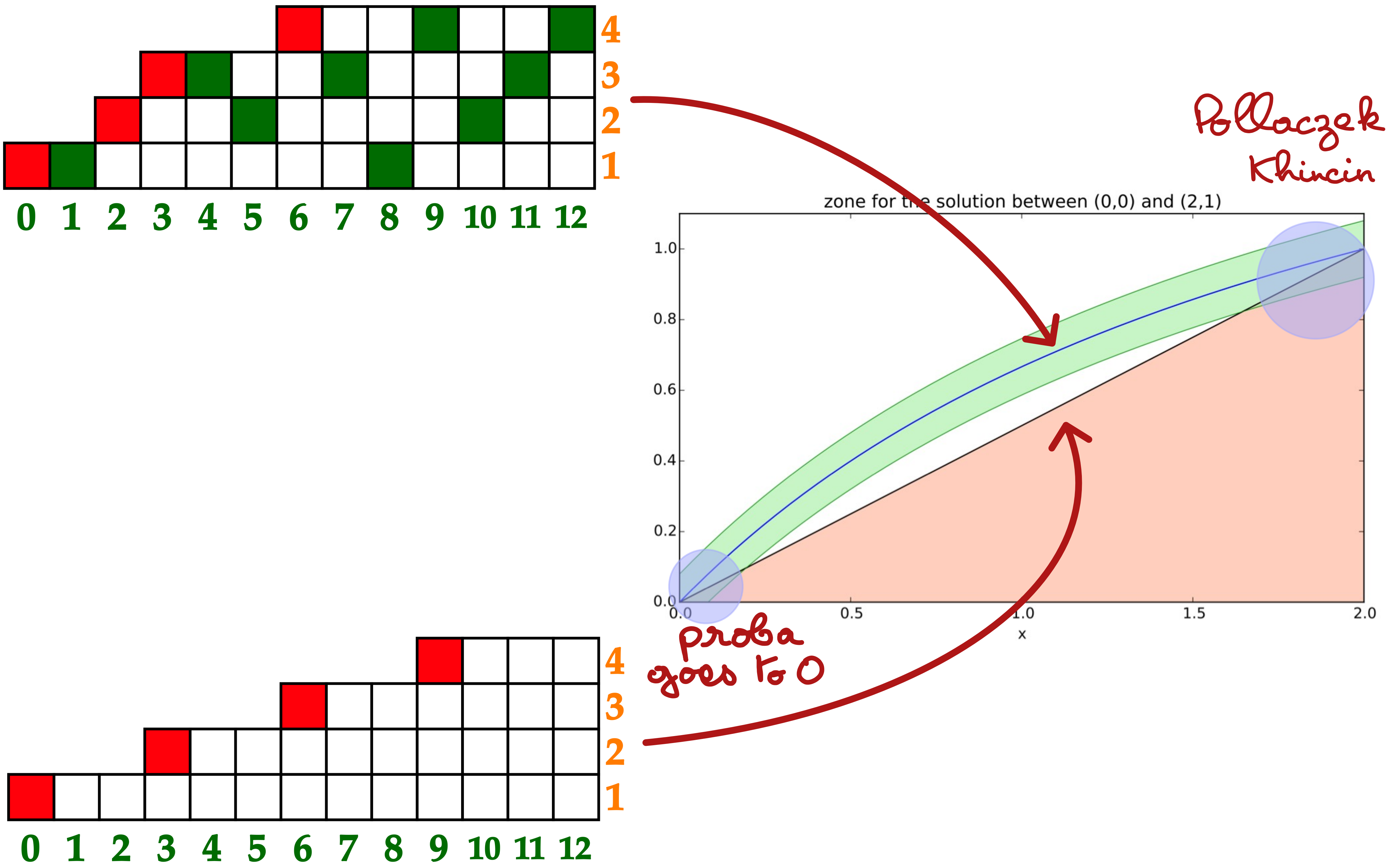

Asymptotics for $\left\{ {m \atop \ell} \right\}$

$$\left\{ {m \atop \ell} \right\} \sim \psi(m,\ell) = \frac{m!(e^{\xi}-1)^{\ell}}{\ell!\,\xi^{m}\sqrt{2\pi m\left(1-\frac{m}{\ell}e^{-\xi}\right)}}.$$

Good 1961

AN ASYMPTOTIC FORMULA FOR THE DIFFERENCES OF THE POWERS AT ZERO

By I. J. Good

Admiralty Research Laboratory, Teddington, England

solved by the above iterative method, thus $\eta_{m+1} = (1+\kappa)\log(1+\eta_m)$. Table II lists the values of $\eta + 1 = (1-\rho)^{-1}$, where ρ is the solution of equation (15).

I am indebted to the Admiralty for permission to publish this paper, and for the use of the computer.

REFERENCES

[1] R. A. Buckingham, *Numerical Methods*, Pitman, London, 1957.
[2] H. E. Daniels, "Saddlepoint approximations in statistics," *Ann. Math. Stat.*, Vol. 25 (1954), pp. 631–650.

Thanks !