

Markov chain Monte Carlo methods have become ubiquitous across science and engineering to model dynamics and explore large combinatorial sets. Over the last 20 years there have been tremendous advances in the design and analysis of efficient sampling algorithms for this purpose. One of the striking discoveries has been the realization that many natural Markov chains undergo phase transitions, whereby they abruptly change from being efficient to inefficient as some parameter of the system is modified. Generating functions can offer an alternative approach to sampling and they play a role in showing when certain Markov chains are efficient or not. We will explore the interplay between Markov chains, generating functions, and phase transitions for a variety of combinatorial problems, including graded posets, Boltzmann sampling, and 3-colorings on \mathbf{Z}^2 .