The impatient collector

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In the coupon collector problem with $n$ items, the collector needs a random number of tries $T_n \simeq n \ln n$ to complete the collection. Also, after $nt$ tries, the collector has secured approximately a fraction $\zeta_\infty(t) = 1-e^{-t}$ of the complete collection, so we call $\zeta_\infty$ the (asymptotic) *completion curve*. In this paper, for $\nu > 0$, we address the asymptotic shape $\zeta(\nu, .)$ of the completion curve under the condition $T_n \leq (1+\nu)\, n$, i.e. assuming that the collection is *completed unlikely fast*. As an application to the asymptotic study of complete accessible automata, we provide a new derivation of a formula due to Koršunov.