

NONLINEAR INTERPOLATION, DIVIDE-AND-CONQUER RECURRENCES, AND FOURIER EXPANSIONS

HWANG HSIEN-KUEI

We study recurrences of the form $f(n) = af(\lfloor n/2 \rfloor) + bf(n - \lfloor n/2 \rfloor) + g(n)$ with $a, b, g(n)$ and initial terms of $f(n)$ given. Such recurrences appear often in numeration systems, combinatorial sequences, analysis of computer algorithms, cellular automata and related areas. We show that the solution satisfies, under very general conditions, the simple IDENTITY $f(n) = n^r P(\log_2 n) + Q(n)$ where P is a continuous periodic function and $r = \log_2(a + b)$. This identity is itself an asymptotic expansion.