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# Deligne-Hitchin moduli spaces and harmonic maps

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Given a compact Riemann surface  $\Sigma$  and the Lie group  $G = SU(2)$ , Hitchin's self-duality equations describe equivariant harmonic maps from  $\Sigma$  into hyperbolic 3-space. The moduli space of solutions admits a natural hyper-Kähler structure, and its twistor space has a complex analytic reincarnation – the Deligne-Hitchin moduli space of  $\lambda$ -connections on  $\Sigma$  (glued with  $\lambda$ -connections on the complex conjugate surface  $\bar{\Sigma}$ ). After a mild introduction I will show that associated families of (equivariant) harmonic maps into symmetric spaces (related to  $G_{\mathbb{C}} = SL(2, \mathbb{C})$  and its real forms) can be considered as holomorphic sections of the Deligne-Hitchin moduli space with certain real symmetries. I show that this description yields a unified treatment of the spectral curve theory for tori, the DPW method and other integrable systems methods for harmonic maps. In the second part of my talk, I will focus on a special class of real holomorphic sections which are related to Willmore surfaces of Babich-Bobenko type – those which are minimal in their intersection with hyperbolic 3-space. As an application, I will indicate the classification of isothermic constrained Willmore tori with Willmore energy below  $8\pi$ . The talk is based on collaborations with F. Beck, I. Biswas, L. Heller, C. Ndiaye and M. Röser.

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