

# View-based query processing

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# Introduction

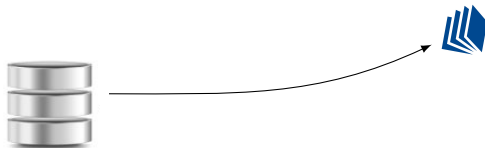
# View-based query processing

General setting



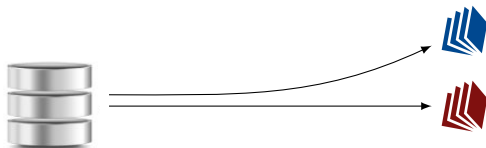
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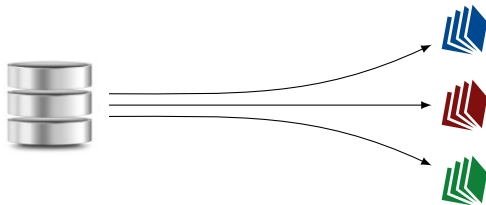
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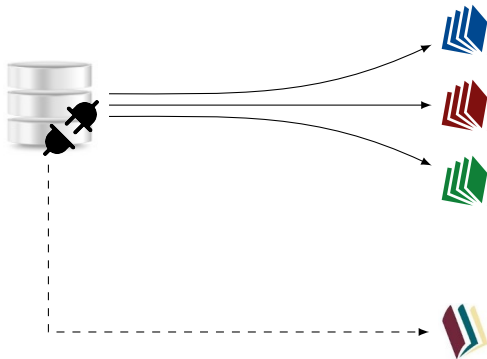
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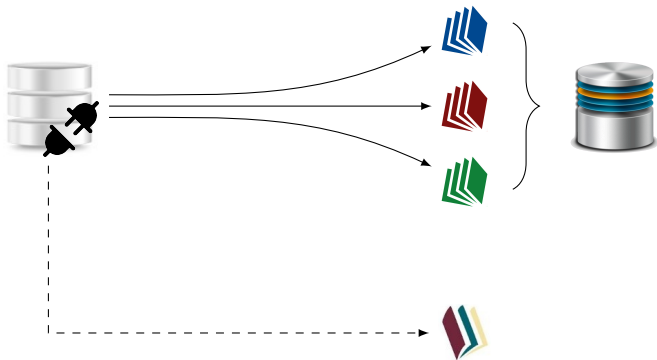
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General setting



# View-based query processing

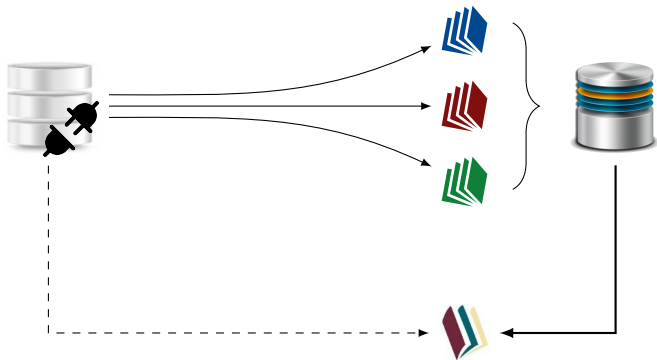
General setting





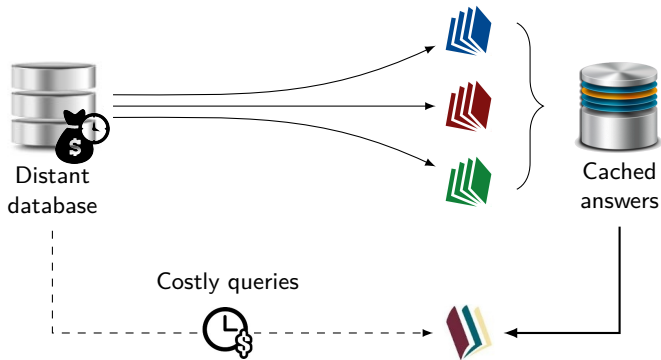
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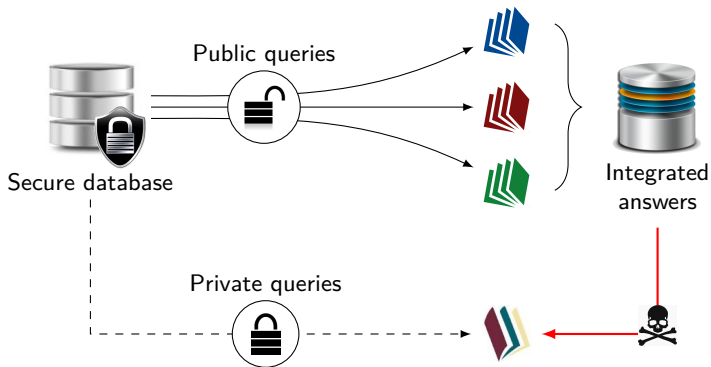
# View-based query processing

Scenario : query optimization and caching



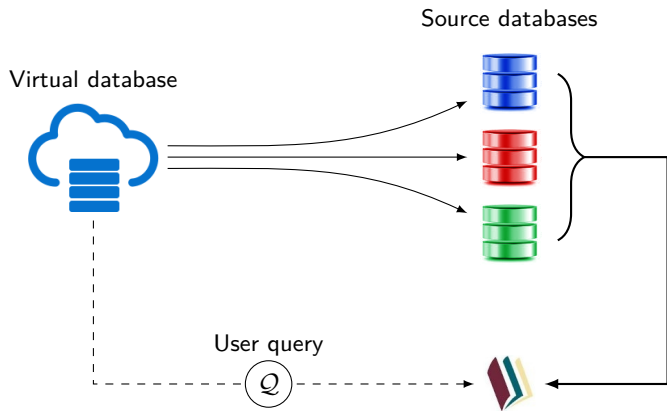
# View-based query processing

Scenario : data leak prevention



# View-based query processing

Scenario : virtual data integration



# Views

# View definition, view image, view instance

Let  $\sigma$  and  $\tau$  be two database schemas.

- **View definition** (or simply **view**): A view definition  $\mathbf{V}$  from  $\sigma$  to  $\tau$  is a set of queries over  $\sigma$  indexed by  $\tau$ :

$$\mathbf{V} = \{Q_b \mid b \in \tau\}$$

such that:

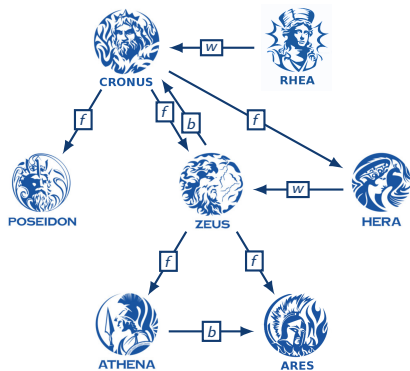
$$\forall b \in \tau, \text{arity}(b) = \text{arity}(Q_b)$$

- **View instance**: A view instance  $E$  is a database over  $\tau$ .
- **View image**: Given a database  $D$  over  $\sigma$  and a view  $\mathbf{V}$  from  $\sigma$  to  $\tau$ , the view image of  $D$ ,  $\mathbf{V}(D)$ , is a view instance such that:

$$\forall b \in \tau, \bar{x} \in D, \quad \bar{x} \in b(\mathbf{V}(D)) \Leftrightarrow \bar{x} \in Q_b(D)$$

# Example: view definition, view image, view instance

$$\sigma = \{b, f, w\}$$

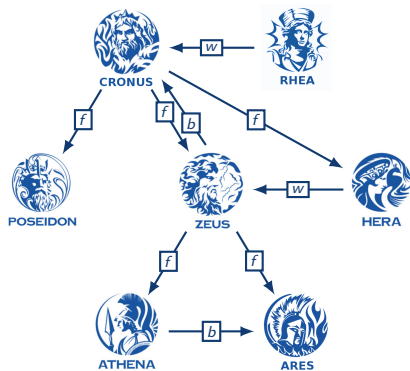


Database  $D$  over  $\sigma$

# Example: view definition, view image, view instance

$$\sigma = \{b, f, w\}$$

$$\tau = \{g, s\}$$

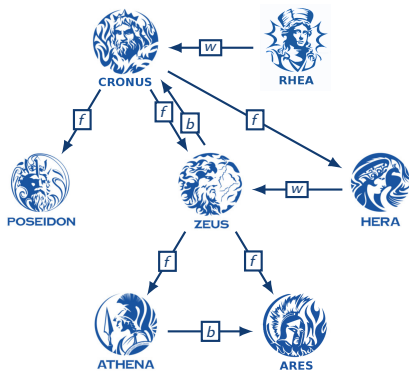


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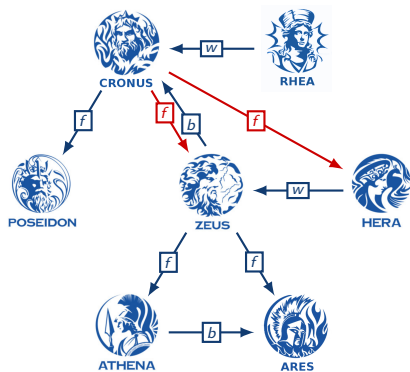
$$\sigma = \{b, f, w\} \quad \mathbf{V} = \left\{ \begin{array}{l} Q_g(x, y) = x \xrightarrow{w} z \xrightarrow{f} z' \xrightarrow{f} y \\ Q_s(x, y) = x \xleftarrow{f} z \xrightarrow{f} y \wedge x \neq y \end{array} \right\} \quad \tau = \{g, s\}$$



Database  $D$  over  $\sigma$

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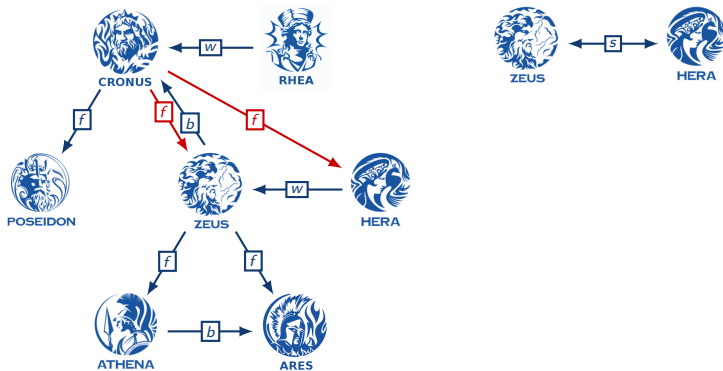
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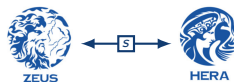
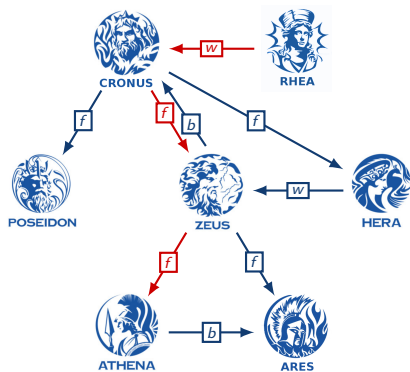
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Database  $D$  over  $\sigma$

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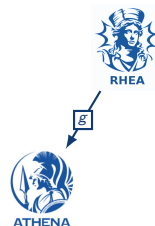
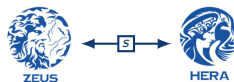
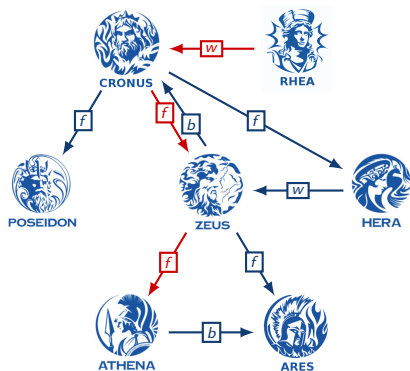
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Database  $D$  over  $\sigma$

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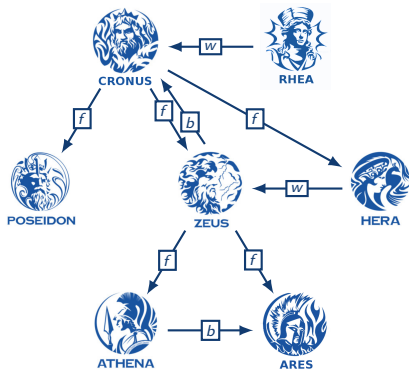
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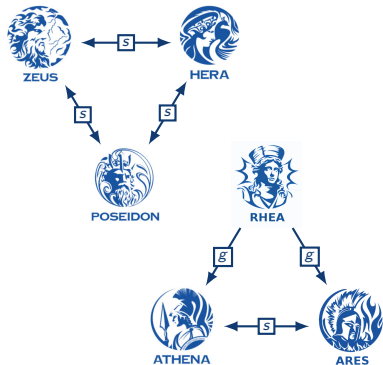
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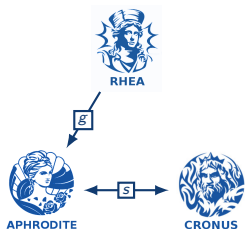
Database  $D$  over  $\sigma$



View image  $\mathbf{V}(D)$  over  $\tau$

# Example: view definition, view image, view instance

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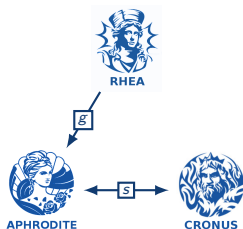


View instance  $E$  over  $\tau$

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Is  $E$  the image of some database  $D$  through  $\mathbf{V}$ ?



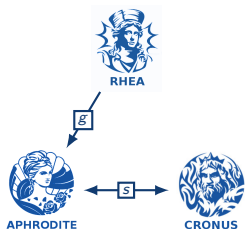
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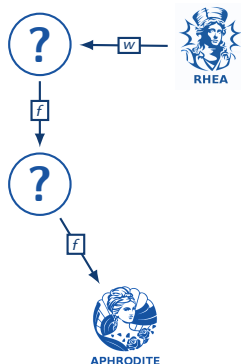
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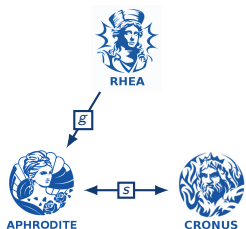
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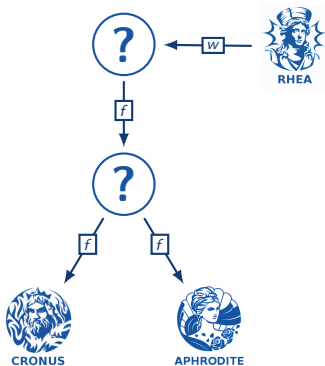
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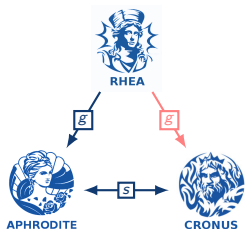
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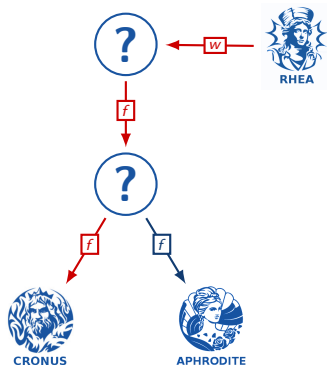
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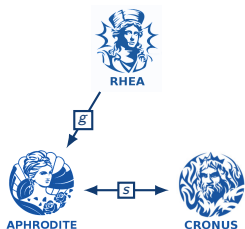
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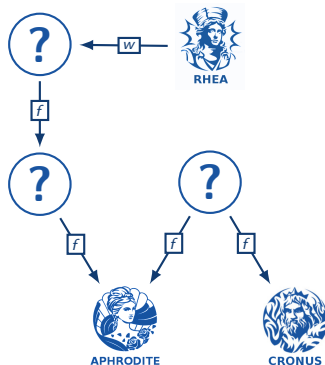
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View instance  $E$  over  $\tau$



# What happened there?

## ■ Myth 1:

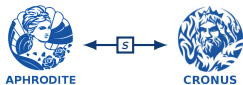


## ■ Myth 2:



# What happened there?

## ■ Myth 1:



( *because Aphrodite is the daughter of Uranus*  
*And so is Cronus* )

## ■ Myth 2:



# What happened there?

## ■ Myth 1:



( *because Aphrodite is the daughter of Uranus*  
*And so is Cronus* )

## ■ Myth 2:



( *because Aphrodite is the daughter of Zeus*  
*And Zeus is the son of Rhea* )

# What happened there?

## ■ Myth 1:



( *because Aphrodite is the daughter of Uranus*  
*And so is Cronus* )

## ■ Myth 2:



( *because Aphrodite is the daughter of Zeus*  
*And Zeus is the son of Rhea* )

We failed **virtual data integration** → the two myths are **incompatible**



# Testing view consistency

PROBLEM : VIEW CONSISTENCY

INPUT : A view  $\mathbf{V}$  from  $\sigma$  to  $\tau$ , a view instance  $E$

QUESTION : Is there some  $D$  over  $\sigma$  such that  $\mathbf{V}(D) = E$ ?

# Testing view consistency

PROBLEM	:	VIEW CONSISTENCY FOR LANGUAGE $\mathcal{L}$
INPUT	:	An $\mathcal{L}$ -view $\mathbf{V}$ from $\sigma$ to $\tau$ , a view instance $E$
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# Testing view consistency

## ■ Combined complexity:

PROBLEM	:	VIEW CONSISTENCY FOR LANGUAGE $\mathcal{L}$
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## ■ Data complexity:

Let  $\mathbf{V}$  be a fixed view from  $\sigma$  to  $\tau$  in some language  $\mathcal{L}$ :

PROBLEM	:	VIEW CONSISTENCY( $\mathbf{V}$ )
INPUT	:	A view instance $E$
QUESTION	:	Is there some $D$ over $\sigma$ such that $\mathbf{V}(D) = E$ ?

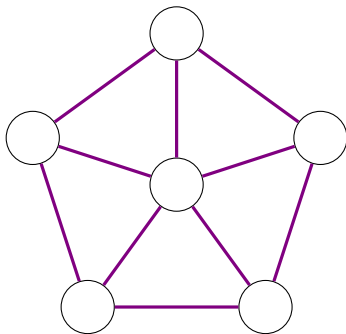
## Example: how hard is testing consistency?

$$\sigma = \{c, e, p\} \qquad \mathbf{v} = \left\{ \begin{array}{c} \\ \\ \end{array} \right\}$$

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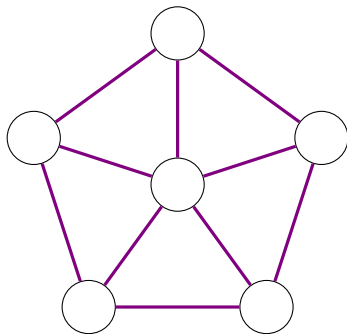
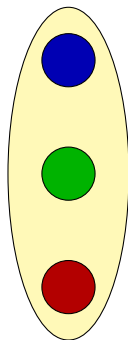
$$\mathbf{v} = \left\{ \begin{array}{c} Q_{\text{edge}}(x, y) = e(x, y) \end{array} \right\}$$



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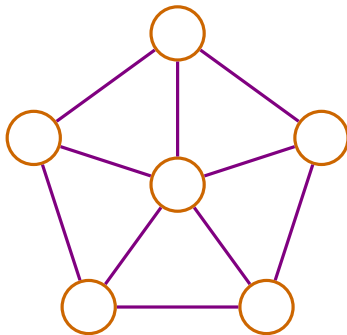
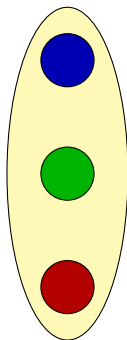
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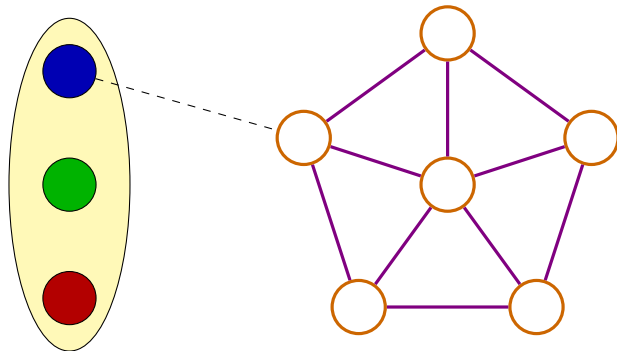
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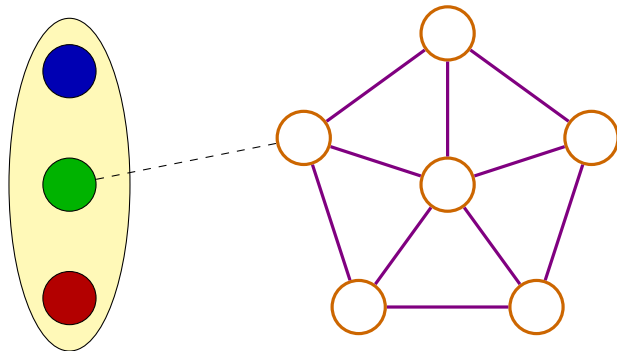
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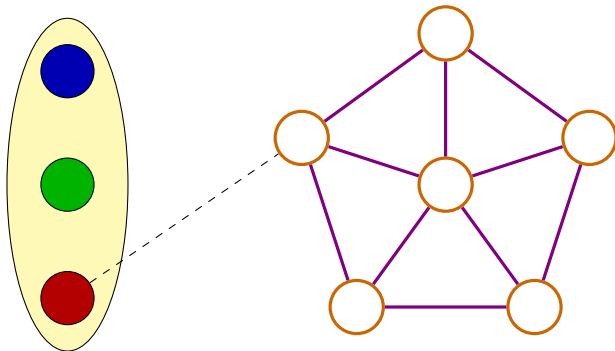
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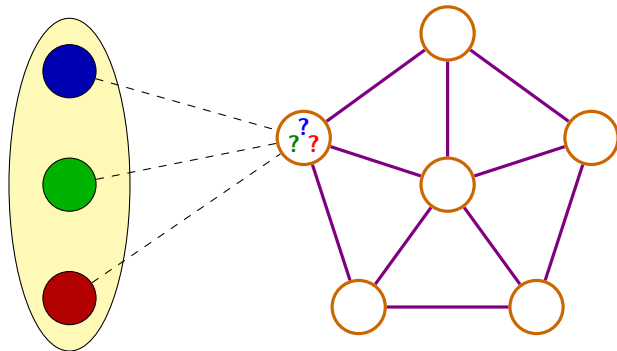
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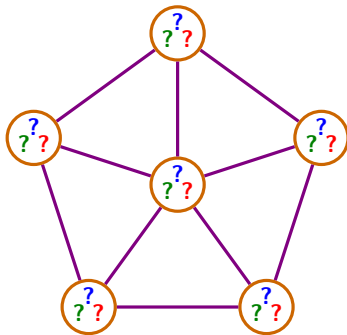
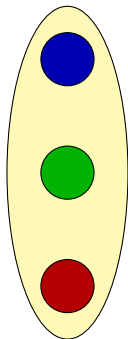
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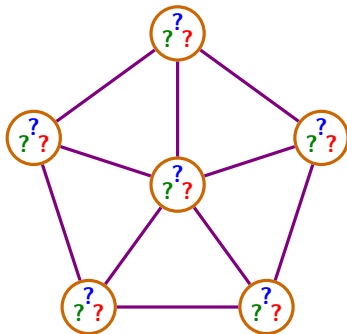
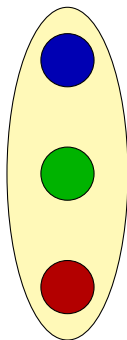
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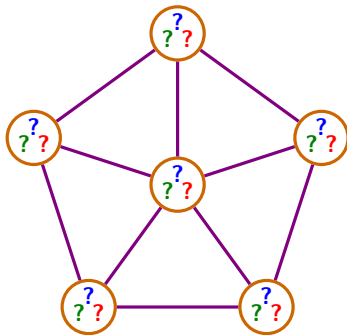
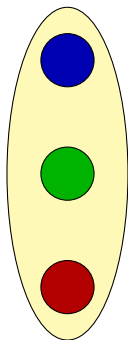
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# Example: how hard is testing consistency?

$$\sigma = \{c, e, p\}$$

$$\mathbf{v} = \left\{ \begin{array}{l} Q_{\text{edge}}(x, y) = e(x, y) \\ Q_{\text{palette}}(x) = p(x) \\ Q_{\text{color}}(x) = \exists z \cdot p(z) \wedge c(x, z) \\ Q_{\text{error}}(x, y) = \exists z \cdot c(x, z) \wedge c(y, z) \wedge e(x, y) \end{array} \right\}$$

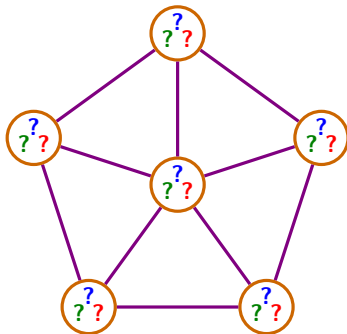
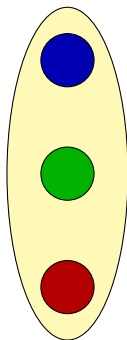


$Q_{\text{error}}$  should show if there is a coloring error... but it is empty.

## Example: how hard is testing consistency?

$$\sigma = \{c, e, p\}$$

$$\mathbf{v} = \left\{ \begin{array}{l} Q_{\text{edge}}(x, y) = e(x, y) \\ Q_{\text{palette}}(x) = p(x) \\ Q_{\text{color}}(x) = \exists z \cdot p(z) \wedge c(x, z) \\ Q_{\text{error}}(x, y) = \exists z \cdot c(x, z) \wedge c(y, z) \wedge e(x, y) \end{array} \right\}$$



$Q_{\text{error}}$  should show if there is a coloring error... but it is empty.

The view instance is consistent iff the **graph** is 3-colorable!

# How hard is testing consistency?

Short answer: it's **hard**, even for simple languages and in data complexity.



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There exists a fixed view **V** defined using **conjunctive** queries such that VIEW CONSISTENCY(**V**) is **NP-complete**.

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The problem quickly becomes **undecidable** for more expressive languages.

## Theorem [Abiteboul, Duschka]

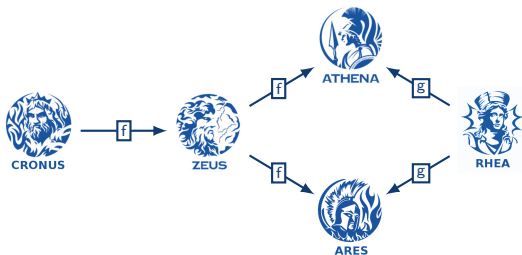
There exists a fixed view  $\mathbf{V}$  defined using **Datalog** queries such that VIEW CONSISTENCY( $\mathbf{V}$ ) is **undecidable**.

*This also holds for context-free path queries, first-order queries...*

## Certain answers

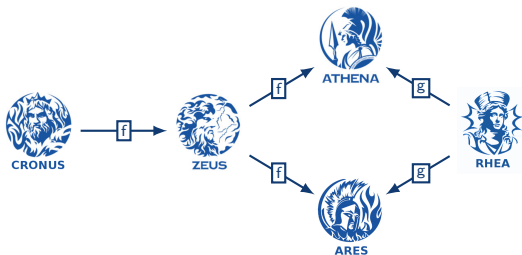
## Example: answering queries using views

$$\sigma = \{b, f, w\} \quad \mathbf{V} = \left\{ \begin{array}{l} Q_g(x, y) = x \xrightarrow{w} z \xrightarrow{f} z' \xrightarrow{f} y \\ Q_f(x, y) = x \xrightarrow{f} y \end{array} \right\} \quad \tau = \{f, g\}$$



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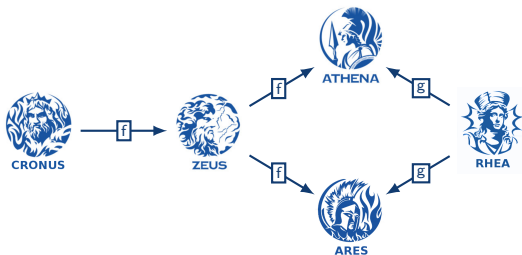
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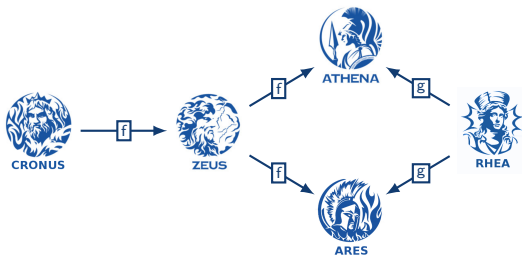


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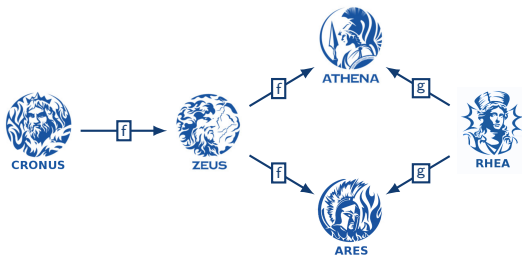


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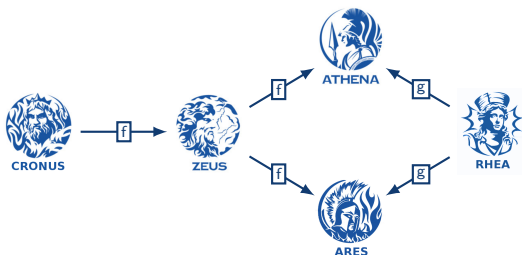
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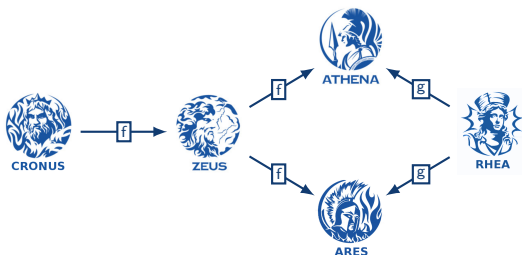


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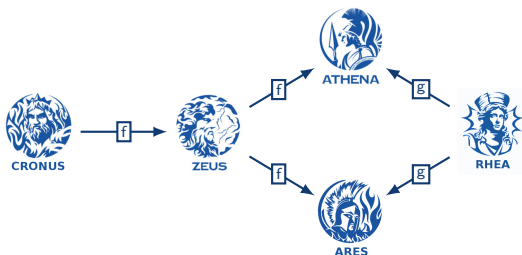


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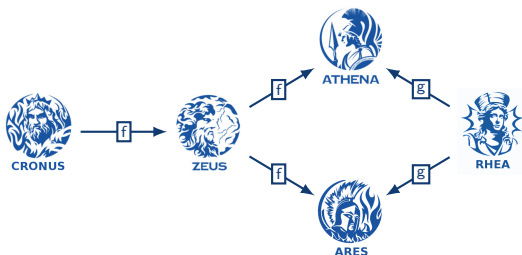


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→ Cronus is a candidate; could there be another?

# Answering queries using views

Given:  $\left\{ \begin{array}{l} \mathbf{V} : \text{view from } \sigma \text{ to } \tau \\ E : \text{view instance over } \tau \\ Q : \text{query over } \sigma \end{array} \right.$

- Certain answers:

$$\text{cert}_{Q,\mathbf{V}}(E) = \bigcap_{D \mid \mathbf{V}(D)=E} Q(D)$$

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# The problem(s) of computing certain answers

PROBLEM : CERTAIN ANSWERS

INPUT : A view  $\mathbf{V}$  from  $\sigma$  to  $\tau$ ,  
A query  $Q$  over  $\sigma$ ,  
A view instance  $E$  and  $\bar{u} \in E$

QUESTION :  $\bar{u} \in \text{cert}_{Q,\mathbf{V}}(E)$ ?

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# The problem(s) of computing certain answers

## ■ Combined complexity:

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## ■ Data complexity:

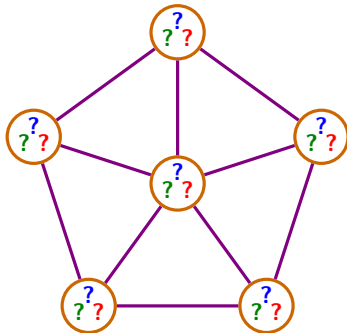
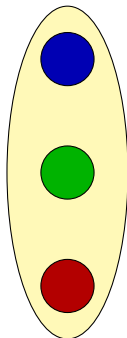
Let  $\mathbf{V}$  be a fixed view from  $\sigma$  to  $\tau$  and  $Q$  be a fixed query over  $\sigma$ :

PROBLEM	:	CERTAIN ANSWERS( $Q, \mathbf{V}$ )
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# Example: how hard is computing certain answers?

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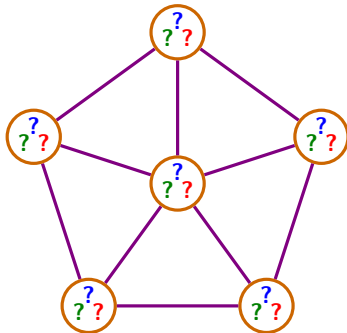
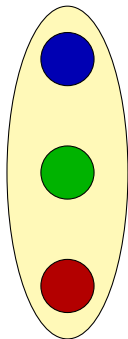
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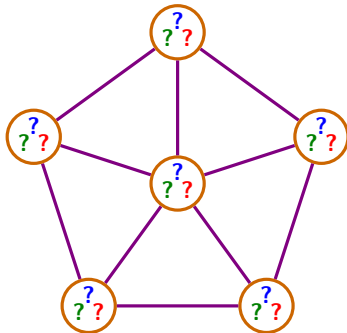
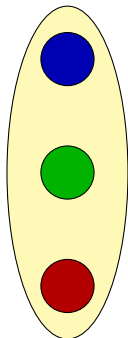


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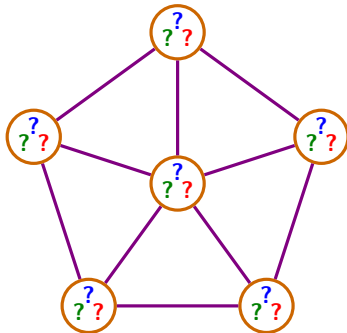
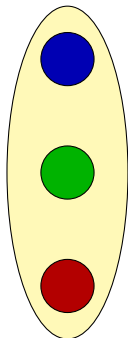
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- **Sound view:** we can always invent more colors!

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Some results from [Abiteboul, Duschka'98]:

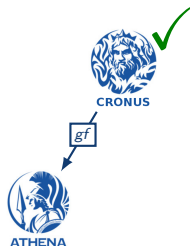
query \ view	CQ	CQ $\neq$	Datalog	FO
CQ	P <del>TIME</del> /coNP	coNP	P <del>TIME</del> /coNP	Undec.
CQ $\neq$	P <del>TIME</del> /coNP	coNP	P <del>TIME</del> /coNP	Undec.
Datalog	coNP/ <del>Undec.</del>	Undec.	Undec.	Undec.
FO	Undec.	Undec.	Undec.	Undec.

Complexity of answering queries using **sound** or **exact** views.

## Determinacy and rewriting

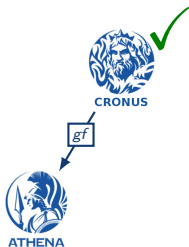
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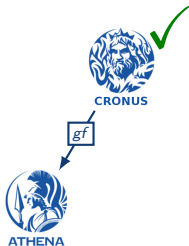
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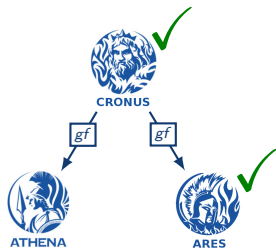
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And nothing else: possible and certain answers coincide.

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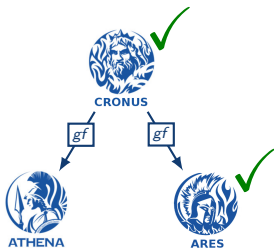


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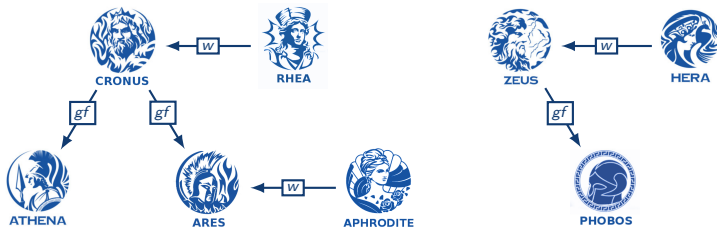
There is no way to match husbands and wives...  
Nothing is certain and every match is possible.

## Example: determinacy and rewriting (2)

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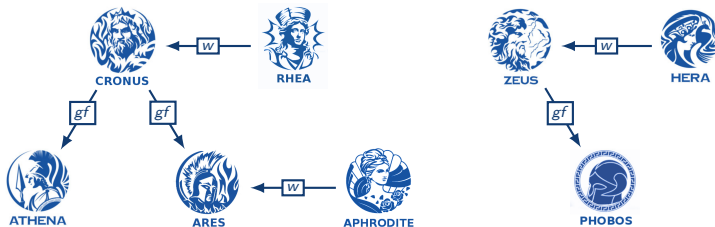
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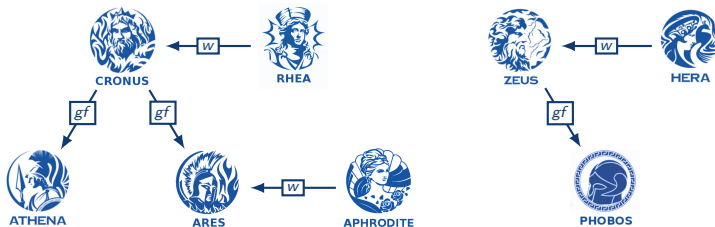
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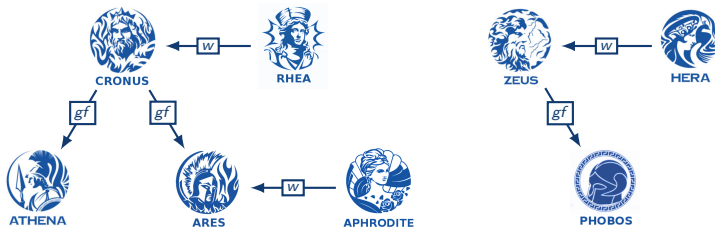


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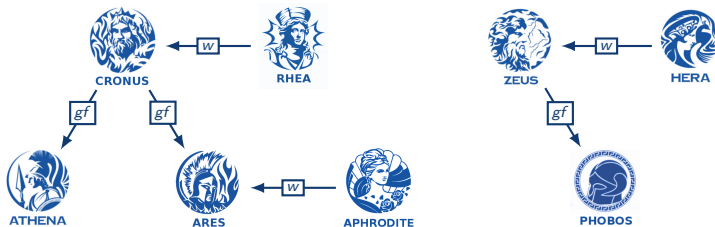


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- Better yet: this is a **static** property of  $\mathbf{V}$  and  $Q$ .

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- Better yet: this is a **static** property of  $\mathbf{V}$  and  $Q$ .
- $Q$  can be **rewritten** as  $R(x, y) = x \xrightarrow{w} z \xrightarrow{gf} y$  over  $\tau$ .

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Thus possible and certain answers coincide both with  $Q(D)$ .

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The existence of a rewriting immediately implies that  $\mathbf{V} \twoheadrightarrow Q$ .

## Example: proving non-determinacy

$$\sigma = \{a\} \qquad \mathbf{V} = \left\{ Q_{a_3}(x, y) = x \xrightarrow{a} z \xrightarrow{a} z' \xrightarrow{a} y \right\} \qquad \tau = \{a_3\}$$

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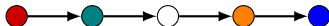
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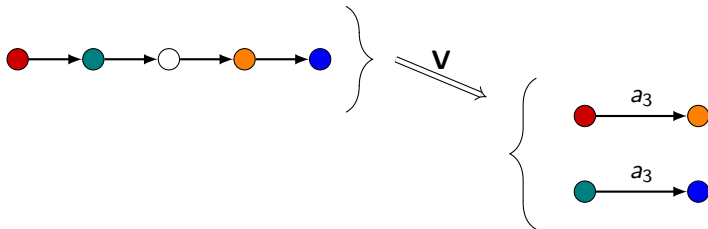
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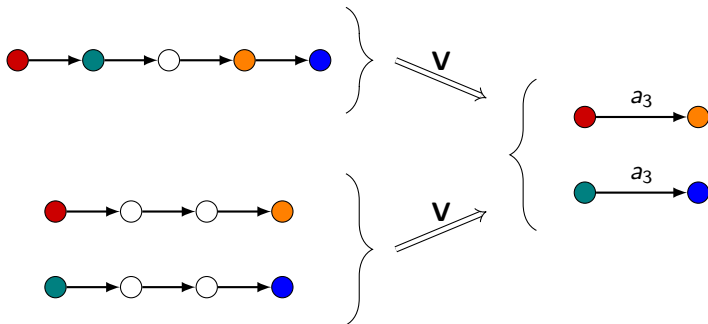
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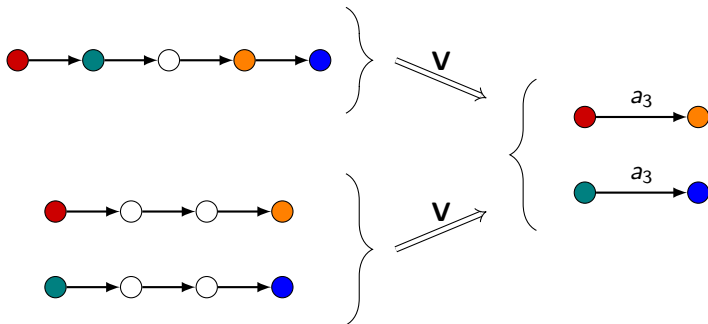
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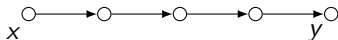
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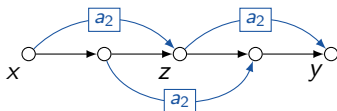
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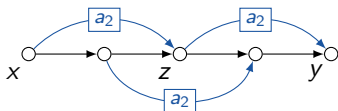
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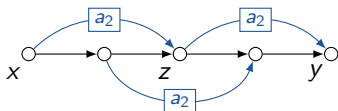
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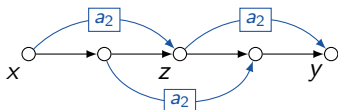
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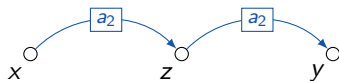
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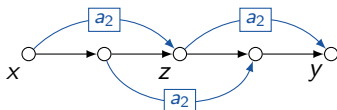
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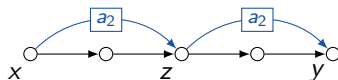
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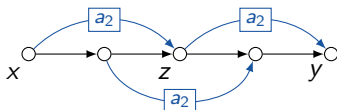
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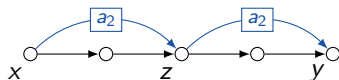
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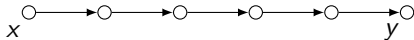
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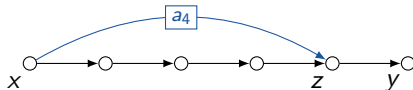
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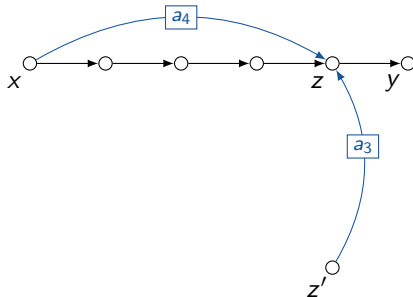
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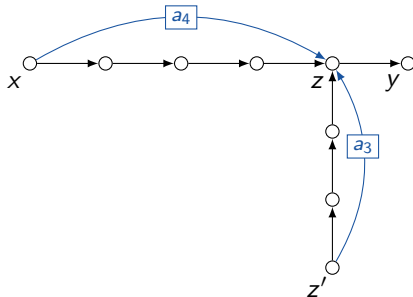
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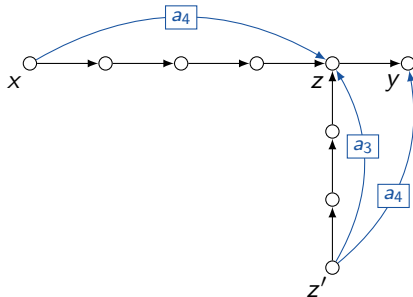
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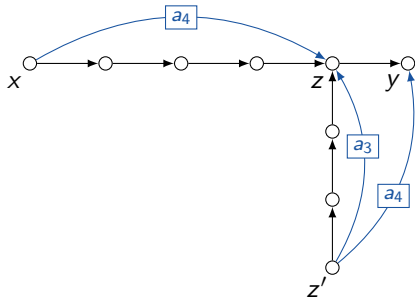
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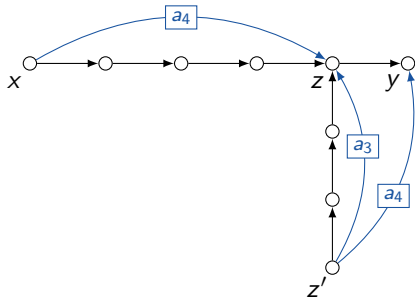
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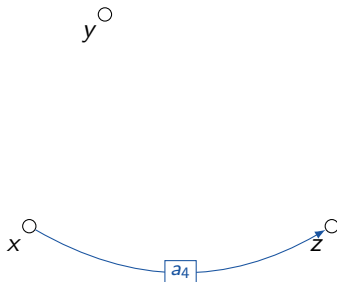
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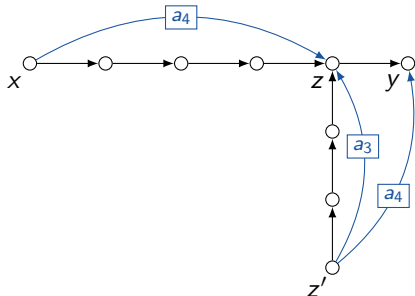
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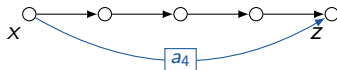
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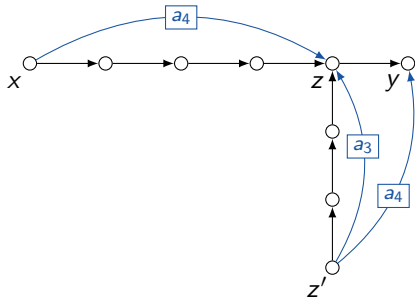


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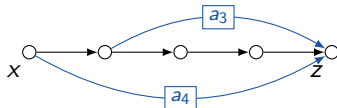
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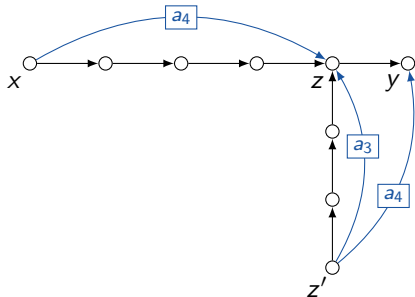
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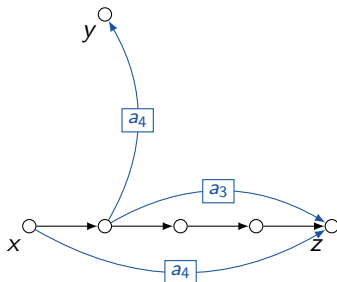
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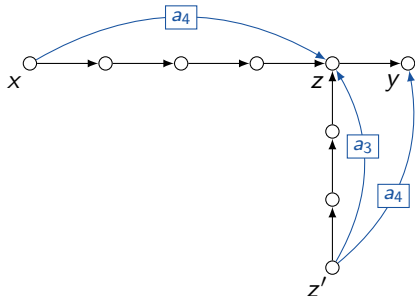


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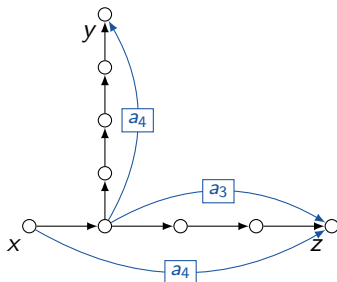
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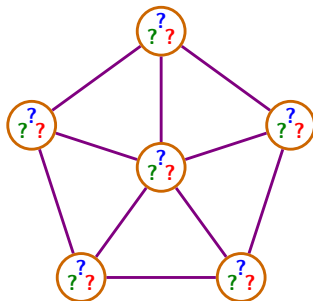
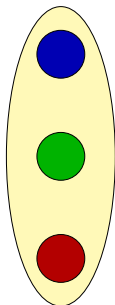
Rewritings can differ in **behavior** and **complexity** outside of view images.



# Example: different rewritings of varying complexity

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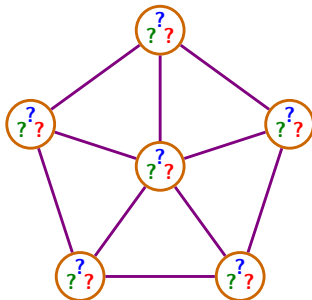
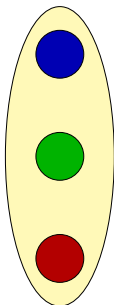
$$\mathbf{v} = \left\{ \begin{array}{l} Q_{\text{edge}}(x, y) = e(x, y) \\ Q_{\text{palette}}(x) = p(x) \\ Q_{\text{color}}(x) = \exists z \cdot p(z) \wedge c(x, z) \\ Q_{\text{error}}(x, y) = \exists z \cdot c(x, z) \wedge c(y, z) \wedge e(x, y) \end{array} \right\}$$



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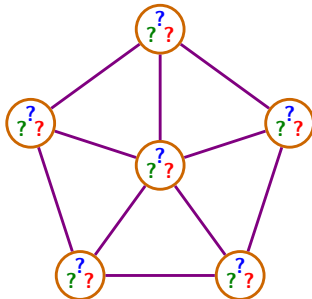
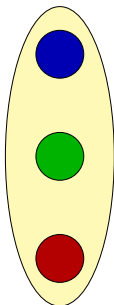


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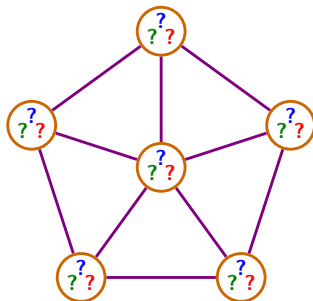
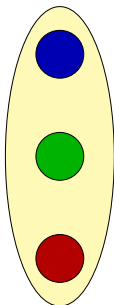
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(check if the graph is 3-colorable)

$R_2(x) = \text{palette}(x) \wedge \text{error}()$   
(trust the view instance)

## Some problems around determinacy and rewritings

PROBLEM	:	DETERMINACY FOR LANGUAGES $\mathcal{L}$ AND $\mathcal{L}'$
INPUT	:	An $\mathcal{L}$ -view $\mathbf{V}$ and an $\mathcal{L}'$ -query $Q$
QUESTION	:	Does $\mathbf{V} \twoheadrightarrow Q$ ?

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QUESTION	:	Is there a rewriting of $Q$ using $\mathbf{V}$ satisfying $\mathcal{P}$ ?

# Some problems around determinacy and rewritings

PROBLEM	:	DETERMINACY FOR LANGUAGES $\mathcal{L}$ AND $\mathcal{L}'$
INPUT	:	An $\mathcal{L}$ -view $\mathbf{V}$ and an $\mathcal{L}'$ -query $Q$
QUESTION	:	Does $\mathbf{V} \twoheadrightarrow Q$ ?

PROBLEM	:	$\mathcal{P}$ -REWRITING FOR LANGUAGES $\mathcal{L}$ AND $\mathcal{L}'$
INPUT	:	An $\mathcal{L}$ -view $\mathbf{V}$ and an $\mathcal{L}'$ -query $Q$ st $\mathbf{V} \twoheadrightarrow Q$
QUESTION	:	Is there a rewriting of $Q$ using $\mathbf{V}$ satisfying $\mathcal{P}$ ?

## Example:

- Is there a rewriting that can be expressed in first-order logic?
- Is there a rewriting with PTIME evaluation complexity?
- Is there a rewriting that is monotone?

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Some results over graphs:

- [Gluch et al'19]: Determinacy is undecidable for finite RPQs.
- [F., Segoufin, Sirangelo'15]: Monotone rewritings of RPQ queries using RPQ views can be expressed in Datalog.  
(Existence is EXPSpace-complete, using [Calvanese et al'02])

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## QUESTION 3

Is determinacy decidable for chain queries and disjunctive chain views?

One last example

## Example: disjunctive chain queries – the Chase

$$\sigma = \{a\}$$
$$\tau = \{(2), (1, 2)\}$$

$$\mathbf{v} = \left\{ \begin{array}{l} Q_2(x, y) = x \xrightarrow{a^2} y \\ Q_{1,2}(x, y) = (x \xrightarrow{a} y) \vee (x \xrightarrow{a^2} y) \end{array} \right\}$$

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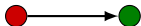
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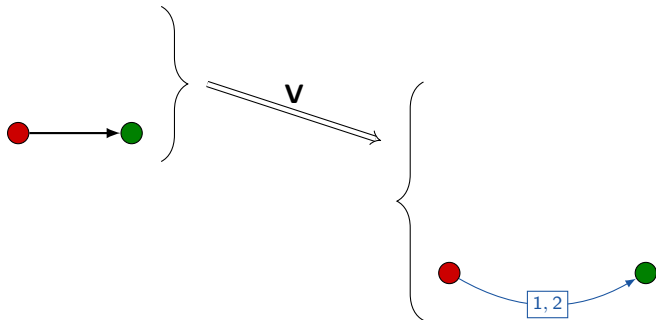


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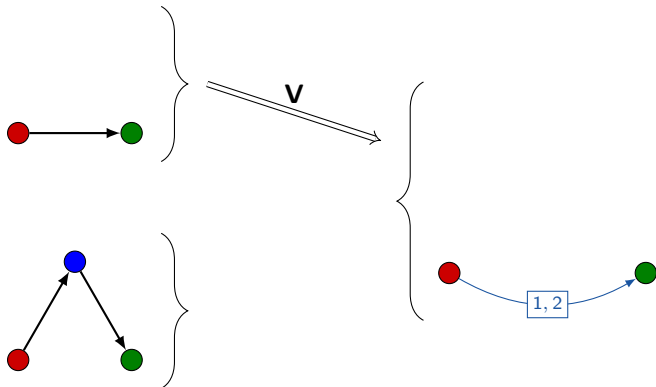


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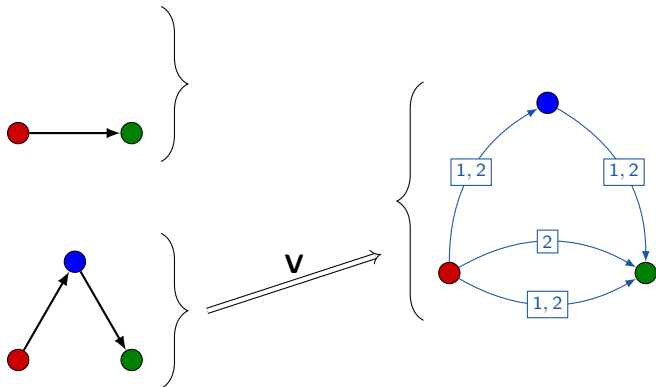
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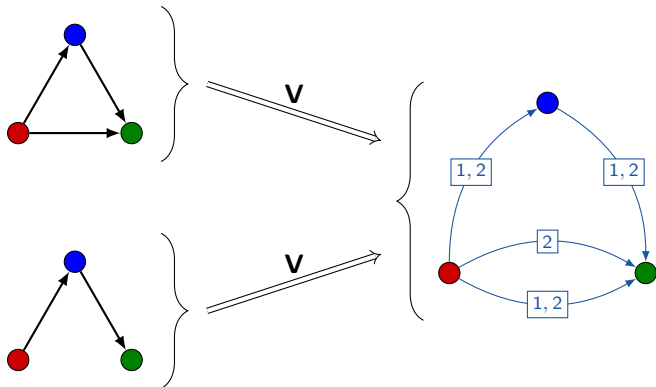
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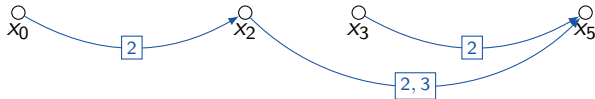
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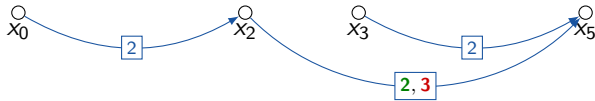
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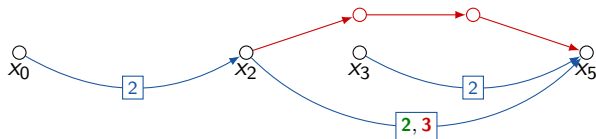
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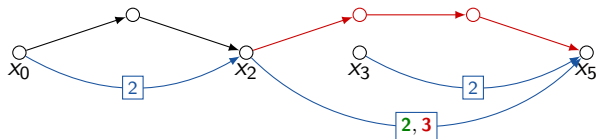
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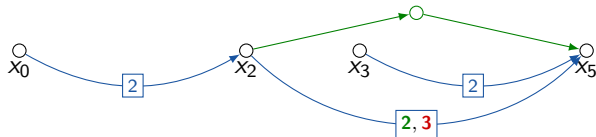
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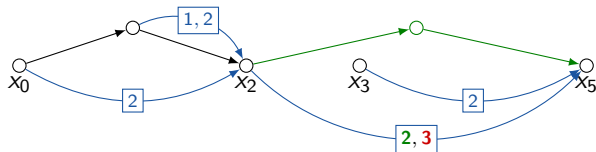
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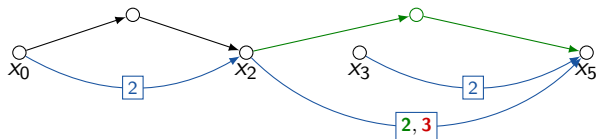
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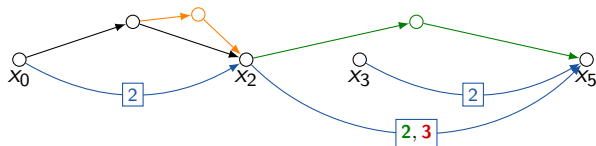
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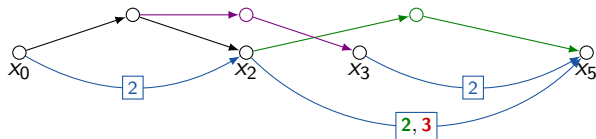
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If you think you have an elegant proof, come talk to me!

## Announcement

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THANK YOU!