ITERATED WEAK INVARIANCE PRINCIPLE
FOR
SLOWLY MIXING DYNAMICAL SYSTEMS

ASSUMPTIONS

\((\Lambda, \mathcal{A}, \mu)\) \hspace{1cm} \text{PROBABILITY SPACE}

\(T : \Lambda \rightarrow \Lambda\) \hspace{1cm} \text{ENDOMORPHISM}

\(\text{(TRANSFER OPERATOR } P)\)

\(v \in L^\infty(\Lambda, \mathbb{R}^d), \int v \, d\mu = 0\) \hspace{1cm} \text{OBSERVABLES}

\(\sum_{n \geq 1} \|P^n v\|_1 < \infty\) \hspace{1cm} \text{MIXING}

QUESTION

If \(W_n \rightarrow_w W\) in \(D([0, \infty), \mathbb{R}^d)\)
then does it follow that \(X_n \rightarrow_w X\)?

ANSWER

\(d = 1\): \hspace{1cm} \text{YES [WZ65], STRATONOVICH}

\(d > 1\): \hspace{1cm} \text{NOT ALWAYS}

THE PROBLEM

\(W_n \rightarrow_w W \implies \int W_n^i \circ dW_n^i \rightarrow_w \int W^i \circ dW^i\)

THEOREM

\(W_n, W : [0, \infty) \rightarrow \mathbb{R}^{d \times d}\)

\(W_n^{ij}(t) := \int_0^t W_n^i \circ dW_n^j\)

\(W^{ij}(t) := \int_0^t W^i \circ dW^j + t \sum_{r \geq 1} \int v^i v^j \circ T^r \, d\mu\)

\((W_n, W_n) \rightarrow_w (W, W) \text{ in } D\)

REMARKS

- The invertible case is also true
- This recovers the WIP of [DR00] and the CLT of [Liv95]
- [KM16] has this result under the stronger mixing assumption

\[\sum_{n \geq 1} \|P^n v\|_2 < \infty\]

OBJECTIVE

Which conditions guarantee that the ODEs

\[dX_n = f(X_n)dt + g(X_n)dW_n\]

converge to the SDE

\[dX = f(X)dt + g(X)dW\]

where \(g(X)dW\) is an appropriate stochastic integral

REFERENCES


