ITERATED WEAK INVARIANCE PRINCIPLE FOR SLOWLY MIXING DYNAMICAL SYSTEMS

ASSUMPTIONS

$(\Lambda, \mathcal{A}, \mu)$ PROBABILITY SPACE

$T : \Lambda \to \Lambda$ ENDOMORPHISM (TRANSFER OPERATOR P)

$v \in L^\infty(\Lambda, \mathbb{R}^d)$, $\int v \, d\mu = 0$ OBSERVABLES

$\sum_{n\geqslant 1} \|P^n v\|_1 < \infty$ MIXING

DEFINITIONS

COVARIANCE

$\Sigma \in \mathbb{R}^{d\times d}$

$\Sigma^{ij} := \int v^i v^j \, d\mu + \sum_{r\geqslant 1} \int v^i v^j \circ T^r + v^i \circ T^r v^j \, d\mu$

PROCESSES

$W_n(t) := n^{-1/2} \sum_{j=0}^{[nt]-1} v \circ T^j$

W d- DIMENSIONAL BROWNIAN MOTION WITH MEAN ZERO AND COVARIANCE Σ

SPACES

$D := D([0,\infty), \mathbb{R}^d \times \mathbb{R}^{d\times d})$ CÀDLÀG

OBJECTIVE

WHICH CONDITIONS GUARANTEE THAT THE ODEs

$$dX_n = f(X_n)dt + g(X_n)dW_n$$

CONVERGE TO THE SDE

$$dX = f(X)dt + g(X)dW$$

WHERE $g(X)dW$ IS AN APPROPRIATE STOCHASTIC INTEGRAL

QUESTION

IF $W_n \to_w W$ IN $D([0,\infty), \mathbb{R}^d)$

THEN DOES IT FOLLOW THAT $X_n \to_w X$?

ANSWER

d=1: YES [WZ65], STRATONOVICH

d > 1: NOT ALWAYS

THE PROBLEM

$$W_n \to_w W \;\not\Rightarrow\; \int W_n^i \circ dW_n^j \to_w \int W^i \circ dW^j$$

THEOREM

$W_n, \mathbb{W} : [0,\infty) \to \mathbb{R}^{d\times d}$

$\mathbb{W}_n^{ij}(t) := \int_0^t W_n^i \, dW_n^j$

$\mathbb{W}^{ij}(t) := \int_0^t W^i \, dW^j + t \sum_{r\geqslant 1} \int v^i v^j \circ T^r \, d\mu$

$$(W_n, \mathbb{W}_n) \to_w (W, \mathbb{W}) \text{ IN } D$$

REMARKS

- THE INVERTIBLE CASE IS ALSO TRUE
- THIS RECOVERS THE WIP OF [DR00] AND THE CLT OF [Liv95]
- [KM16] HAS THIS RESULT UNDER THE STRONGER MIXING ASSUMPTION $$\sum_{n\geqslant 1} \|P^n v\|_2 < \infty$$

REFERENCES

[DR00] J. Dedecker and E. Rio. On the functional central limit theorem for stationary processes. Ann. de l'I.H.P. **36**:1-34. 2000.

[KM16] D. Kelly and I. Melbourne. Smooth approximation of stochastic differential equations. Ann. Prob. **44**:479-520. 2016.

[Liv95] C. Liverani. Central limit theorem for deterministic systems. Pitman research notes in mathematics series. **19**:1030-1070. 1995.

[WZ65] E. Wong and M. Zakai. On the convergence of ordinary integrals to stochastic integrals. Ann. Math. Stat. **36**:1560-1564. 1965.

THE UNIVERSITY OF WARWICK

MATT GALTON

IAN MELBOURNE

MASDOC