Gibbs random walks in random environment

by Romain AIMINO, Universidade do Porto, Portugal

I will introduce a particular class of random processes in random environment with exponential weak memory. I will explain why they are relevant for dynamical systems and I will discuss a few of their basic ergodic properties. This is a joint work with Carlangelo Liverani.

Resonances of Nilmanifold Automorphisms, Equidistribution of Nilflows

by Oliver BUTTERLEY

We study the spectra of transfer operators associated to partially hyperbolic automorphisms on the Heisenberg nilmanifold. The spectral information is used to study the parabolic flows (called nilflows in this case) which are the ones renormalised by the given automorphism. In particular we consider the regularity of solutions to the cohomological equation and the deviation of ergodic averages. (Joint work with Lucia Simonelli.)

FCLT for toral automorphisms along the orbits of random walks

by Guy COHEN, Ben-Gurion University, Israel

Let (ζ_k) be $\mathbb{Z}^2$-valued i.i.d. random variables on $(\Omega, \mathbf{P})$, with $\mathbb{E}(\zeta_0) = 0$ and $\mathbb{E}|\zeta_0|^2 < \infty$. Let (Z_n) be the associate random walk, defined by $Z_0 = (0, 0)$ and $Z_n = \zeta_0 + \cdots + \zeta_{n-1}$. Assume that the vector space which is generated by $(\underline{\ell} : \mathbf{P}(\zeta_0 = \underline{\ell}) > 0)$ is $\mathbb{R}^2$. Let $\mathbb{Z}^2 \ni \underline{\ell} \mapsto A^{\underline{\ell}}$ be a totally ergodic $\mathbb{Z}^2$-action by automorphisms on $\mathbb{T}^\rho$, $\rho > 1$, with the Lebesgue measure (hence, defined by commuting $\rho \times \rho$ matrices with coefficients in $\mathbb{Z}$ and determinant ± 1). For f a real function on $\mathbb{T}^\rho$, put $S_n(f, \omega) := \sum_{k=0}^{n-1} A^{Z_k(\omega)} f$. For $f \in AC_0(\mathbb{T}^\rho)$, we present a FCLT for $(\frac{1}{\sqrt{n \log n}} S_{[nt]}(f, \omega))_{t \in [0,1]}$ for a.e. ω, that is a quenched FCLT. This is a joint work with Jean-Pierre Conze.

ASIP with rates for some non uniformly expanding maps

by Christophe CUNY, Université de Brest, France

Let (X, Σ, μ, T) be an ergodic dynamical system and $f \in L^2(\mu)$. Let $(r_n)_{n\in\mathbb{N}}$ be an unbounded non decreasing sequence such that $r_n = o(\sqrt{n \log\log n})$. We say that $(f \circ T^n)_{n\in\mathbb{N}}$ satisfies the strong invariance principle with rate r_n if there exists another probabilty space on which one can build a process $(X_n)_{n\in\mathbb{N}}$ with same distribution as $(f \circ T^n)_{n\in\mathbb{N}}$ and a sequence $(W_n)_{n\in\mathbb{N}}$ of iid gaussian variables such that $|X_1 + \ldots + X_n - (W_1 + \ldots + W_n)| = o(r_n)$, $\mu -$a.s. Until a paper by Berkes, Liu and Wu (2014), where arbitrary polynomial rates are obtained, the best available rates satisfied $r_n = n^{1/4}$ (up to logarithmic terms). We will present results, based on ideas of Berkes, Liu and Wu and on a recent work by Korepanov (2018), where optimal rates of arbitrary polynomial order are obtained. Joint work with Jérôme Dedecker, Alexey Korepanov and Florence Merlevède.

Diffusion limit for a slow-fast Standard Map

by Jacopo DE SIMOI, University of Toronto, Canada

Consider the map $(x, z) \mapsto (x + \epsilon^{-\alpha}\sin(2\pi x) + \epsilon^{-(1+\alpha)}z, z + \epsilon \sin(2\pi x))$, which is conjugate to the Chirikov standard map with a large parameter. For suitable α, we obtain a central limit theorem for the slow variable z for a (Lebesgue) random initial condition. The result is proved by conjugating to the Chirikov standard map and utilizing the formalism of standard pairs. Our techniques also yield for the Chirikov standard map a related limit theorem and a "finite-time" decay of correlations result. This is joint work with Alex Blumenthal and Ke Zhang.

The extremal index and the cluster size

by Ana Cristina MOREIRA FREITAS, University of Porto, Portugal

We consider stochastic processes arising from dynamical systems by evaluating an observable function along the orbits of the system. The existence of an Extremal Index less than 1 is associated to the occurrence of periodic phenomena, which is responsible for the appearance of clusters of exceedances. If the observable achieves a global maximum at a single point of the phase space, then, under certain conditions, in the absence of clustering, the point processes of exceedances converge to a standard Poisson process. In the presence of clustering, the point processes converge to a compound Poisson process, in which the Poisson times are marked by the cluster size.
The Extremal Index usually coincides with the reciprocal of the mean of the limiting cluster size distribution. We build dynamically generated stochastic processes with an Extremal Index for which that relation does not hold. The mechanism used to build such counterexamples is based on considering observable functions maximised at at least two points of the phase space, where one of them is an indifferent periodic point.

Limit theorems for random fields 1 - weak

by Davide GIRAUDO, Ruhr-Universität Bochum, Germany

The understanding of the asymptotic behaviour of partial sums on rectangles of strictly stationary random fields is an important problem in probability theory. It is in general done by using a so-called partial sum process, which consists of random elements of the space of continuous functions on the unit cube $[0,1]^d$, where $d \geq 1$ is an integer.

We will define the orthomartingales and mention a functional central limit theorem obtained by Volný (2015). We will proceed by giving a sufficient condition for orthomartingale approximation and present a multidimensional equivalent of the Maxwell and Woodroofe condition (2000).

Growth of normalizing sequences in limit theorems

by Sébastien GOUËZEL, CNRS, Université de Nantes, France

Assume that a renormalized Birkhoff sum $S_n f/B_n$ converges in distribution to a nontrivial limit. What can one say about the sequence B_n? Most natural statements in the literature involve sequences B_n of the form $B_n = n^\alpha L(n)$, where L is slowly varying. We will discuss the possible growth rate of B_n both in the probability preserving case and the conservative case. In particular, we will describe examples where B_n grows superpolynomially, or where B_{n+1}/B_n does not tend to 1.

Local escape rates for ϕ-mixing systems

by Nicolai HAYDN, University of South California, USA

If one places a hole of positive measure in an ergodic dynamical system, then almost every point will eventually hit the hole and disappear. The exponential decay rate of the left-over set is the escape rate to the hole. Naturally a smaller hole will have a smaller escape rate. However, if one divides the escape rate by the size (measure) of the hole and takes a limit as the size goes to zero, then one obtains the local escape rate. In this talk we show that if the invariant measure is -mixing (with respect to a generating partition) then the local escape rate is equal to one at every point except at periodic points, where it is given by one minus the extremal index. We apply this result to Young towers, equilibrium states for Axiom A systems, interval maps and conformal maps.

Stable limits for Markov chains and contractivity properties of transition operators

by Adam JAKUBOWSKI, Nicolaus Copernicus University, Pologne

We study limit theorems for partial sums of instantaneous functions of a homogeneous Markov chain on a general state space. The summands are heavy-tailed and the limits are stable distributions. The conditions imposed on the transition operator P of the Markov chain ensure that the limit is the same as if the summands were independent. Such a scheme admits a physical interpretation, as given in Jara et al. (*Ann. Appl. Probab.*, 19 (2009), 2270–2300). We considerably extend the results of Jara et al., *ibid.* and Cattiaux and Manou-Abi (*ESAIM Probab. Stat.*, 18 (2014), 468–486). We show that the theory holds under the assumption of operator uniform integrability in L^2 of P (a notion introduced by Wu (*J. Funct. Anal.*, 172 (2000), 301–376)) *plus* the L^2-spectral gap property. If we strengthen the uniform integrability in L^2 to the hyperboundedness, then the L^2-spectral gap property can be relaxed to the strong mixing at geometric rate (in practice: to geometric ergodicity). We provide an example of a Markov chain on a countable space that is uniformly integrable in L^2 (and admits an L^2-spectral gap), while it is not hyperbounded. Moreover, we show by example that hyperboundedness is still a weaker property than ϕ-mixing, what enlarges the range of models of interest. What makes our assumptions working is a new, efficient version of the Principle of Conditioning that operates with conditional characteristic functions rather than predictable characteristics. This is a joint work with Mohamed El Machkouri and Dalibor Volný.

Statistical stability on short time scale

by Alexey KOREPANOV, University of Exeter, England

A parametrized family of dynamical systems is statistically stable if their physical invariant measures change continuously with the parameter. The physical invariant measures capture the almost sure limit of Birkhoff averages, as the time goes to infinity.

I will talk about statistical behavior of Birkhoff averages for nearby quadratic maps, in a situation where the statistical stability breaks down in a variety of ways: even the physical invariant measure may not exist. This is a joint work with Neil Dobbs.

A strong law of large numbers for the boundary of the range the simple random walk in two dimensions

by Zemer KOSLOFF, The Hebrew University of Jerusalem, Israël

Dvoretsky and Erdős have shown that the size the range of the simple two dimensional random walk stopped at time n divided by $\frac{\pi n}{\log(n)}$ tends to 1 almost surely as n tends to infinity. We show that there exists a positive constant C such that the size of the boundary of the range up to time n is almost surely $\frac{cn}{(\log n)^2}(1+o(1))$. This also holds in the case of two dimensional aperiodic random walks in the standard domain of attraction of the normal distribution and some one dimensional random walks in the domain of attraction of the Cauchy distribution.

I will discuss the proof of this result and if time permits a quantitative result of the Fölner asymptotic of ranges of other recurrent random walks and the almost sure failure of the Fölner property of the ranges in the case of transient random walks. This is joint work with George Deligiannidis (Oxford).

Multiplicative ergodicity of Laplace transforms for additive functional of Markov chains with application to age-dependent branching process

by Sana LOUHICHI, Université Grenoble Alpes, France

We study the exponential growth of bifurcating processes with ancestral dependence. We suppose here that the lifetimes of the cells are dependent random variables, that the numbers of new cells are random and dependent. Lifetimes and new cellss numbers are also assumed to be dependent. We illustrate our results by examples, including some Markov models. Our approach is related to the behaviour of the Laplace transform of nonnegative additive functional of Markov chains and require weak moment assumption (no exponential moment is needed). This talk is based on common works with Bernard Ycart (2015) and with Loïc Hervé & Francoise Pène (2017).

Mixing and the local central limit theorem for hyperbolic dynamical systems

by Péter NANDORI, University of Maryland, USA

We present a convenient joint generalization of mixing and the local version of the central limit theorem (MLLT) for probability preserving dynamical systems. We verify that MLLT holds for several examples of hyperbolic systems by reviewing old results for maps and presenting new results for flows. Then we discuss applications such as proving various mixing properties of infinite measure preserving systems. Based on joint work with Dmitry Dolgopyat.

On the second Borel-Cantelli lemma for weakly dependent stationnary sequences (joint work with J. Dedecker and F. Merlevède)

by Emmanuel RIO, Université de Versailles, France

In this talk, we give critera implying the second Borel-Cantelli lemma for stationnary sequences satisfying various types of weak dependence conditions, including absolute regularity as well as weaker notions of dependence. We also give some applications to Markov chains and dynamical systems.
This is a joint work with J. Dedecker and F. Merlevède.

Asymptotic behaviour of infinite coupled map systems

by Fanni SÉLLEY, Hungarian Academy of Sciences Rényi Institute, Hungary

Coupled map systems of infinite size arise as the limit of systems describing the behavior of finitely many interacting units. The phase space of our system is the (flat) circle $\mathbb{T} = \mathbb{R}/\mathbb{Z}$, and the system state is represented by a density function $f : \mathbb{T} \to \mathbb{R}_{\geq 0}$. The dynamics is a composition of two circle maps, an expanding map T and a coupling map of meanfield-type, which we shall denote by Φ_f^{ε}. The parameter ε is called the coupling strength: $\varepsilon = 0$ corresponds to an uncoupled system, while larger vales of this parameter mean stronger coupling. Our main objective is to study the asymptotic behavior of sequences of densities representing the state of the system, for various values of ε. In this talk we show that if we assume some regularity conditions on T, the system has a unique invariant density (in a suitable class of functions) for *sufficiently weak coupling.* Furthermore, all initial densities from this suitable class converge to the unique invariant density with exponential speed. We further demonstrate that a different behavior is possible for *sufficiently strong coupling*: some initial distributions approach a moving point mass in any sensible metric on spaces of measures. This can be interpreted as the perfect synchronization of the coupled map system.

The joint spectrum and limit theorems for random matrix products

by Çagri SERT, ETH Zürich, Switzerland

We will start by introducing a deterministic limit object associated to a compact set of matrices called the joint spectrum. We will then turn to random matrix products and talk about two new results where the joint spectrum appears naturally: the first one concerns a refinement of a classical result due to Furstenberg, Kesten, Le Page, Guivarc'h, Raugi, Benoist, Quint etc., on the properties of Lyapunov exponents. The second one is about large deviations principles for random matrix products (partly joint work with Emmanuel Breuillard).

Limit theorems for Almost Anosov flows

by Dalia TERHESIU, University of Exeter, England

An almost Anosov flow is a flow having continuous flow-invariant splitting of the tangent bundle with exponential expansion/contraction in the unstable/stable direction, except for a finite number (in our case a single) periodic orbits. Roughly, almost Anosov flows are perturbed Anosov flows, where the perturbation is local around these periodic orbits, making them neutral. For this type of flows, we obtain limit theorems (stable, standard and non-standard CLT) for a large class of (unbounded) observables. I will present these results stressing on the method of proof. This is joint work with H. Bruin and M. Todd.

An application of probabilistic potential theory to dynamical systems

by Damien THOMINE, Université de Paris-Sud Orsay, France

Objects from potential theory (Laplacian, discrete Laplacian, Green's function, etc.) are closely related to objects from the theory of Markov chains (Brownian motion, random walks, potential kernel, etc.). This relation has proved to be very fruitful in both directions. For instance, it allows the computation of *hitting probabilities* in a Markov chain: given two states, the probability that, starting from the first state, the chain hits the second state.

In this talk, I will present some of these correspondences, and outline how they can be applied to dynamical systems. As an application, in the particular context of $\mathbb{Z}^d$-extensions of Markov maps, the asymptotic behaviour of these hitting probabilities can be computed.

Pressure and limit theorems: the infinite measure case

by Mike TODD, University of St Andrews, Scotland

Sarig related the pressure function, for dynamical systems with a finite equilibrium state and observables with heavy tails, to non-standard limit laws (CMP 2006). Part of the theory was to pass between the form of the pressure function, and the limit theorems, for the original system to a related induced scheme. In this talk Ill present joint work with Henk Bruin and Dalia Terhesiu in the infinite measure case. Applying renewal theory techniques, we show that relation between the original and induced pressure functions changes from being linear, in the finite case, to being a power law in the infinite case. Ill give applications to interval maps including Manneville-Pomeau maps and Fibonacci maps.

Lévy diffusion for Lorentz gas with flat points

by Hong-Kun ZHANG, University of Massachusetts, Amherst, USA

We investigate the diffusion and statistical properties of Lorentz gas with cusps at flat points. This is a modification of dispersing billiards with cusps. The decay rates are proven to depend on the degree of the flat points, which varies from n^{-a}, for $a \in (0, \infty)$. The stochastic processes driven by these systems enjoy stable law and have super-diffusion driven by Lévy process. This is a joint work with Paul Jung and Françoise Pene

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On the quenched CLT for stationary random fields under projective criteria

by Na ZHANG, University of Cincinnati, USA

This talk is motivated by random evolutions which do not start from equilibrium, in a recent work, Peligrad and Volný (2018) showed that the quenched CLT (central limit theorem) holds for ortho-martingale random fields. In this paper, we study the quenched CLT for a class of random fields larger than the ortho-martingales. To get the results, we impose sufficient conditions in terms of projective criteria under which the partial sums of a stationary random field admit an ortho-martingale approximation. More precisely, the sufficient conditions are of the Hannan's projective type. As applications, we establish quenched CLT's for linear and nonlinear random fields with independent innovations. This is a joint work with Lucas Reding and Magda Peligrad.

Iterated weak invariance principle for slowly mixing dynamical systems

by Matt GALTON, University of Warwick, England

Deviation inequalities for martingales

by Davide GIRAUDO, Ruhr-Universität Bochum, Germany

We establish deviation inequalities for the maxima of partial sums of a martingale differences sequence, and of an orthomartingale differences random field. These inequalities can be used to give rates for linear regression and the law of large numbers.

Quenched CLT for stationary random fields

by Lucas REDING, Université de Rouen Normandie, France

In a recent work, Peligrad and Volný (2018) showed that the quenched CLT (central limit theorem) holds for ortho-martingale random fields. In this paper, we study the quenched CLT for general random fields. To get the results, we establish sufficient conditions in terms of projective criteria under which the partial sum of a stationary random field admits an ortho-martingale approximation. As applications, we establish quenched CLT for linear and nonlinear random fields with independence.

On the Nadaraya-Watson regression estimator for irregularly spaced data

by Lucas REDING, Université de Rouen Normandie, France

Quenched limit theorems via complex cone contraction

by Julien SEDRO, Université de Paris-Sud Orsay, France
