Bayesian Inference in Many Dimensions: Examples from Macroeconomics and Finance

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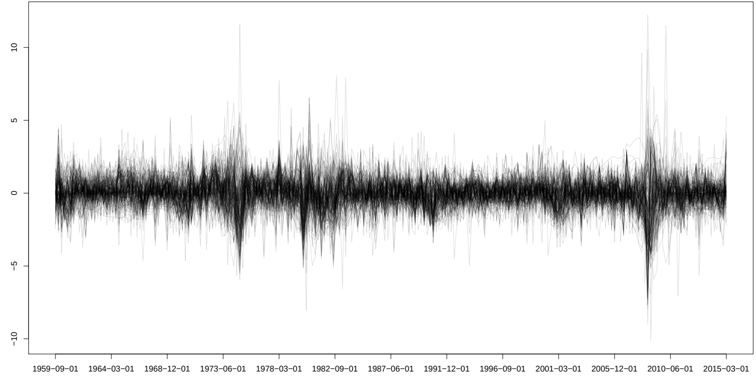

based on joint work with

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OeNB

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Insper São Paulo

Bayesians Statistics in the Big Data Era | CIRM | Marseille Luminy | November 28, 2018

Three simple steps in 25 minutes (or so)

1. Step 1: Time-Varying Covariance
2. Step 2: Vector Autoregression with Time-Varying Covariance
3. Step 3: Thresholded Time-Varying Parameter Vector Autoregression with Time-Varying Covariance Matrix

Factor stochastic volatility

Suppose that $\boldsymbol{y}_t \in \mathbb{R}^m$, $t = 1, \ldots, T$,

$$\boldsymbol{y}_t \sim \mathcal{N}_m(\boldsymbol{0}, \boldsymbol{\Omega}_t),$$

where

$$\boldsymbol{\Omega}_t = \boldsymbol{\Lambda} \boldsymbol{V}_t \boldsymbol{\Lambda} + \boldsymbol{\Sigma}_t$$

and

- $\boldsymbol{\Sigma}_t = \text{diag}(\sigma^2_{1t}, \ldots, \sigma^2_{mt})$ and $\boldsymbol{V}_t = \text{diag}(\sigma^2_{m+1,t}, \ldots, \sigma^2_{m+q,t})$
- $\boldsymbol{\Lambda}$: $m \times q$ factor loadings matrix
- AR(1) processes for the log variances (non-linear state space model)

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> $\boldsymbol{\Lambda}$: $m \times q$ factor loadings matrix

> AR(1) processes for the log variances (non-linear state space model)

Known as the factor stochastic volatility model (see e.g. Pitt and Shephard, 1999; Aguilar and West, 2000; Kastner et al., 2017) with latent variable representation

$$\boldsymbol{\varepsilon}_t \sim \mathcal{N}_m(\boldsymbol{\Lambda} \boldsymbol{f}_t, \boldsymbol{\Sigma}_t), \quad \boldsymbol{f}_t \sim \mathcal{N}_q(\mathbf{0}, \boldsymbol{V}_t)$$

> Off-diagonal entries of $\boldsymbol{\Omega}_t$ exclusively stem from the volatilities of the q factors

> Diagonal entries of $\boldsymbol{\Omega}_t$ are allowed to feature idiosyncratic deviations driven by the elements of $\boldsymbol{\Sigma}_t$

> Reduces the number of free elements in $\boldsymbol{\Omega}_t$ from $m(m+1)/2$ to $m(q+1)$

Sparse factor models

Factor models are a sparse representation of $\mathbf{\Omega}$ and $\mathbf{\Omega}^{-1}$. To achieve additional sparsity, use shrinkage priors a.k.a. penalized likelihood.

- Point mass priors
  - Basic factor model (West, 2003; Carvalho et al., 2008; Frühwirth-Schnatter and Lopes, 2018)
  - Bayesian dedicated factor analysis (Conti et al., 2014)
  - Sparse dynamic factor models (Kaufmann and Schumacher, 2018)
- Continuous shrinkage priors, e.g. in sparse Bayesian infinite factor models (Bhattacharya and Dunson, 2011)
- Latent thresholding approaches (Nakajima and West, 2013a; Zhou et al., 2014)
- ...

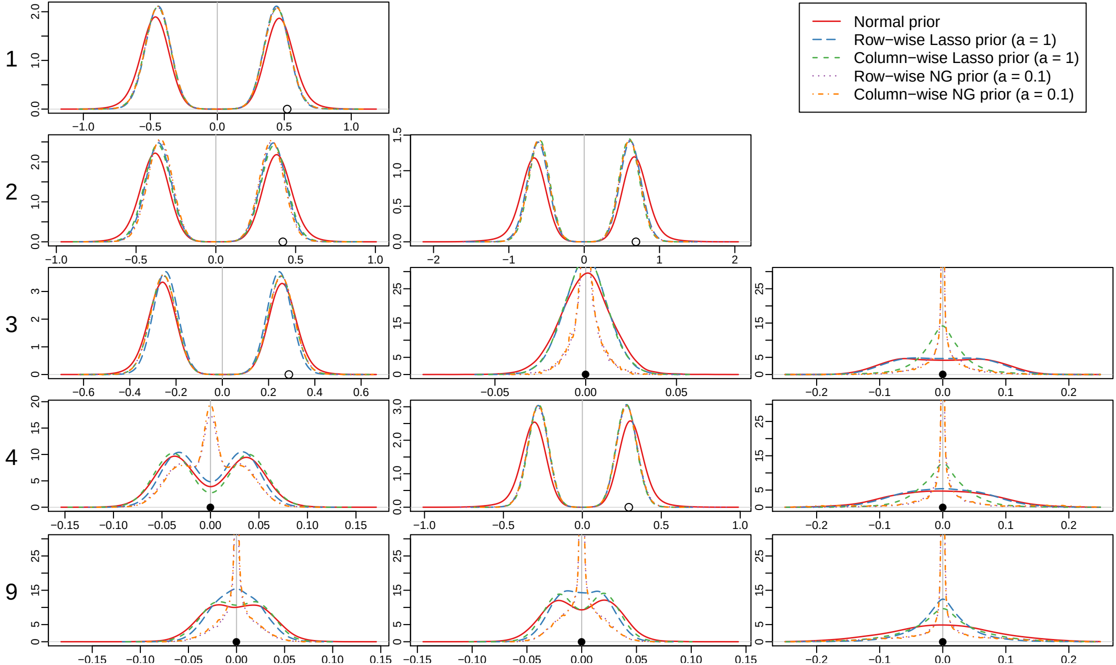
Sparse factor SV models

Use continuous shrinkage priors such as the Normal-Gamma prior (Griffin and Brown, 2010):

$$\Lambda_{ij}|\lambda_j, \tau_{ij} \sim \mathcal{N}\left(0, \tau_{ij}^2/\lambda_j^2\right), \quad \lambda_j^2 \sim \mathcal{G}\left(c, d\right), \quad \tau_{ij}^2 \sim \mathcal{G}\left(a, a\right).$$

- $\mathbb{V}(\Lambda_{ij}|\lambda_j^2) = 1/\lambda_j^2$
- Excess kurtosis of Λ_{ij} is $3/a$ if it exists
- Shrink globally (column-wise) through λ_j^2 (or row-wise through λ_i^2, industry-wise, ...)
- Adjust locally (element-wise) through τ_{ij}: $\tau_{ij} < 1$ more, $\tau_{ij} > 1$ less shrinkage
- Bayesian Lasso (Park and Casella, 2008) arises for $a = 1$

Marginal factor loadings posteriors, $r_{\text{true}} = 2$, $r = 3$, $m = 10$, $T = 1000$



Application to S&P 500 members

- Only firms which have been listed from November 1994 onwards, resulting in $m = 300$ stock prices on 5001 days, ranging from 11/1/1994 to 12/31/2013.
- Data was obtained from Bloomberg Terminal in January 2014.
- Investigate $T = 5000$ demeaned percentage log-returns.
- Time-varying covariance matrix with 45150 nontrivial elements on 5000 days can be well explained by 4 factors with many factor loadings shrunken to 0.

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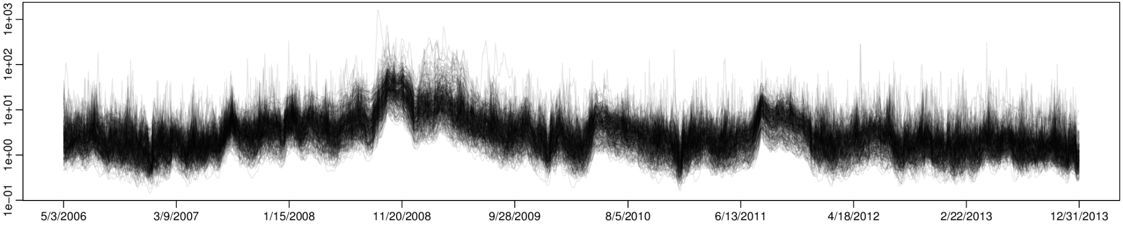
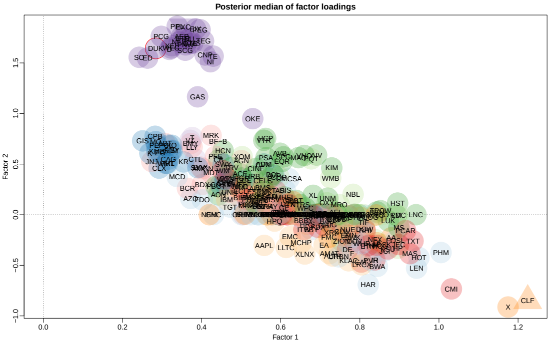
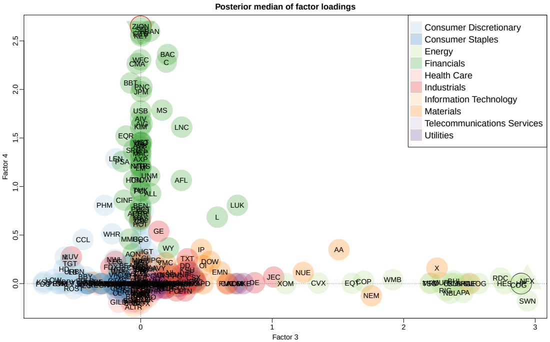
Marginal conditional variances (posterior mean, log scale)

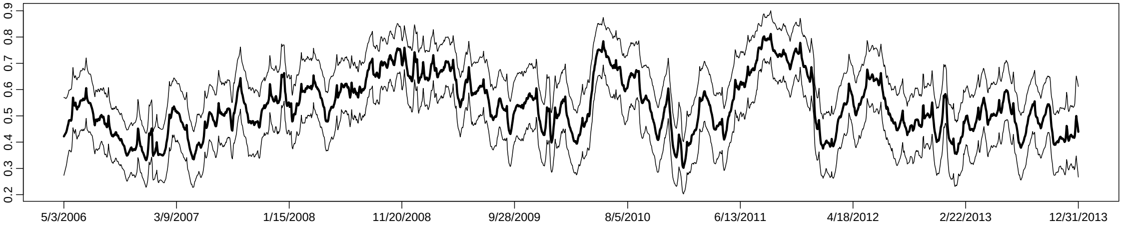


- Substantial co-movement (generally)
- Idiosyncratic deviations (at certain stretches in time)

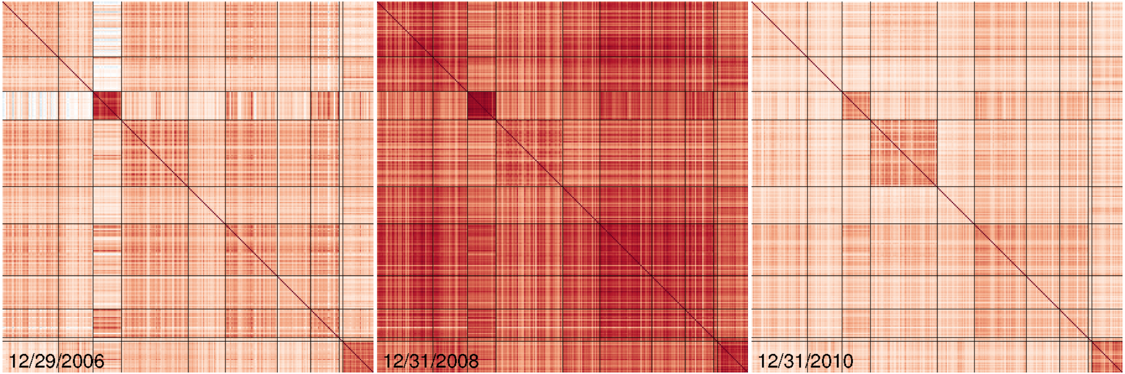
Median factor loadings and joint communalities (mean $\pm$ 2 sd)







Mean posterior correlations



Video

Works well for

- > density predictions
- > minimum variance portfolio constructions

(Kastner, 2018)

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The VAR model

Suppose that $\boldsymbol{y}_t \in \mathbb{R}^m, t = 1, \ldots, T$, follows a zero-mean heteroskedastic VAR(p) process,

$$\boldsymbol{y}_t = \boldsymbol{A}_1 \boldsymbol{y}_{t-1} + \cdots + \boldsymbol{A}_p \boldsymbol{y}_{t-p} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}_m(\mathbf{0}, \boldsymbol{\Omega}_t)$$

> $\boldsymbol{A}_j$ ($j = 1, \ldots, p$): $m \times m$ matrices of autoregressive coefficients

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- $\boldsymbol{A}_j$ ($j = 1, \dots, p$): $m \times m$ matrices of autoregressive coefficients
- $\boldsymbol{x}_t = (\boldsymbol{y}'_{t-1}, \dots, \boldsymbol{y}'_{t-p})'$ and a $m \times mp$ coefficient matrix $\boldsymbol{B} = (\boldsymbol{A}_1, \dots, \boldsymbol{A}_p)$ to rewrite the model more compactly as

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{x}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}_m(\mathbf{0}, \boldsymbol{\Omega}_t).$$

(Including an intercept is straightforward but omitted here for simplicity of exposition)

Shrinking the coefficients

Dirichlet-Laplace prior (Bhattacharya et al., 2015) for $\boldsymbol{b} = \text{vec}(\boldsymbol{B})$. In what follows, we impose the DL prior on each of the $K = m^2 p$ elements of $\boldsymbol{b}$, for $j = 1, \ldots, K$,

$$b_j \sim \mathcal{N}(0, \psi_j \vartheta_j^2 \zeta)$$

> ψ_j are local scaling parameters $\psi_j \sim \text{Exp}(1/2)$
> $\boldsymbol{\vartheta} = (\vartheta_1, \ldots, \vartheta_K)' \in \mathcal{S}^{K-1} = \{\boldsymbol{\vartheta} : \vartheta_j \geq 0, \sum_{j=1}^{K} \vartheta_j = 1\}$, $\boldsymbol{\vartheta} \sim \text{Dir}(a, \ldots, a)$
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Nice features:

- strong shrinkage with plenty of flexibility
- mimicking "real" spike and slab priors, but much lower computational burden
- implementation is simple, only requires one single structural hyperparameter a (smaller $a \Rightarrow$ stronger spike)
- good contraction guarantees in stylized models when $a = K^{-(1+\epsilon)}$ with ϵ small

MCMC at a glance

- Local shrinkage parameters $\psi_j|\bullet \sim iG(\vartheta_j\zeta/|b_j|, 1)$, $j = 1, \dots, K$ via efficient and stable rejection sampler from Hörmann and Leydold (2013), R-package `GIGrvg`
- Global shrinkage parameter $\zeta|\bullet \sim \mathcal{GIG}\left(K(a-1), 1, 2\sum_{j=1}^{K} |b_j|/\vartheta_j\right)$, `GIGrvg`
- Scaling parameters ϑ_j by first sampling L_j from $L_j|\bullet \sim \mathcal{GIG}(a-1, 1, 2|b_j|)$, and then setting $\vartheta_j = L_j/\sum_{i=1}^{K} L_i$ (Bhattacharya et al., 2015), `GIGrvg`
- Conditionally "univariate" SV states and parameters of volatility processes via auxiliary mixture sampling (Omori et al., 2007) with ASIS (Yu and Meng, 2011) via R-package `stochvol`
- Factors and factor loadings: "deep interweaving" (Kastner et al., 2017) via R-package `factorstochvol`
- VAR parameters, see next slide

Sampling the VAR parameters

Exploiting the data augmentation representation for the factor SV model, the model can be cast as a system of m conditionally unrelated regression models

$$z_{it} := y_{it} - \boldsymbol{\Lambda}_{i\bullet}\boldsymbol{f}_t = \boldsymbol{B}_{i\bullet}\boldsymbol{x}_t + \eta_{it}, \quad i = 1, \dots, m, \quad t = 1, \dots, T$$

Full conditional posteriors:

$$\boldsymbol{B}'_{i\bullet}|\bullet \sim \mathcal{N}(\boldsymbol{b}_i, \boldsymbol{Q}_i), \quad \boldsymbol{Q}_i = (\tilde{\boldsymbol{X}}'_i\tilde{\boldsymbol{X}}_i + \boldsymbol{\Phi}_i^{-1})^{-1}, \quad \boldsymbol{b}_i = \boldsymbol{Q}_i(\tilde{\boldsymbol{X}}'_i\tilde{\boldsymbol{z}}_i)$$

- $\boldsymbol{\Phi}_i$ is the respective $k \times k$ diagonal submatrix of $\boldsymbol{\Phi} = \zeta \times \text{diag}(\psi_1\vartheta_1^2, \dots, \psi_K\vartheta_K^2)$
- $\tilde{\boldsymbol{X}}_i$ is a $T \times k$ matrix with typical row t given by $\boldsymbol{X}_t/\sigma_{it}$
- $\tilde{\boldsymbol{z}}_i$ is a T-dimensional vector with the tth element given by z_{it}/σ_{it}.

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$\Rightarrow$ allows for equation by equation estimation: each draw costs $O(m^4p^3 + Tm^3p^2)$ – as opposed to $O(m^6p^3)$ in the naive approach (cf. Carriero et al., 2015)

Yet another approach

In typical macro data $T \lesssim 200$ … even faster sampling is possible via an algorithm proposed by Bhattacharya et al. (2016) for univariate regressions, applied to each equation:

1. Sample independently $\boldsymbol{u}_i \sim \mathcal{N}(\mathbf{0}_k, \boldsymbol{\Phi}_i)$ and $\boldsymbol{\delta}_i \sim \mathcal{N}(\mathbf{0}, \boldsymbol{I}_T)$
2. Use $\boldsymbol{u}_i$ and $\boldsymbol{\delta}_i$ to construct $\boldsymbol{v}_i = \tilde{\boldsymbol{X}}_i \boldsymbol{u}_i + \boldsymbol{\delta}_i$
3. Solve $(\tilde{\boldsymbol{X}}_i \boldsymbol{\Phi}_i \tilde{\boldsymbol{X}}_i' + \boldsymbol{I}_T)\boldsymbol{w}_i = (\tilde{\boldsymbol{z}}_i - \boldsymbol{v}_i)$ for $\boldsymbol{w}_i$
4. Set $\boldsymbol{B}_{i\bullet}' = \boldsymbol{u}_i + \boldsymbol{\Phi}_i \tilde{\boldsymbol{X}}_i' \boldsymbol{w}_i$

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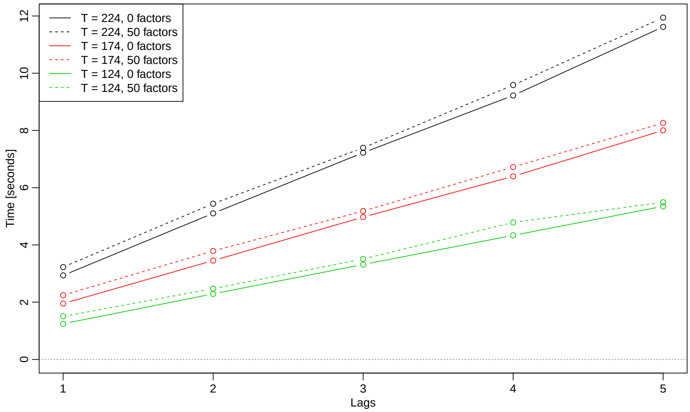
Cost is now $O(m^2 T^2 p)$, e.g. for $m = 215$ we have:

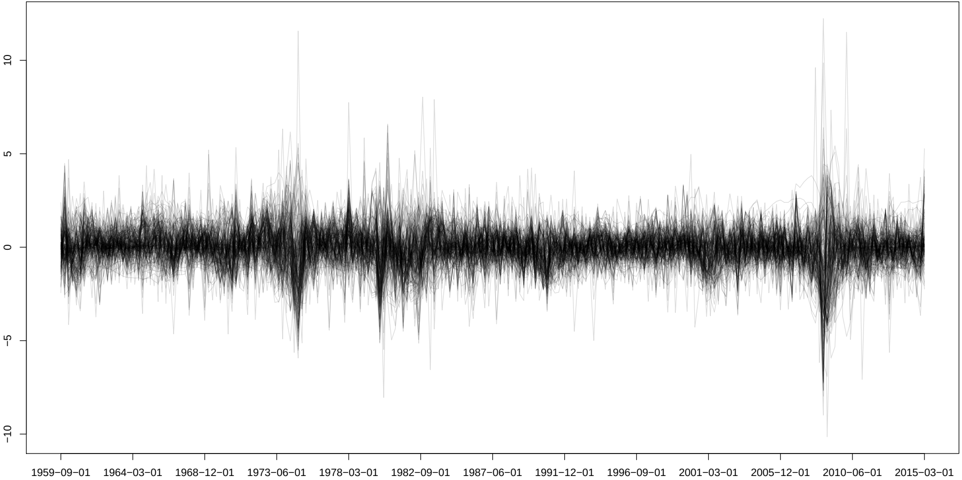


…and thanks to Aki's I now know that there is even more room for improvement (Nishimura and Suchard, 2018; Zhang et al., 2018)!

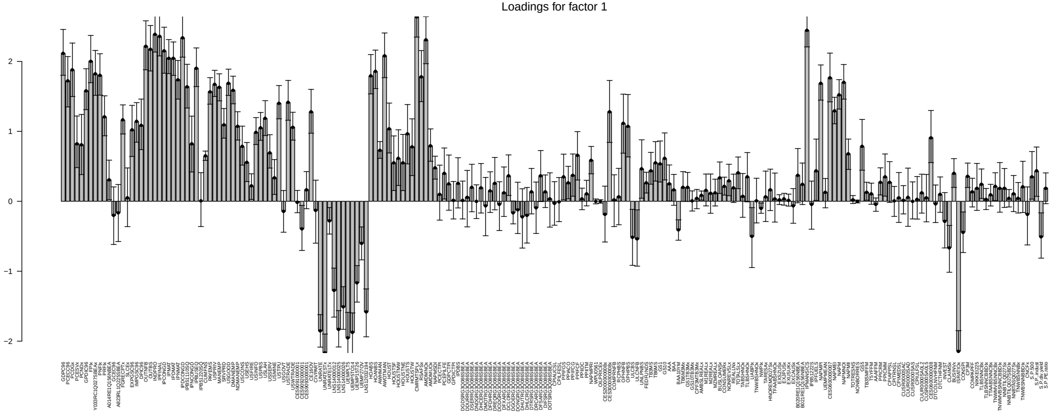
Modeling the US economy

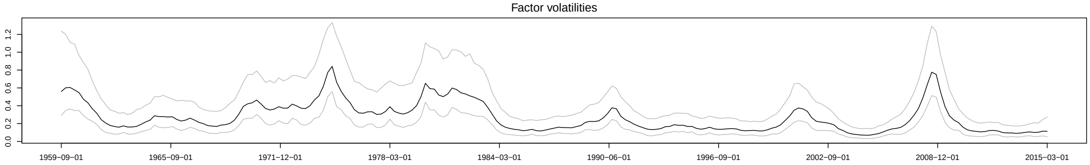
Quarterly dataset from McCracken and Ng (2016) including suggested transformations

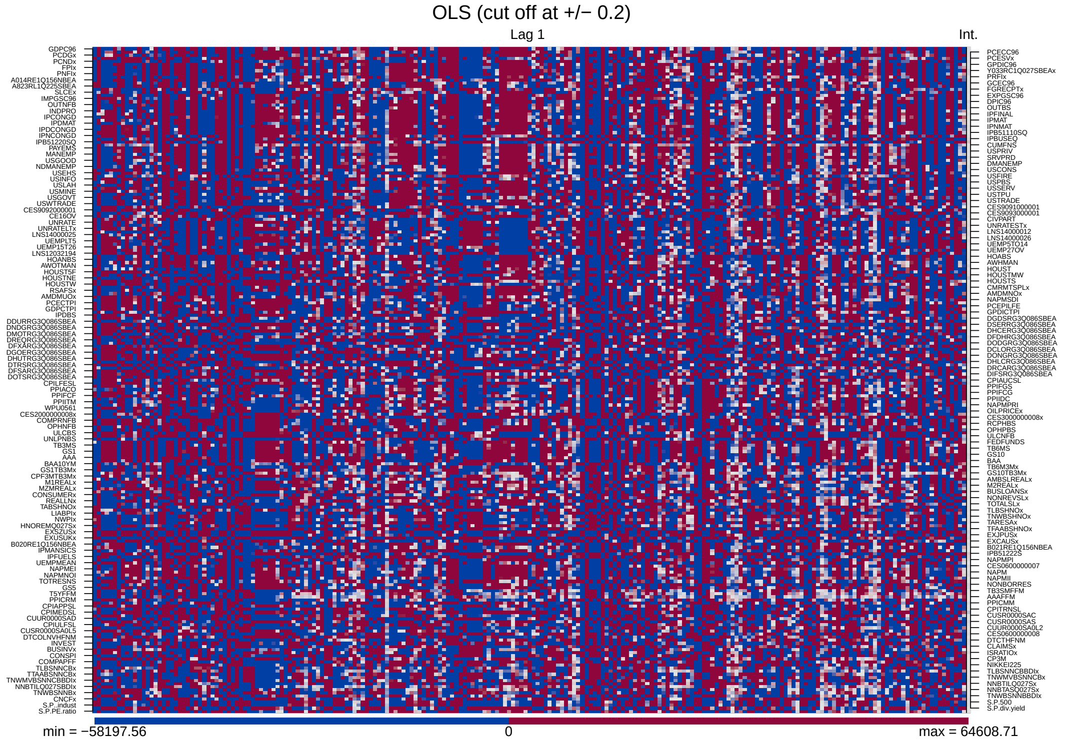
- Ranging from 1959:Q1 to 2015:Q4, $m = 215$
- Component-wise standardization (zero mean, variance one)
- For presentation purposes: One lag, one factor



Factor loadings and factor volatilities







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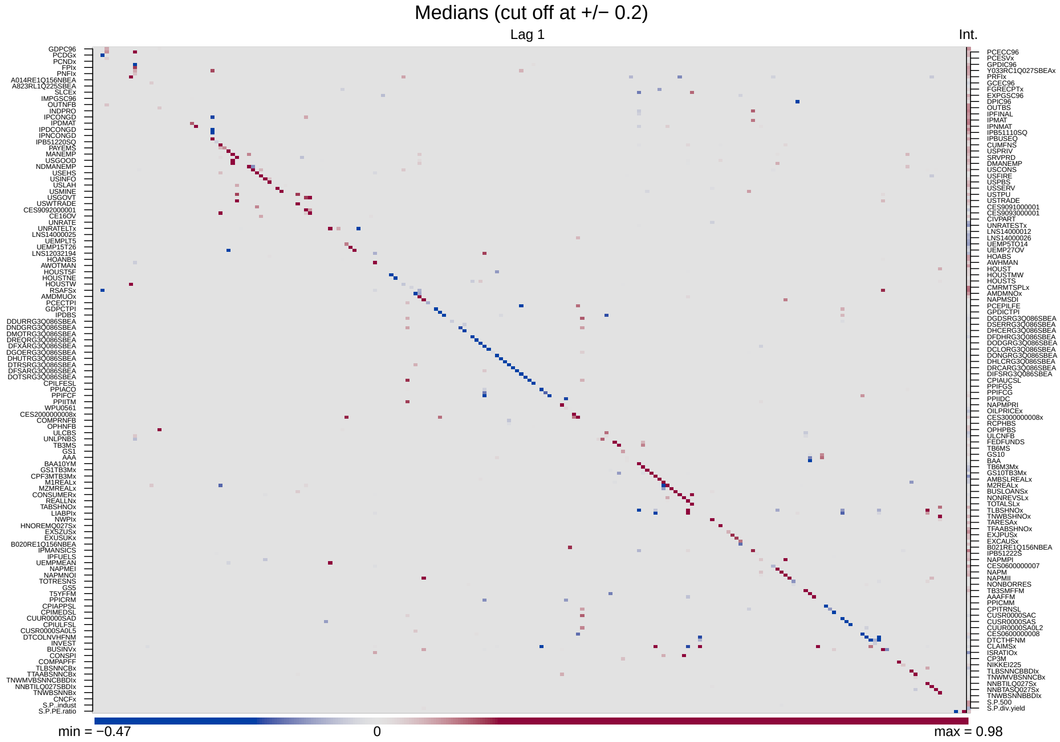
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USPRIV
SRVPRD
DMANEMP
USCONS
USFIRE
USPBS
USSERV
USTPU
USTRADE
CES9091000001
CES9093000001
CIVPART
UNRATESTx
LNS14000012
LNS14000026
UEMP5TO14
UEMP27OV
HOABS
AWHMAN
HOUST
HOUSTMW
HOUSTS
CMRMTSPLx
AMDMNOx
NAPMSDI
PCEPILFE
GPDICTPI
DGDSRG3Q086SBEA
DSERRG3Q086SBEA
DHCERG3Q086SBEA
DFDHRG3Q086SBEA
DODGRG3Q086SBEA
DCLORG3Q086SBEA
DONGRG3Q086SBEA
DHLCRG3Q086SBEA
DRCARG3Q086SBEA
DIFSRG3Q086SBEA
CPIAUCSL
PPIFGS
PPIFCG
PPIIDC
NAPMPRI
OILPRICEx
CES3000000008x
RCPHBS
OPHPBS
ULCNFB
FEDFUNDS
TB6MS
GS10
BAA
TB6M3Mx
GS10TB3Mx
AMBSLREALx
M2REALx
BUSLOANSx
NONREVSLx
TOTALSLx
TLBSHNOx
TNWBSHNOx
TARESAx
TFAABSHNOx
EXJPUSx
EXCAUSx
B021RE1Q156NBEA
IPB51222S
NAPMPI
CES0600000007
NAPM
NAPMII
NONBORRES
TB3SMFFM
AAAFFM
PPICMM
CPITRNSL
CUSR0000SAC
CUSR0000SAS
CUUR0000SA0L2
CES0600000008
DTCTHFNM
CLAIMSx
ISRATIOx
CP3M
NIKKEI225
TLBSNNCBBDIx
TNWMVBSNNCBx
NNBTILQ027Sx
NNBTASQ027Sx
TNWBSNNBBDIx
S.P.500
S.P.div.yield
min = −0.47
0
max = 0.98

Three simple steps in 25 minutes (or so)

1. Step 1: Time-Varying Covariance
2. Step 2: Vector Autoregression with Time-Varying Covariance
3. Step 3: Thresholded Time-Varying Parameter Vector Autoregression with Time-Varying Covariance Matrix

Making the VAR coefficients time-varying I

- Especially in larger dimensional contexts (think of time-varying parameter VARs), allowing for too much flexibility quickly leads to overfitting issues
- Questions such as "should all parameters or only a subset of parameters drift over time?" or "do parameters evolve according to a random walk process or are their dynamics better captured by model that implies relatively few, large breaks?" typically arise
- Literature offers several solutions (e.g. McCulloch and Tsay, 1993; Gerlach et al., 2000; Giordani and Kohn, 2008; Koop et al., 2009; Frühwirth-Schnatter and Wagner, 2010; Koop and Korobilis, 2012; Koop and Korobilis, 2013; Belmonte et al., 2014; Kalli and Griffin, 2014; Eisenstat et al., 2016; Bitto and Frühwirth-Schnatter, 2018)

Making the VAR coefficients time-varying II

- Mixture innovation models provide a flexible means of shrinking time variation of the latent states within a state space model
  - Generally straightforward to implement and quite flexible (allows for large swings in the parameters over certain time intervals)
  - However, not feasible in large applications (think of densely parameterized time varying parameter VAR models)
- Combine the literature on threshold and latent threshold models (Nakajima and West, 2013b; Neelon and Dunson, 2004) with the literature on mixture innovation models (McCulloch and Tsay, 1993; Carter and Kohn, 1994; Gerlach et al., 2000; Koop and Potter, 2007; Giordani and Kohn, 2008)
- We assume that the indicator that controls the mixture component used is determined by the absolute period-on-period change of the states $\rightarrow$ small changes are effectively set equal to zero
- Provides a great deal of flexibility while keeping the additional computational burden tractable

Econometric framework

We consider the following dynamic regression model (TVP model),

$$y_t = \boldsymbol{x}_t' \boldsymbol{\beta}_t + u_t, \;\; u_t \sim \mathcal{N}(0, \sigma^2),$$
$$\beta_{jt} = \beta_{j,t-1} + e_{jt}, \;\; e_{jt} \sim \mathcal{N}(0, \vartheta_j),$$

where

> $\boldsymbol{x}_t$ is a $K \times 1$ vector of explanatory variables

> $\boldsymbol{\beta}_t$ is a $K \times 1$ vector of time varying regression coefficients

> σ^2 and ϑ_j $(j = 1, \ldots, K)$ are innovation variances

This specification assumes that parameters evolve according to a random walk with small movements governed by ϑ_j.

The threshold mixture innovation model (Huber et al., forthcoming)

By contrast, the threshold mixture innovation model (TTVP model) assumes that

$$e_{jt} \sim \mathcal{N}(0, \theta_{jt}),$$
$$\theta_{jt} = s_{jt}\vartheta_{j1} + (1 - s_{jt})\vartheta_{j0},$$

with $\vartheta_{j1} \gg \vartheta_{j0}$.

As in standard mixture innovation models, the indicators s_{jt} are (unconditionally) Bernoulli distributed. In contrast to those models, we however assume that

$$s_{jt}|\Delta\beta_{jt}, d_j = \begin{cases} 1 & \text{if} \quad |\Delta\beta_{jt}| > d_j, \\ 0 & \text{if} \quad |\Delta\beta_{jt}| \leq d_j, \end{cases}$$

where d_j is a coefficient-specific threshold to be estimated. Note that we "only" have one extra parameter per equation (and the indicators are conditionally deterministic).

Can use standard MCMC with one additional griddy Gibbs (Ritter and Tanner, 1992) step per equation.

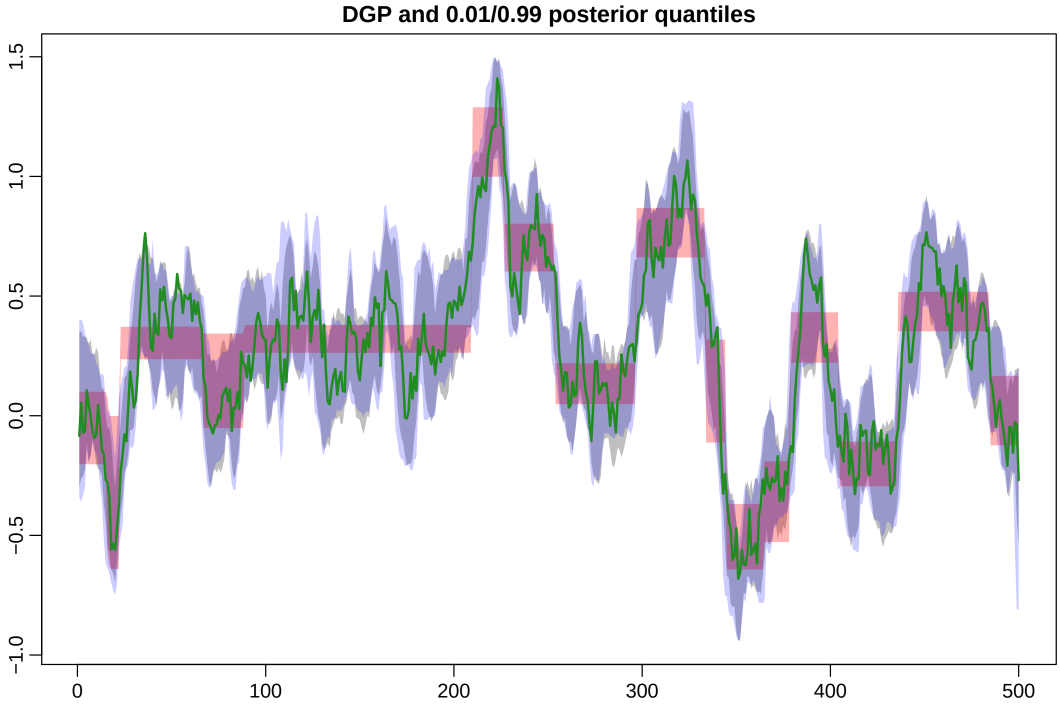
Four illustrative examples

Consider four simple DGPs with $K = 1$: TVP, many, few, and no breaks. Independently for all t, we generate $x_{1t} \sim \mathcal{U}(-1, 1)$ and set $\beta_{1,0} = 0$.

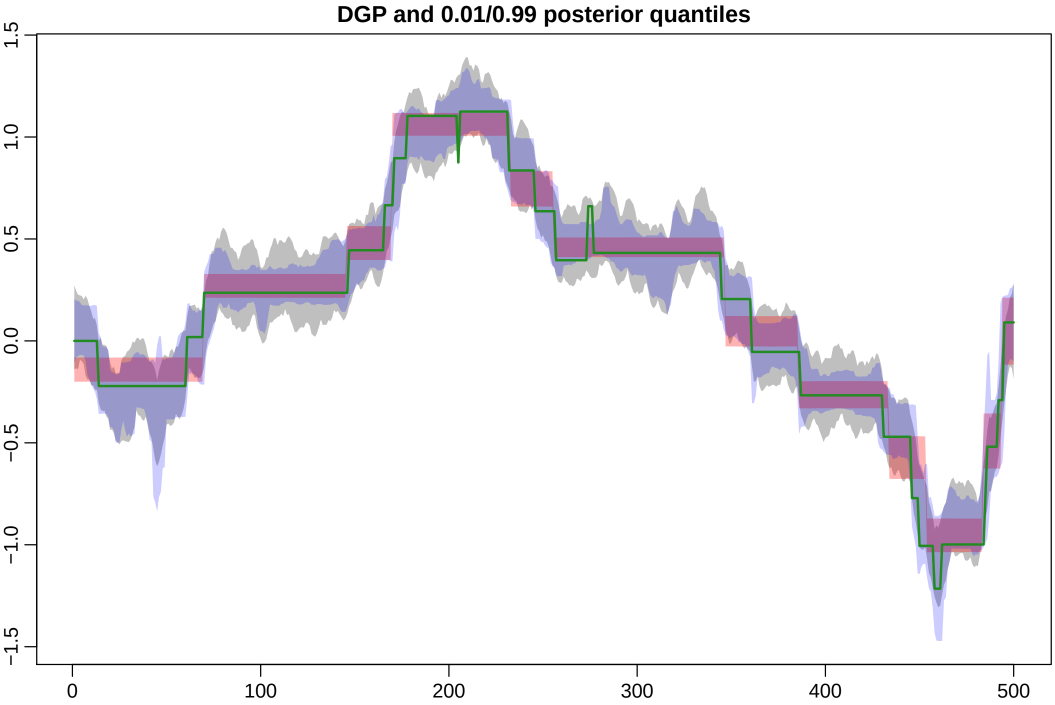
In order to assess how different models perform in recovering the latent processes, we run:

- standard TVP model
- mixture innovation model estimated using the algorithm outlined in Gerlach et al. (GCK, 2000)
- TTVP

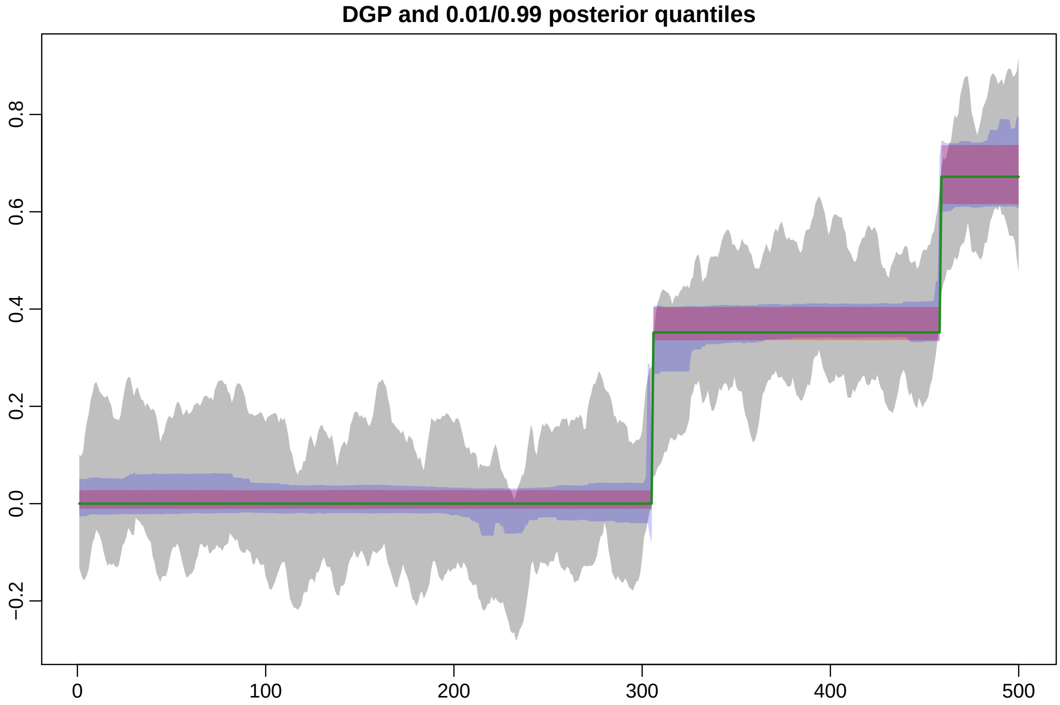
To ease comparison between the models, we impose a similar prior setup for all models.

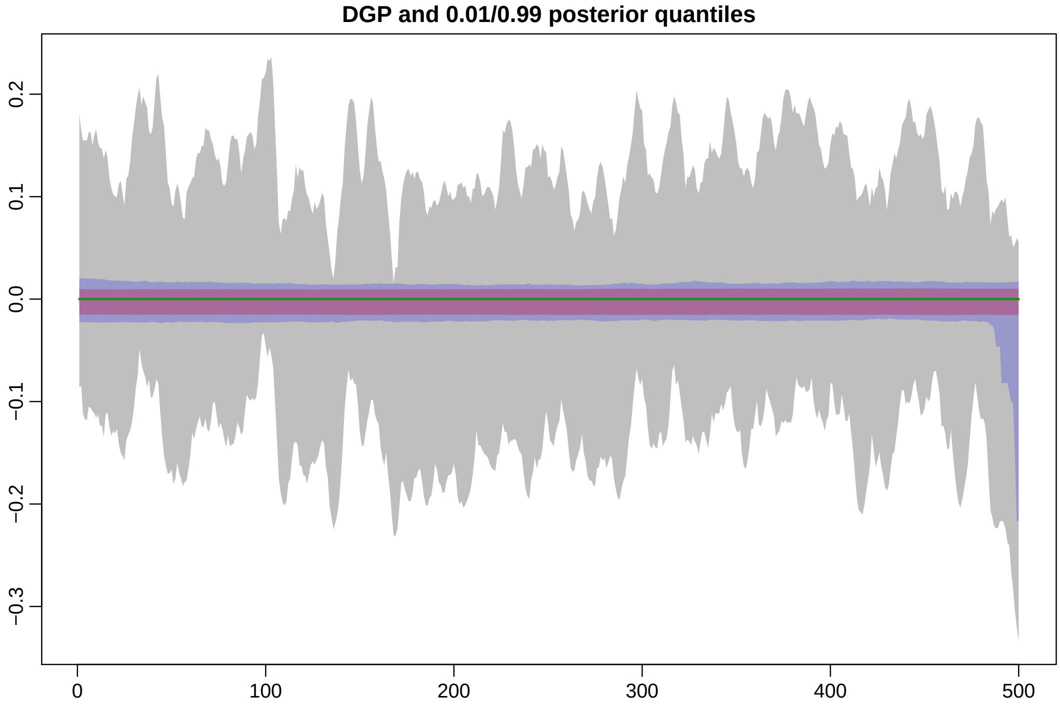


DGP and 0.01/0.99 posterior quantiles
−1.0
−0.5
0.0
0.5
1.0
1.5
0
100
200
300
400
500



DGP and 0.01/0.99 posterior quantiles
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
100
200
300
400
500


DGP and 0.01/0.99 posterior quantiles
0.8
0.6
0.4
0.2
0.0
−0.2
0
100
200
300
400
500


DGP and 0.01/0.99 posterior quantiles
0.2
0.1
0.0
−0.1
−0.2
−0.3
0
100
200
300
400
500

Generalization to the multivariate case

It's straightforward to generalize the framework to a TTVP-VAR-SV model:

$$\boldsymbol{y}_t = \boldsymbol{B}_{1t}\boldsymbol{y}_{t-1} + \cdots + \boldsymbol{B}_{Pt}\boldsymbol{y}_{t-P} + \boldsymbol{u}_t,$$

where

- $\boldsymbol{B}_{pt}$ $(p = 1, \ldots, P)$ are $m \times m$ matrices of thresholded dynamic coefficients
- $\boldsymbol{u}_t \sim \mathcal{N}(\boldsymbol{0}_m, \boldsymbol{\Sigma}_t)$ with $\boldsymbol{\Sigma}_t = \boldsymbol{V}_t\boldsymbol{H}_t\boldsymbol{V}_t'$ being a time-varying variance-covariance matrix
- $\boldsymbol{V}_t$ is lower triangular with unit diagonal; free elements have thresholded dynamic coefficients
- $\boldsymbol{H}_t = \text{diag}(e^{h_{1t}}, \ldots, e^{h_{mt}})$ with $h_{it} = \mu_i + \rho_i(h_{i,t-1} + \mu_i) + \nu_{it},\ \nu_{it} \sim \mathcal{N}(0, \zeta_i)$

This structure imposes order-dependence but allows to estimate the model equation-by-equation with the same techniques as before.

Forecasting the US term structure of interest rates

We use monthly Fama-Bliss zero coupon yields obtained from the CRSP database as well as the dataset described in Gürkaynak et al. (2007). The data spans the period from 1960:M01 to 2014:M12 and the maturities included are 1, 2, 3, 4, 5, 7, and 10 years. Moreover, we include 3 lags of the endogenous variables.

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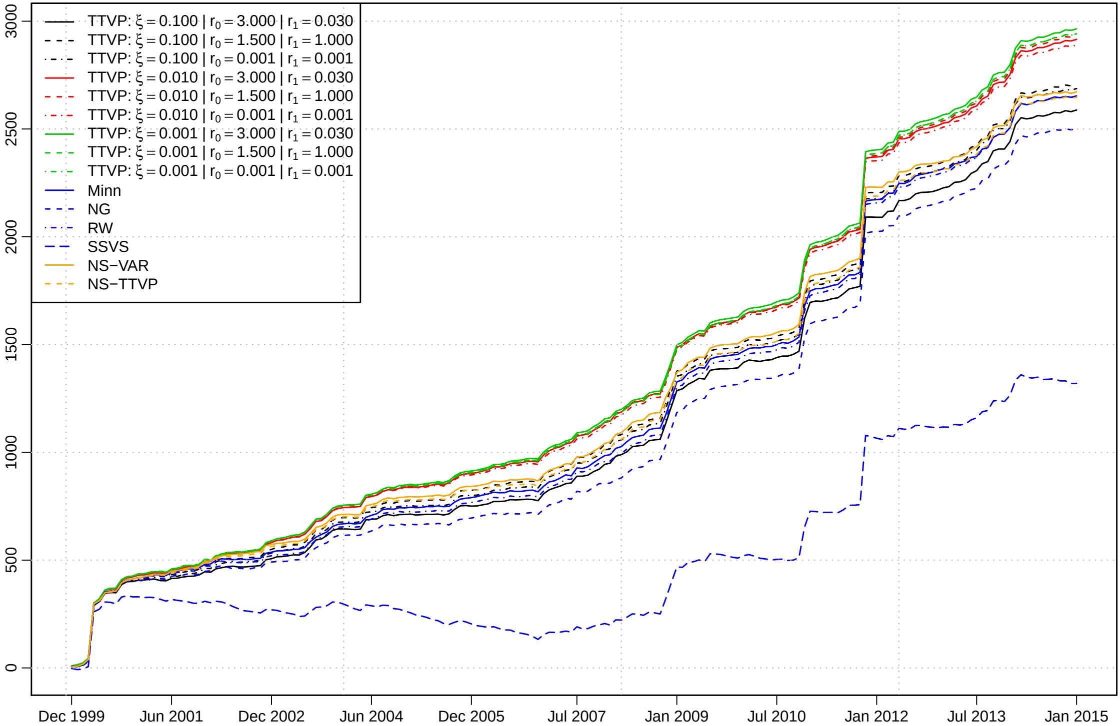
Competitors (all models include SV):

- Benchmark: TVP-VAR as in Primiceri (2005).
- Three constant parameter VAR models:
  - Minnesota-type VAR (Doan et al., 1984)
  - Normal-Gamma (NG) VAR (Huber and Feldkircher, 2018)
  - VAR coupled with an SSVS prior (George et al., 2008)
- Nelson-Siegel (NS, Nelson and Siegel, 1987) VAR as in Diebold and Li (2006)
- NS-TTVP-VAR
- Random walk



TTVP: ξ = 0.100 | r0 = 3.000 | r1 = 0.030
TTVP: ξ = 0.100 | r0 = 1.500 | r1 = 1.000
TTVP: ξ = 0.100 | r0 = 0.001 | r1 = 0.001
TTVP: ξ = 0.010 | r0 = 3.000 | r1 = 0.030
TTVP: ξ = 0.010 | r0 = 1.500 | r1 = 1.000
TTVP: ξ = 0.010 | r0 = 0.001 | r1 = 0.001
TTVP: ξ = 0.001 | r0 = 3.000 | r1 = 0.030
TTVP: ξ = 0.001 | r0 = 1.500 | r1 = 1.000
TTVP: ξ = 0.001 | r0 = 0.001 | r1 = 0.001
Minn
NG
RW
SSVS
NS−VAR
NS−TTVP
0
500
1000
1500
2000
2500
3000
Dec 1999
Jun 2001
Dec 2002
Jun 2004
Dec 2005
Jul 2007
Jan 2009
Jul 2010
Jan 2012
Jul 2013
Jan 2015

Wrap-up

Step 1: Sparse factor SV models

- "Hybrid approach" to dynamic covariance modeling, i.e. parsimony through factor structure, plus additional shrinkage
- "Plug-and-play" friendly due to the modular nature of MCMC

Step 2: High-dimensional VARs

- Equation-by-equation estimation of VARs possible due to latent variable representation of residual covariance matrix
- Faster algorithm(s) for sampling VAR coefficients if $T \lesssim mp$
- Continuous global-local shrinkage priors (DL prior and friends) help to cure the curse of dimensionality and provide a viable alternative to Minnesota-type priors
- Multivariate SV is crucial for macro application and improves joint density predictions markedly

Step 3: TTVP

- Computationally feasible variant of a mixture innovation model
- Allows for jumps in the parameters if the conditional absolute change exceeds a threshold value to be estimated
- Works well for term structure modeling, especially during crisis times

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