STRUCTURE OF 3-MANIFOLD GROUPS

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Abstracts

Mladen Bestvina

The Farrell-Jones conjecture for free-by-cyclic groups

Abstract: The Farrell-Jones conjecture for a given group is an important conjecture in manifold theory. I will review some of its consequences and will discuss a class of groups for which it is known, for example 3-manifold groups. Finally, I will discuss a proof that free-by-cyclic groups satisfy FJC, answering a question of Lück. This is joint work with Koji Fujiwara and Derrick Wigglesworth

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Steven Boyer

Deforming foliations in branched covers and the L-space conjecture

Abstract: We discuss joint work with Ying Hu concerning the left-orderability of the fundamental groups of cyclic branched covers of links which admit co-oriented taut foliations. In particular, we do this for cyclic branched covers of fibered knots in integer homology 3-spheres and cyclic branched covers of closed braids in the 3-sphere. The latter allows us to complete the proof of the L-space conjecture for closed, connected, orientable, irreducible 3-manifolds containing a genus 1 fibered knot. We also prove that the universal abelian cover of a manifold obtained by generic Dehn surgery on a hyperbolic fibered knot in an integer homology 3-sphere admits a co-oriented taut foliation and has left-orderable fundamental group, even if the surgered manifold does not.

François Dahmani

Mapping Class Groups do not have deep relations (between Dehn twists)

Abstract: We investigate what relations can exist between high powers of Dehn twists in a Mapping Class Group of a closed hyperbolic orientable surface. To analyse this, we look for a presentation of the group generated by all k-th powers of Dehn twists. It is similar to looking for a presentation of the normal closure of a power of a Dehn twist. Using Bestvina-Bromberg-Fujiwara projection complexes, and a method of composite (or somewhat simultaneous) rotating families, we prove that there are no other relations than the one we can expect from the sole observation that Dehn twists about disjoint curves commute, and Dehn twists about curves that are images of one-another by an element of the group, are conjugate.

Thomas Delzant

Product set growth in hyperbolic geometry

Abstract: Generalizing results of Razborov and Safin, we prove that for every hyperbolic group there exists a constant $a > 0$ such that for every finite subset U that is not contained in a virtually cyclic subgroup The cardinal $|U^n|$ is greater than $|U|^{\frac{n+1}{2}}$. Similar estimates are established for groups acting acylindrically on trees or hyperbolic spaces.

Cornelia Drutu Badea
Median geometry for lattices

Abstract: In this talk I shall explain how various types of lattices of isometries of (products of) hyperbolic spaces present various degrees of compatibility with the median geometry and its discrete version, the CAT(0) cubical complex geometry.

A byproduct of these results is that they provide evidence that Rips-type theorems cannot hold for actions on median spaces, not even under strong hypotheses.

Roberto Frigerio
Profinite completions of fundamental groups and discrete approximations of simplicial volume

Abstract: The simplicial volume is a homotopy invariant of closed manifolds defined by Gromov in 1982. A long-standing conjecture by Gromov asserts that, for aspherical manifolds, the vanishing of the simplicial volume implies the vanishing of the Euler characteristic. Gromov's conjecture is known to hold if one replaces the classical simplicial volume with its integral version. In this talk I describe an approach to the conjecture that makes use of integral approximations of the simplicial volume in towers of coverings, as well as of ergodic actions of the fundamental group on suitable probability spaces.
Joint work with C. Loeh, C. Pagliantini and R. Sauer

David Gabai
The 4-Dimensional Light Bulb Theorem

Abstract: We generalize the classical light bulb theorem to four dimensions. I.e. a smooth 2-sphere in $S^2 \times S^2$ that intersects $S^2 \times 0$ once and is homologous to $0 \times S^2$ is smoothly isotopically standard.

Daniel Groves
Homomorphisms to 3-manifold groups and other families

Abstract: We are interested in the structure of the set of homomorphisms from a fixed (but arbitrary) finitely generated group G to the groups in some fixed family (such as the family of 3-manifold groups).

I will explain what one might hope to say in different situations, and explain some applications to relatively hyperbolic groups and acylindrically hyperbolic groups, and some hoped-for applications to 3-manifold groups.

This is joint work with Michael Hull and joint work in preparation with Michael Hull and Hao Liang.

Jonathan Hillman
Poincaré duality in dimension 3

Abstract: Poincaré duality complexes model the homotopy types of closed manifolds. In the lowest dimensions the correspondence is precise: every connected PD_n-complex with $n \leqslant 2$ is homotopy equivalent to S^1 or to a closed surface. Every PD_3-complex has an essentially unique factorization as a connected sum of indecomposables, and these are either aspherical or have virtually free fundamental group. There are many examples of the latter type which are not homotopy equivalent to 3-manifolds, but the possible groups are largely known. However the question of whether every aspherical PD_3-complex is homotopy equivalent to a 3-manifold remains open.

We shall outline the work which lead to this reduction to the aspherical case, mention briefly remaining problems in connection with indecomposable virtually free fundamental groups, and consider how we might try to show that PD_3-groups are 3-manifold groups.

Sang-hyun Kim
Diffeomorphism groups of critical regularity

Abstract: For each real number $a = k + c >= 1$ where $k = [a]$, we let $\text{Diff}^a(S^1)$ denote the group of C^k–diffeomorphisms on S^1 whose k–th derivatives are c-Hölder–continuous. For each $a \geqslant 1$, we construct a fg group G_a inside $\text{Diff}^a(S^1)$ such that G_a admits no injective homomorphisms into $\bigcup_{b>a} \text{Diff}^b(S^1)$. We also have a dual result: there exists a finitely generated group H_a inside $\bigcap_{b<a} \text{Diff}^b(S^1)$ such that H_a admits no injective homomorphisms into $\text{Diff}^a(S^1)$. This result was previously known for $a = 1$, and partially for $a = 1.5$ and $a = 2$ (Joint work with Thomas Koberda)

Sylvain Maillot
One-ended 3-manifolds without locally finite toric decompositions

Abstract: We describe a new family of examples of open, irreducible 3-manifolds containing an infinite family of canonical tori which is not isotopic to a locally finite one. Our examples are obtained as double-covers of π-orbifolds with underlying space $\mathbf{R}^3$ and are easier to visualize that previous examples.

Alan Reid
Profinite rigidity in low dimensions

Abstract: A finitely generated residually finite group G is called profinitely rigid if whenever a finitely generated residually finite group H has a profinite completion isomorphic to that of G, then H is isomorphic to G. Although by now there are many constructions of groups that are not profinitely rigid, there seems to be a growing sense that when G is a free group, surface group or the fundamental group of a finite volume hyperbolic 3-manifold, things are different and these will be profinitely rigid. In this lecture we will survey recent progress on this topic in the context of 3-manifold groups.

Emily Stark

The visual boundary of hyperbolic free-by-cyclic groups

Abstract: Given an automorphism of the free group, we consider the mapping torus defined with respect to the automorphism. If the automorphism is atoroidal, then the resulting free-by-cyclic group is hyperbolic by work of Brinkmann. In addition, if the automorphism is fully irreducible, then work of Kapovich–Kleiner proves the boundary of the group is homeomorphic to the Menger curve. However, their proof is very general and gives no tools to further study the boundary and large-scale geometry of these groups. In this talk, I will explain how to construct explicit embeddings of non-planar graphs into the boundary of these groups whenever the group is hyperbolic. Along the way, I will illustrate how our methods distinguish free-by-cyclic groups which are the fundamental group of a 3-manifold. This is joint work with Yael Algom-Kfir and Arnaud Hilion.

Bena Tshishiku

Groups with Bowditch boundary a 2-sphere

Abstract: Bestvina-Mess showed that the duality properties of a group G are encoded in any boundary that gives a Z-compactification of G. For example, a hyperbolic group with Gromov boundary an n-sphere is a $PD(n+1)$ group. For relatively hyperbolic pairs (G, P), the natural boundary – the Bowditch boundary – does not give a Z-compactification of G. Nevertheless we show that if the Bowditch boundary of (G, P) is a 2-sphere, then (G, P) is a $PD(3)$ pair. This is joint work with Genevieve Walsh.

Gareth Wilkes

Relative cohomology, profinite completions and 3-manifold decompositions

Abstract: I will describe the properties of the cohomology theory of group pairs and its recent extension to profinite groups. This will then be used to illustrate the detection of the prime and JSJ decompositions of 3-manifolds via the finite quotients of the fundamental group. If time permits, I will also discuss a connection to the residual properties of mapping class groups.

Henry Wilton

Negative immersions for one-relator groups

Abstract: The class of one-relator groups has many parallels with the class of fundamental groups of Haken 3-manifolds: they are aspherical in a suitable sense, and enjoy a hierarchy which was famously used by Magnus to solve the word problem. However, unlike the 3-manifold case, there is a huge menagerie of pathological examples, there is no geometrization theorem, and many fundamental problems remain unsolved.

I will explain new results which show that the subgroup structure of a one-relator group $F/''r''$ is governed by the "primitivity rank" $\pi(w)$: the smallest rank of a subgroup of the free group F that contains w as an imprimitive element. A sample theorem is that every subgroup of a one-relator group $F/''w''$ with fewer than $\pi(w)$ relators is free.

This joint work with Larsen Louder.