Embedding braid groups in mapping class groups

Alan McLeay – University of Glasgow

Joint w/ Ty Ghaswala – University of Manitoba

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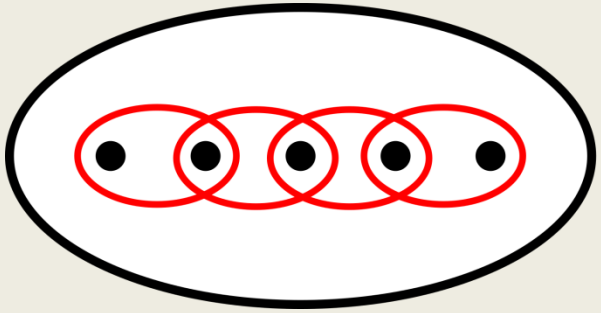

$$H_{a_i} \leftarrow \sigma_i$$

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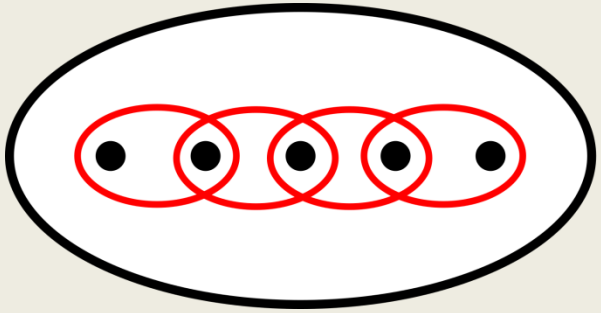
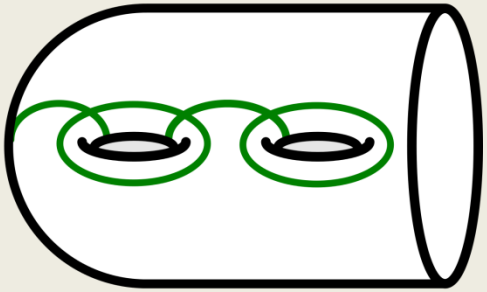
$$\mathrm{Mod}(D_5) \leftarrow B_5 \rightarrow \mathrm{Mod}(S_{2,1})$$





$$H_{a_i} \leftarrow \sigma_i \rightarrow T_{b_i}$$

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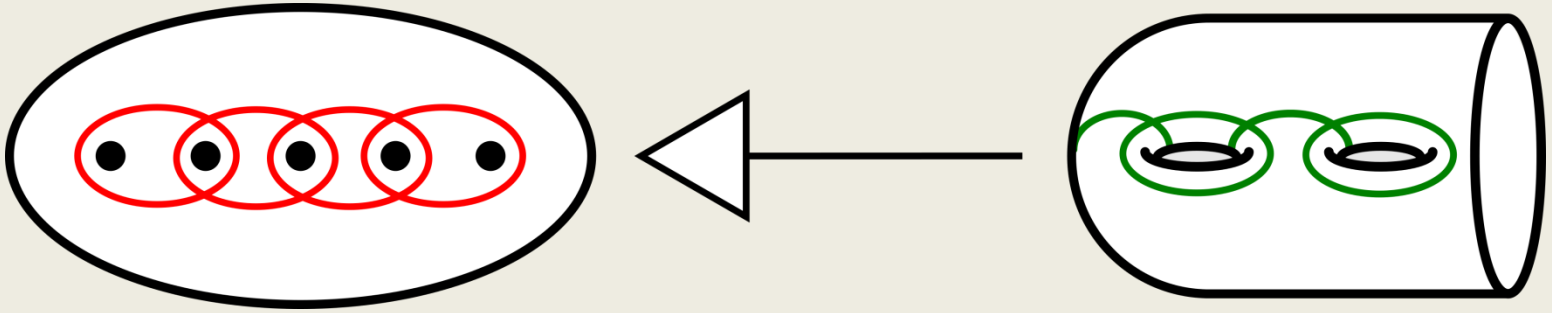

$$H_{a_i} \leftarrow \sigma_i \rightarrow T_{b_i}$$

Geometric embeddings:

Send σ_i to a twist.

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(Wajnryb)

Are there any non-geometric embeddings?

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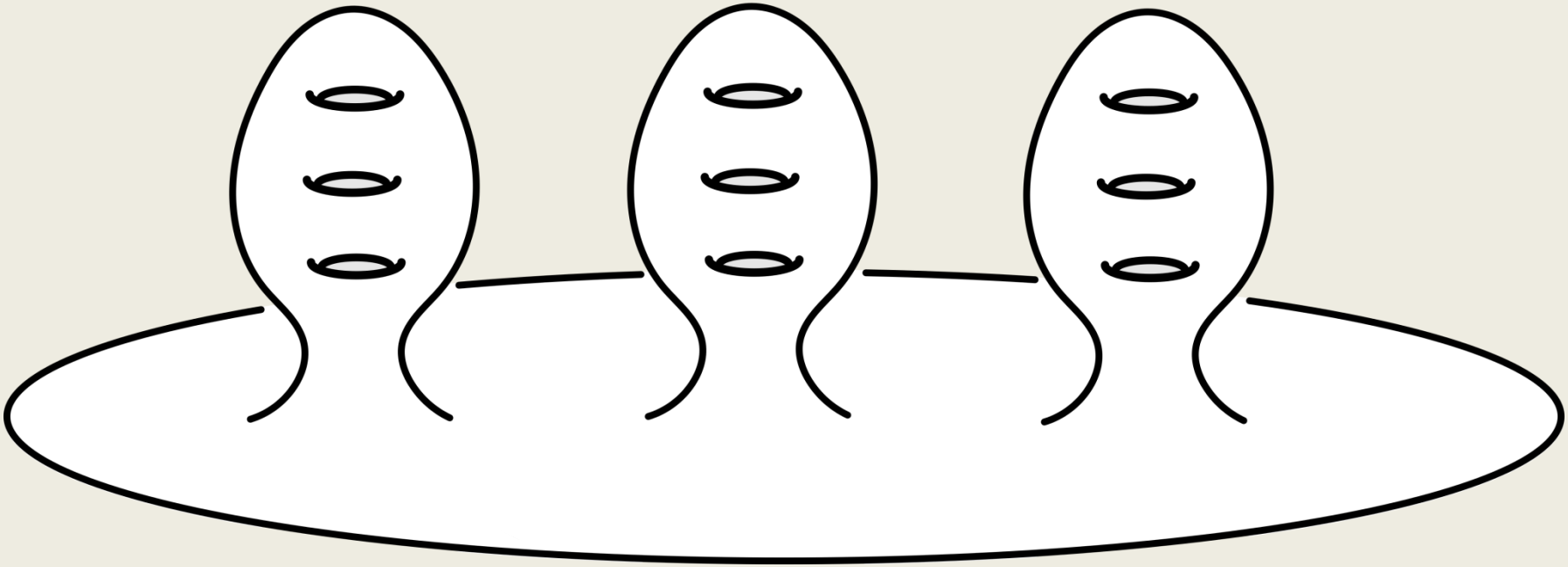
Yes.

Example



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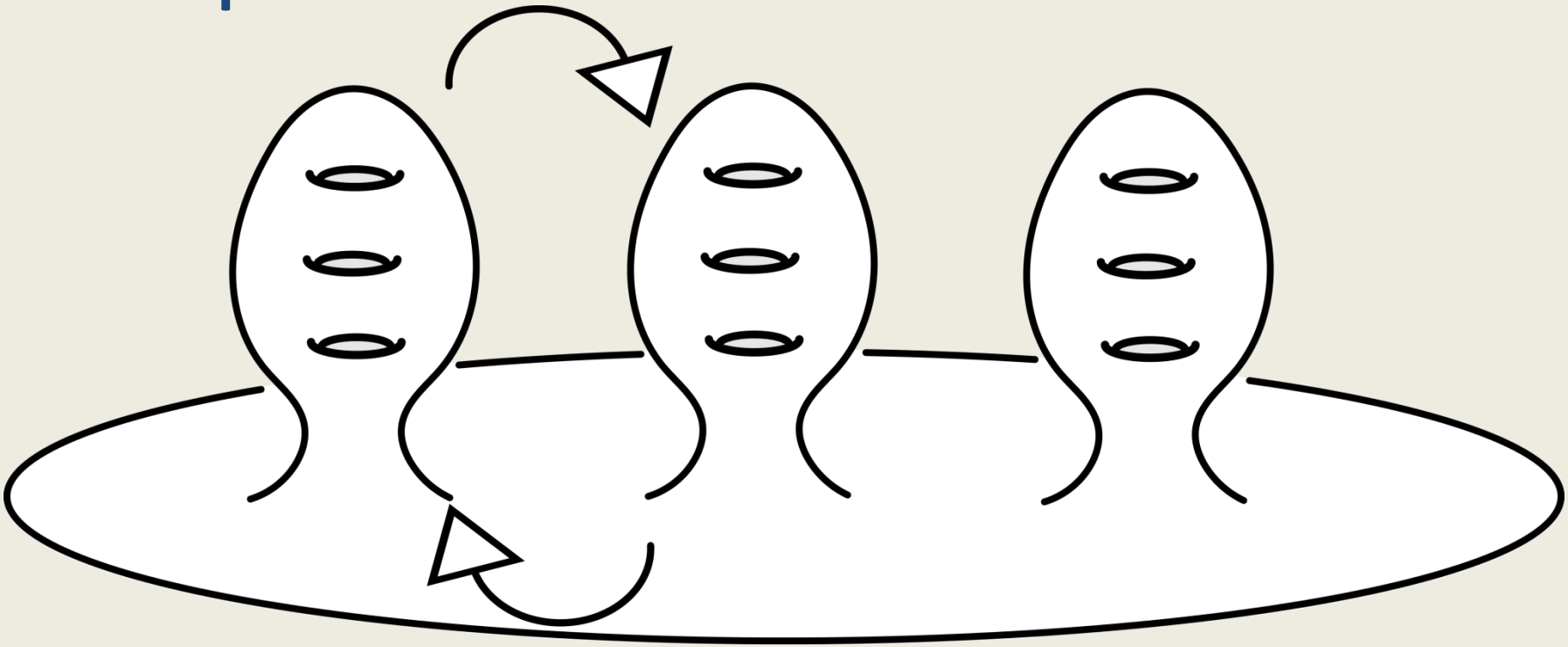
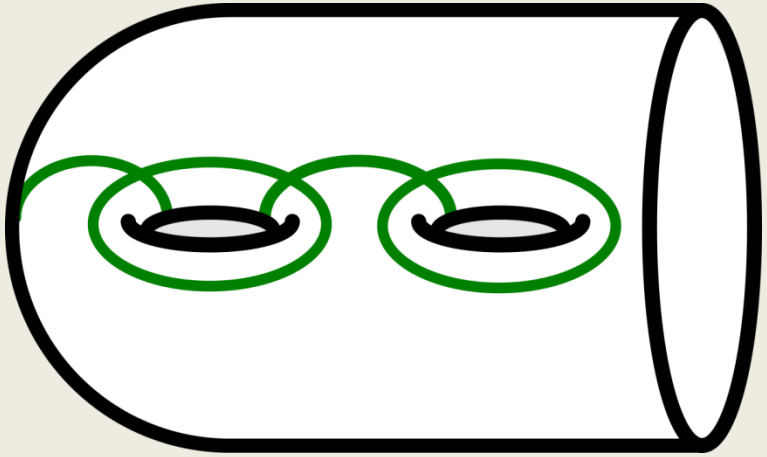
Example



Your NEW favourite embedding!

We send each σ_i to a product of Dehn twists about curves forming a k-chain.

Your NEW favourite embedding!



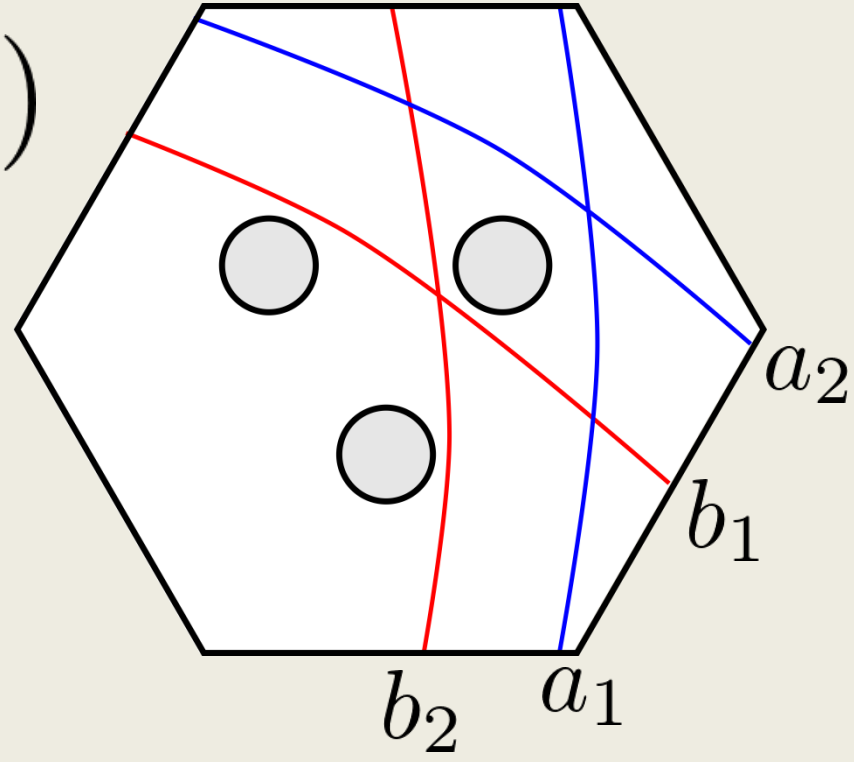
$$\sigma_i \to T_{c_1} T_{c_2} T_{c_3} T_{c_4}$$

Your NEW favourite embedding!

$$B_3 \to \mathrm{Mod}(S_{1,3})$$

$$\sigma_1 \to T_{a_1} T_{a_2}$$

$$\sigma_2 \to T_{b_1} T_{b_2}$$



Your NEW favourite embedding!

$$B_3 \to \mathrm{Mod}(S_{1,3})$$

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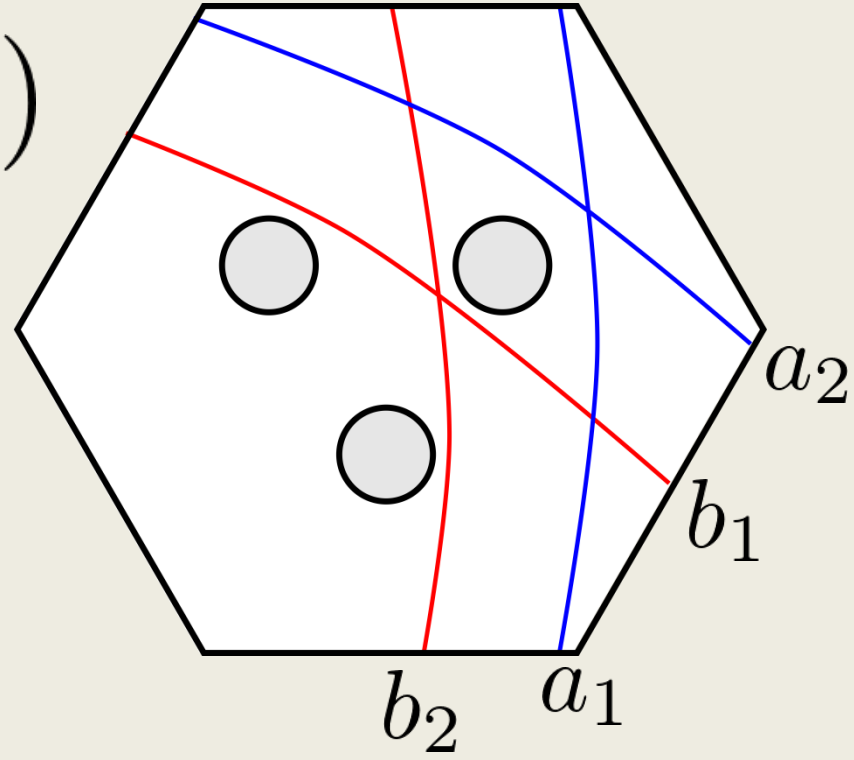

$$(T_{a_1} T_{a_2})(T_{b_1} T_{b_2})(T_{a_1} T_{a_2}) = (T_{b_1} T_{b_2})(T_{a_1} T_{a_2})(T_{b_1} T_{b_2})$$

Ask me about…

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Birman-Hilden Theorem (new proof!)

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Complete classification of finite-sheeted, regular covering spaces.

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Burau representation of braid groups.

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Fundamental groupoid (automorphisms).