The Teichmüller TQFT

Faddeev's quantum dilogarithm: $(b \in \mathbb{C} \text{ s.t. } \hbar^{-1/2} = b + b^{-1})$

$$\Phi_\hbar(x) := \exp\left(\int_{z \in \mathbb{R}+i0^+} \frac{e^{-2izx}\, dz}{4\sinh(z\mathbf{b})\sinh(z\mathbf{b}^{-1})z}\right)$$

Tetrahedral operator : $p_\hbar(T)(\alpha) \in S'(\mathbb{R}^{X_2})$,

$$\overline{x} \mapsto \frac{\delta\left(x_0(T) - x_1(T) + x_2(T)\right) e^{2\pi i \epsilon(T) x_0(T)(x_3(T) - x_2(T))} e^{\frac{\alpha_3(T)(x_3(T)-x_2(T))}{\hbar^{1/2}}}}{\Phi_\hbar\left((x_3(T) - x_2(T)) - \frac{i}{2\pi\hbar^{1/2}}\epsilon(T)(\alpha_2(T) + \alpha_3(T))\right)^{\epsilon(T)}}$$

Partition function of the triangulation X :

$$Z_\hbar(X, \alpha) = \int_{\overline{x} \in \mathbb{R}^{X_2}} d\overline{x} \prod_{T \in X_3} p_\hbar(T)(\alpha)(\overline{x}) \quad \in \mathbb{C}$$