REPRESENTATION SPACES, TEICHMÜLLER THEORY, AND THEIR RELATIONSHIP WITH 3-MANIFOLDS FROM THE CLASSICAL AND QUANTUM VIEWPOINTS

CIRM, LUMINY, 29 JANUARY - 2 FEBRUARY, 2018

Abstracts

Stéphane Baseilhac

On the quantum Teichmüller invariants of surface diffeomorphisms

Abstract: I will review the construction of intertwiners of representations of the quantum Teichmüller spaces associated to surface diffeomorphisms, and discuss related open problems. This is joint work with Riccardo Benedetti and Philippe Roche.

Fathi Ben Aribi

The Teichmüller TQFT volume conjecture for twist knots

Abstract: In 2014, Andersen and Kashaev defined an infinite-dimensional TQFT from the Quantum Teichmüller theory. This Teichmüller TQFT is an invariant of triangulated 3-manifolds, in particular knot complements.

The associated volume conjecture stated that the Teichmüller TQFT of an hyperbolic knot complement contains the volume of the knot as a certain asymptotical coefficient, and Andersen-Kashaev proved this conjecture for the first two hyperbolic knots.

In this talk I will present the construction of the Teichmüller TQFT and how we approached this volume conjecture for the infinite family of twist knots: in particular, we proved the conjecture for several new examples of knots, up to 13 crossings.

Joint work with Eiichi Piguet-Nakazawa.

Tetsuya Ito

On a characterizaion of strongly quaipositive links

Abstract: A braid is strongly quasipositive if it is a product of positvie band generators $a_{i,j}$, a braid expressed as a boundary of a positively twisted band connecting the i-th and the j-th strand. One of an important property of strongly quasipositive braids is that its closure has a sharp Bennequin bound – when a knot K is a closure of strongly quasipositive n-braid we have $2g(K)-1=-n+e$, where e denotes the exponent sum of the braid.

In this talk, assuming the largeness of the FDTC (Fractional Dehn twist coefficient) we show that the converse – the sharpness of the Bennequin inequality implies the strong quasipositivity. If time permit, we will dicuss a generalization of the results for closed braids in general open book decomposition of 3-manifolds.

This is a joint work with Keiko Kawamuro (Univ. Iowa).

Julien Marché

Differential equation for the Reidemeister Torsion

Abstract: The Reidemeister torsion may be viewed as a volume form on the character variety of a 3-manifold with boundary. I will explain a conjectural differential equation that this form should satisfy, motivated by the study of the asymptotical behaviour of quantum invariants.

Kimihiko Motegi

L-space knots in twist families and satellite L-space knots

Abstract: Twisting a knot K in S^3 along a disjoint unknot c produces a twist family of knots $\{K_n\}$ indexed by the integers. Comparing the behaviors of the Seifert genus $g(K_n)$ and the slice genus $g_4(K_n)$ under twistings, we prove that if $g(K_n) - g_4(K_n) < C$ for some constant C for infinitely many integers $n > 0$ or $g(K_n)/g_4(K_n) \to 1$ as $n \to \infty$, then either the winding number of K about c is zero or the winding number equals the wrapping number. As an application, if $\{K_n\}$ contains infinitely many L-space knots, then the latter must occur. We further develop this to show that if K_n is an L-space knot for infinitely many integers $n > 0$ and infinitely many integers $n < 0$, then c is a braid axis. We then use this to show that satellite L-space knots are braided satellites.
This is joint work with Ken Baker.

Delphine Moussard

Finite type invariants of knots in homology 3-spheres

Abstract: For null-homologous knots in rational homology 3-spheres, there are two equivariant invariants obtained by universal constructions à la Kontsevich, one due to Kricker and defined as a lift of the Kontsevich integral, and the other constructed by Lescop by means of integrals in configuration spaces. In order to explicit their universality properties and to compare them, we study a theory of finite type invariants of null-homologous knots in rational homology 3-spheres. We give a partial combinatorial description of the space of finite type invariants, graded by the degree. This description is complete for knots with a trivial Alexander polynomial, providing explicit universality properties for the Kricker lift and the Lescop equivariant invariant and proving the equivalence of these two invariants for such knots.

Jun Murakami

Braided Wirtinger presentation of knots

Abstract: The Wirtinger presentation of a knot group G is reformulated to the conjugate quandle associated with G, and a $\mathrm{PSL}(2)$ representation ρ of G is also determined by the same quandle. The matrix elements of ρ are functions on G and so they form a Hopf algebra, which is presented by the dual of the above conjugate quandle. Here we generalize the Hopf algebra associated with ρ to a braided Hopf algebra, and explain its relationship with the Reidemeister moves. A typical example of the braided Hopf algebra is the braided quantum group $\mathrm{BSL}(2)$ introduced by S. Majid, and some special properties for $\mathrm{BSL}(2)$ case is also explained.
This is a joint work with Roloand van der Veen.

Takahiro Oba
Surfaces in D^4 with the same boundary and fundamental group

Abstract: This talk is concerned with symplectic surfaces in a symplectic 4-disk (D^4, ω) bounded by the same transverse link in the standard contact 3-sphere (S^3, ξ_{st}). There are some examples of transverse links (or knots) bounding more than one symplectic surface up to isotopy. All these surfaces can be distinguished by the fundamental groups of their complements. In this talk, I will present a family of pairs of distinct two symplectic surfaces whose boundaries are the same transverse knot and whose complements have isomorphic fundamental groups. To distinguish the two surfaces of each pair, I take double branched covers branched along them.

Frédéric Palesi
Dynamical decomposition of character varieties of surface groups

Abstract: The Teichmüller space of a closed surface can be embedded in the $\mathrm{PSL}(2, \mathbb{C})$ character variety as a connected component. The mapping class group acts properly discontinuously on the connected component corresponding to Teichmüller space, but the dynamic of the action is much more mysterious on the rest of the character variety. In particular, a famous conjecture of Goldman states that the action is ergodic on each non-Teichmuller connected component of the $\mathrm{PSL}(2, \mathbb{C})$ character variety. There is a similar conjecture for type-preserving representations of punctured surfaces, and these conjectures have only be solved in a few particular cases.

In this talk we focus on surfaces with low complexity, possibly non-orientable and/or with boundaries, and construct open domains of discontinuity in the $\mathrm{SL}(2, \mathbb{C})$ character variety containing indiscrete representations. To do so we use a combinatorial tool which allows us to understand the simple trace spectrum of a representation. As a consequence, we can use this tool to provide a trace reduction algorithm in the real case in order to determine if there is a simple closed curve sent to an elliptic element. This allows us to prove ergodicity in certain cases.

Joan Porti
Volume forms on the $\mathrm{SL}_n(\mathbb{C})$-moduli space of surfaces with boundary

Abstract: For an oriented surface of finite type, we consider the moduli space of representations in a simply connected reductive Lie group (eg $\mathrm{SL}_n(\mathbb{C})$, and also the moduli space relative to the boundary. We relate the complex valued volume forms in those moduli spaces, the relative and the absolute one.
This is joint work with M. Heusener.

Jean Raimbault
Counting manifolds and subgroups

Abstract: I will discuss counting problems for hyperbolic manifolds. A previous example of the kind of results that the talk is about is that the number of arithmetic surfaces of volume $2\pi n$ is roughly of size n^n as $n \to +\infty$. One of the main steps in its proof is estimating the number of subgroups of index n in a surface group. I will explain a similar result on subgroups of right-angled Artin and Coxeter groups.
This is joint work with Hyungryul Baik and Bram Petri.

Takao Satoh

On the Andreadakis conjecture of the automorphism groups of free groups

Abstract: In the mapping class group of a surface, there are two descending central filtrations of the Torelli group. One is called the Johnson filtration, which is defined by using the actions of the mapping class group on the nilpotent quotients of the fundamental group of the surface. The other is the lower central series of the Torelli group. Due to Johnson and Morita, it is known that they are different by a certain "obstractions" coming from topological reasons.

Here, we consider a similar situation for the automorphism group of a free group. The group of automorpshisms which act on the abelianization of the free group trivially is called the IA-automorphism group. This group has two descending central filtrations. One is called the Andreadakis-Johnson filtration, and the other is its lower central series. Andreadakis showed that they are equal for the rank of the free group is two, and conjectured that they coincede with in general. Recently Bartholdi showed that this conjecture is not true for the rank is three.

In this talk, we will talk about a combinatorial group theoretic approach to this problem, and some recent results. In particular, we show that the third subgroups of the two filtrations are equal.

Toshie Takata

On the quantum SU(2) invariant at $q = \exp(4\pi\sqrt{-1}/N)$ ***and the twisted Reidemeister torsion for some closed 3-manifolds***

Abstract: We show that a square root of the Reidemeister torsion appears as a coefficient in the semi-classical approximation of the asymptotic expansion of the quantum $SU(2)$ invariant of M_p at $q = \exp(4\pi\sqrt{-1}/N)$. Further, when $q = \exp(4\pi\sqrt{-1}/N)$, we show that the semi-classical approximation of the asymptotic expansion of the quantum $SU(2)$ invariant of some Seifert 3-manifolds M is presented by a sum of contributions from some of $SL_2\mathbb{C}$ flat connections on M, and square roots of the Reidemeister torsions appear as coefficients of such contributions.
This is a joint work with Tomotada Ohtsuki.

Liam Watson

Heegaard Floer homology via immersed curves

Abstract: The Heegaard Floer homology of a manifold with torus boundary can be expressed as a collection of immersed curves (possibly decorated with local systems). This provides a geometric structure theorem, interpreting the algebraic invariants that arise in bordered Floer theory. From this point of view, the Heegaard Floer homology of a closed manifold obtained by gluing manifolds (with boundary) along a torus may be recovered as the Lagrangian intersection Floer homology of the associated curves. In practice, this reduces gluing problems to simple minimal intersection counts. This talk, which is part of a joint project with J. Hanselman and J. Rasmussen will give an overview of this machinery and describe some of the applications that follow.

Bert Wiest

Algorithmic aspects of braids and subsurface projections

Abstract: Given a braid, how can we detect to which subsurfaces of the punctured disk it has large projections? And how do such subsurface projections play into attempts to find fast solutions to the conjugacy problem in braid groups using Garside theory? I will talk about some aspects of these question, representing joint work with Sandrine Caruso and with Saul Schleimer.

Anton Zorich

Equidistribution of square-tiled surfaces, meanders, and Masur-Veech volumes

Abstract: We show how recent results of the authors on equidistribution of square-tiled surfaces of given combinatorial type allow to compute approximate values of Masur–Veech volumes of the strata in the moduli spaces of Abelian and quadratic differentials by Monte Carlo method.

We also show how similar approach allows to count asymptotical number of meanders of fixed combinatorial type in various settings in all genera. Our formulae are particularly efficient for classical meanders in genus zero.

We construct a bridge between flat and hyperbolic worlds giving a formula for the Masur–Veech volume of the moduli space of quadratic differentials in terms of intersection numbers of $\mathcal{M}_{g,n}$ (in the spirit of Mirzakhani's formula for Weil–Peterson volume of the moduli space of pointed curves).

Joint work with V. Delecroix, E. Goujard, P. Zograf.