INTERNATIONAL WORKSHOP ON GEOMETRIC QUANTIZATION AND APPLICATIONS

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Abstracts

Anton ALEKSEEV

Poisson-Lie duality and Langlands duality via Bohr-Sommerfeld

Abstract: Let G be a connected semisimple Lie group with Lie algebra $\mathfrak{g}$. There are two natural duality constructions that assign to it the Langlands dual group $G^\vee$ (associated to the dual root system) and the Poisson-Lie dual group G^*. Cartan subalgebras of $\mathfrak{g}^\vee$ and $\mathfrak{g}^*$ are isomorphic to each other, but $G^\vee$ is semisimple while G^* is solvable.

In this talk, we explain the following non-trivial relation between these two dualities: the integral cone defined by the Berenstein-Kazhdan potential on the Borel subgroup $B^\vee \subset G^\vee$ is isomorphic to the integral Bohr-Sommerfeld cone defined by the Poisson structure on $K^* \subset G^*$ (the Poisson-Lie dual of the compact form $K \subset G$). The first cone parametrizes canonical bases of irreducible G-modules. The corresponding points in the second cone belong to integral symplectic leaves of K^*.

The talk is based on a joint work with A. Berenstein, B. Hoffman and Y. Li.

Hugues AUVRAY

Bergman kernels on punctured Riemann surfaces

Abstract: In a joint work with X. Ma (Paris 7) and G. Marinescu (Cologne), we obtain refined asymptotics for Bergman kernels computed from singular data on Riemann surfaces. More precisely, we work on the complement of a finite set of points, seen as singularities, on a compact Riemann surface, that we endow with a metric extending Poincaré's cusp metric around the singularities. The polarization line bundle is in turn equipped with a positively curved Hermitian metric, whose curvature is the base metric near the singularities. I shall thus explain how an advanced description of the model geometry (given by Poincaré's metric on the punctured unit disc), and localization techniques in the spirit of Bismut-Lebeau in a weighted analysis context, allow us to describe the Bergman kernels attached to these punctured Riemann surfaces, up to their singularities. I shall also mention an arithmetic interpretation of these results, in terms of modular forms.

Jean-Michel BISMUT
Hypoelliptic Laplacian and coadjoint orbits

Abstract: The hypoelliptic Laplacian is a family of operators, indexed by $b \in \mathbf{R}_+^*$, acting on the total space of the tangent bundle of a Riemannian manifold, that interpolates between the ordinary Laplacian as $b \to 0$ and the generator of the geodesic flow as $b \to +\infty$. These operators are not elliptic, they are not self-adjoint, they are hypoelliptic.

In the talk, I will discuss the role of the hypoelliptic Laplacian in the riugorous proof of localization formulas in equivariant cohomology on the coadjoint orbits of the loop group of reductive groups. In the case of compact Lie groups, we reobtain this way the classical evaluation of the heat kernel in terms of the coroot lattice. For general reductive Lie groups, we will explain the local formulas we obtained for semisimple elliptic orbital integrals.

Arzu BOYSAL
Multiple Bernoulli series and volumes of moduli spaces of flat bundles over surfaces

Abstract: Using Szenes formula for multiple Bernoulli series, we explain how to compute Witten series associated to classical Lie algebras. Particular instances of these series compute volumes of moduli spaces of flat bundles over surfaces, and also certain multiple zeta values.
This is joint work with V. Baldoni and M. Vergne.

Maxim BRAVERMAN
The Atiyah-Patodi-Singer index on manifolds with non-compact boundary

Abstract: We study the index of the APS boundary value problem for a strongly Callias-type operator D on a complete Riemannian manifold M. We use this index to define the relative eta-invariant of two strongly Callias-type operators A and A', which are equal outside of a compact set. Even though in our situation the eta-invariants of A and A' are not defined, the relative eta-invariant behaves as if it were the difference of the eta-invariants of A and A'. We also define the spectral flow of a family of such operators and use it compute the variation of the relative eta-invariant.
It is a joint work with Pengshuai Shi.

Michel BRION
Homogeneous vector bundles over abelian varieties

Abstract: The objects of the talk are the translation-invariant vector bundles over an abelian variety. We will present a representation-theoretic description of these vector bundles, which displays a remarkable analogy with finite-dimensional representations of a compact connected Lie group: the weight lattice is replaced with the dual abelian variety, the Weyl group with the Galois group of the ground field...

Michel DUFLO
Michèle Vergne and the Orbit Method

Abstract: The title is self-explanatory

Nigel HIGSON
On Mackey's parametrization of tempered irreducible representations

Abstract: In the 1970's George Mackey suggested that there ought to be a "correspondence" between "most" irreducible unitary representations of a real reductive group G and the irreducible unitary representations of its Cartan motion group $G_0 = K \ltimes \mathfrak{g}/\mathfrak{k}$ (here K is a maximal compact subgroup of G). Mackey's idea was kept alive by Alain Connes, who noticed a related correspondence in operator K-theory, and eventually a precise bijection was constructed between the irreducible *tempered* unitary representations of G and the irreducible unitary representations of G_0; the final, decisive steps were taken by Alexandre Afgoustidis in his thesis. I'll describe these developments, and then examine the next challenge: to give a conceptual explanation for the phenomenon that Mackey predicted. Here there are some interesting possibilities; among the most promising are conjectual results that seem to fit squarely within C^*-algebra theory.

Louis IOOS
Geometric quantization of symplectic maps and Witten's asymptotic conjecture

Abstract: The process of geometric quantization of a compact symplectic manifold depends on the choice of a complex structure, and a natural way to study this dependence is to consider the spaces of quantum states as a vector bundle over a space of complex structures. This idea is of particular interest in the context of moduli spaces of flat connections over a compact surface and the associated Verlinde bundle over Teichmüller space, for which there exists a canonical projectively flat connection. Parallel transport with respect to this connection allows one to define the Witten-Reshetikhin-Turaev invariant of mapping tori. In this talk, I will establish the asymptotic expansion of this invariant as the level tends to infinity, and compute its first coefficient. This follows from a general semi-classical study of parallel transport in quantum bundles, via the theory of Berezin-Toeplitz quantization.

Toshiyuki KOBAYASHI
Global geometry and analysis on locally symmetric Spaces with indefinite-metric

Abstract: The local to global study of geometries was a major trend of 20th century geometry, with remarkable developments achieved particularly in Riemannian geometry. In contrast, in areas such as preudo-Riemannian geometry, familiar to us as the spacetime of relativity theory, and more generally in pseudo-Riemannian geometry of general signature, surprising little is known about global properties of the geometry even if we impose a locally homogeneous structure.

I plan to explain two projects:

Global geometry : Existence problem of compact locally homogeneous spaces, and deformation theory.

Spectral analysis : Construction of periodic eigenfunctions for the Laplacian for indefinite-metric, and discuss the stability of eigenvalued under deformation of geometric structure.

Shrawan KUMAR

Conformal blocks for Galois covers of algebraic curves

Abstract: We study the space of twisted conformal blocks attached to A-curves S with marked A-orbits and an action of A on a simple Lie algebra $\mathfrak{g}$, where A is a finite group. We prove that if A stabilizes a Borel subalgebra of $\mathfrak{g}$, then Propogation Theorem and Factorization Theorem hold. We endow a projectively flat connection on the sheaf of twisted conformal blocks attached to a smooth family of pointed A-curves; in particular, it is locally free. We also prove that the sheaf of twisted conformal blocks on the stable compactification of Hurwitz stack is locally free. We further identify the space of twisted conformal blocks with the space of global sections of certain line bundles on the stack of A-equivariant principal G-bundles over the curve S, G being the simply-connected group with Lie algebra $\mathfrak{g}$. This generalizes the Verlinde theory of conformal blocks to the twisted setting.
This is a joint work with Jiuzu Hong.

Bingxiao LIU

Selberg's twisted trace formula and asymptotics of equivariant analytic torsion

Abstract: Bismut obtained an explicit geometric formula for the semisimple orbital integrals by using the theory of hypoelliptic Laplacian on the real reductive Lie group G. Here we introduce a twist $\sigma \in \mathrm{Aut}(G)$, and we extend this explicit formula to the corresponding twisted orbital integrals. Then with the help of Selberg's twisted trace formula, we can evaluate the leading term in the asymptotic expansion of the σ-equivariant analytic torsion associated with a sequence of flat vector bundles $F_d|_{d\in\mathbb{N}}$ on a compact locally symmetric space Z. We find that the leading term is given in terms of W-invariants defined by Bismut-Ma-Zhang associated with the fixed point set of σ in Z.

Salah MEHDI

Asymptotics of characters, nilpotent orbits and Dirac index

Abstract: We describe a translation principle for the Dirac index of virtual $(\mathfrak{g}, K)$-modules. To each coherent family of such modules we attach a polynomial, on the dual of the compact Cartan subalgebra, which expresses the dependence of the leading term in the Taylor expansion of the character of the modules. We will explain how this polynomial is related to nilpotent orbits, via the multiplicities of the associated cycle of Harish-Chandra modules.
These results are joint with P. Pandžić, D. Vogan and R. Zierau.

Eva MIRANDA

Geometric quantization of toric and semitoric systems

Abstract: One of the many contributions of Kostant is a rare gem which probably has not been sufficiently explored: a sheaf-theoretical model for geometric quantization associated to real polarizations. Kostant's model works very well for polarizations given by fibrations or fibration-like objects (like integrable systems away from singularities). For toric manifolds where the real polarization is determined by the fibers of the moment map, Kostant's model yields a representation space whose dimension is the number of integer points inside the corresponding Delzant polytope. We will discuss extensions of this model to consider almost toric manifolds and integrable systems with non-degenerate singularities where "unexpected" infinities can show up even if the manifold is compact.

Leonid POLTEROVICH
Quantum footprints of symplectic rigidity

Abstract: We discuss interactions between quantum mechanics and symplectic topology including a link between symplectic displacement energy, a fundamental notion of symplectic dynamics, and the quantum speed limit, a universal constraint on the speed of quantum-mechanical processes.
Joint work with Laurent Charles.

Pablo RAMACHER
Singular reduction and quantization

Abstract: We study singular situations arising in equivariant spectral geometry and cohomology using resolution of singularities. We derive a local Weyl law for the reduced spectral function of an invariant elliptic operator on a compact manifold carrying the action of a compact Lie group, and characterize its caustic behaviour near singular orbits. From this we deduce pointwise and L^p-bounds for eigenfunctions, showing that the orbit structure is reflected in the shape of eigenfunctions. Based on these results, we prove subconvex bounds for Hecke–Maass forms on compact arithmetic quotients in the eigenvalue and isotypic aspect, as well as an equivariant quantum ergodicity theorem. To conclude, we report on an ongoing project devoted to the derivation of residue formulae in equivariant cohomology for general symplectic manifolds, and present first results for S^1-actions. As we shall explain, our work relies on the description of the asymptotic behaviour of certain oscillatory integrals of Witten-type.

András SZENES
Grothendieck polynomials and K-theoretic Thom polynomials

Abstract: We present a residue calculus for stable Grothendieck polynomials, and using these new formulas, we initiate the study of the Grothendieck expansion of K-theoretic Thom polynomials of contact singularities.
This is joint work with richard Rimanyi.

Shu SHEN
Fried conjecture on locally symmetric spaces

Abstract: The relation between the spectrum of the Laplacian and the closed geodesics on a closed Riemannian manifold is one of the central themes in differential geometry. Fried conjectured that the analytic torsion, which is an alternating product of regularized determinants of the Hodge Laplacians, equals the zero value of the dynamical zeta function of Ruelle. We will explain a proof of this conjecture in the case where the underlying manifold is a closed locally symmetric space of arbitraire rank. The proof is based on the explicit formula for semisimple orbital integrals due to Bismut and on the classification of unitary representations of reductive groups with vanshing cohomology due to Vogan, Zuckermann, and Salamanca-Riba.

Sue TOLMAN

Beyond semitoric

Abstract: A compact four dimensional completely integrable system $f\colon M \to \mathbb{R}^2$ is **semitoric** if it has only non-degenerate singularities, without hyperbolic blocks, and one of the components of f generates a circle action. Semitoric systems have been well studied and have many nice properties; for example, the fibers $f^{-1}(x)$ are connected. Unfortunately, although there are many interesting examples of semitoric systems, the class has some limitations. For example, there are blowups of $S^2 \times S^2$ with Hamiltonian circle actions that cannot be extended to semitoric system. We show that, by allowing certain degenerate singularities, we can expand the class of semitonic systems but still prove that $f^{-1}(x)$ is connected. We hope that this class will be large enough to include not only all compact four manifolds with Hamiltonian circle actions, but more generally all complexity one spaces.
Based on joint work with D. Sepe.

David VOGAN

Associated varieties and geometric quantization

Abstract: Geometric quantization seeks to find and understand something close to a bijection between *coadjoint orbits* (a classical notion) for a Lie group G and *irreducible unitary representations* (a quantum notion) of that same Lie group. We know how to do this in many cases, but there seems to be a lot of interesting mathematics hidden in the cases we do not understand. The notion of *associated variety* is an elementary one, easy to understand, and it provides something like a"left inverse" to the geometric quantization functor we seek. I will explain an algorithm for computing the associated variety of any irreducible representation of a real reductive group, and say some of what that tells us about quantization.

San VU NGOC

Analyticity of the Bergman projection

Abstract: Recently, Rouby gave a precise description, in the semiclassical limit, of the spectrum of a class of non-selfadjoint pseudo-differential operators on the real line, with analytic symbols. This results opens interesting questions relating the complex spectrum to the symplectic geometry of the phase space, and it was natural to investigate the generalization of the analytic microlocal tools to the setting of Berezin-Toeplitz quantization.

As a first step, I will present a recent result on the structure of the weighted Bergman projection on $\mathbb{C}^n$ or on a compact Kähler manifold with high powers of a prequantum line bundle, when the weight (or curvature) is analytic. In this case, the projection is an elliptic Fourier integral operator acting on the class of analytic symbols. As a corollary, we prove a conjecture of Zelditch about the analyticity of the semiclassical asymptotics of the Bergman kernel.
It is a joint work with with Ophélie Rouby and Johannes Sjöstrand.

Michael WALTER

Moment polytopes and noncommutative optimization

Abstract: We will revisit the problem of characterizing moment polytopes from an algorithmic perspective. We will discuss how to relate moment polytopes to noncommutative optimization and describe how this point of view, which can be traced back to Kirwan's gradient flow, leads to efficient new algorithms. Of particular interest will be the moment polytopes for the tensor product action of products of general linear groups, which capture the asymptotic support of the Kronecker coefficients.

Siye Wu

Deformation of the prequantum action

Abstract: We construct a family of star products of functions on polarised sections of the prequantum line bundle over as a deformation of the standard prequantum action in geometric quantisation. We show that the star products on symplectic vector spaces are compatible with the flat connection defined by intertwining operators on functions and the projectively flat connection on the bundle of Hilbert spaces. We also discuss star products of fermionic systems.

Weiping ZHANG

Enlargeability and positive scalar curvature on foliations

Abstract: We report recent progress concerning positive scalar curvature on foliations, including generalizations of previous results to the enlargeable foliated spaces.
