Non Standard Diffusions in Fluids, Kinetic Equations and Probability

CIRM, December 10-14 2018

Ricardo Alonso: Emergence of exponentially weighted L^p-norms and Sobolev regularity for the Boltzmann equation.

Abstract: We consider the homogeneous Boltzmann equation for Maxwell and hard potentials, without cutoff, and study the appearance and propagation of L^p-norms, including polynomial and exponential weights. Propagation of Sobolev regularity with such weights is also considered. Classical and novel ideas are combined to elaborate an elementary argument that proves the result in the full range of integrability $p \in [1, \infty]$ and singularity $s \in (0, 1)$. For the case $p = \infty$, we use an adaptation of the classical level set method by De Giorgi.

Stephen Cameron: Lipschitz regularization for fractional mean curvature flow.

Abstract: The s-perimeter of a set E is given by the $\dot{W}^{s,1}$ norm of its characteristic function for s in $(0, 1)$. The first variation of this functional gives the s-mean curvature H_s, the fractional, non-local analog of typical mean curvature. We show that if your initial surface is bounded between two hyperplanes, then after evolving for a fixed finite time under fractional mean curvature flow the surface becomes a Lipschitz graph. The proof is inherently nonlocal in nature, and in fact the theorem is false for classical mean curvature flow.

Ludovic Cesbron: Derivation of confined non-local diffusion equation from kinetic models

Abstract: We derive non-local diffusion equations confined to a bounded spatial domain as fractional diffusion limits of kinetic equations with heavy-tailed equilibria. We will consider either a fractional Vlasov-Fokker-Planck or a Linear Boltzmann kinetic model, set in a spatially bounded domain with absorption, specular reflections or diffusive boundary conditions. Using an adapted moment method, we will show how we can derive, from these models, confined versions of the fractional heat equation. In particular, we will see that unlike the classical setting where both reflective boundary conditions lead to the homogeneous Neumann condition in the diffusion limit, the non-local framework is much more sensitive to the choice of kinetic boundary condition.

Peter Constantin: Singularities in Fluids.

Abstract: I will start by considering time dependent hypersurfaces immersed in Euclidean space and computing the evolution of geometric quantities such as the first and second fundamental form, curvatures, area and volume enclosed. I will give examples of geometric and hydrodyamic evolution.
I will then present two recent results: a proof of an old conjecture regarding slender jet breakup, and a rigorous proof of finite or infinite time pinchoff in an equally old model of Hele-Shaw necks. I will also briefly describe a recent result about the inviscid limit in the vortex sheet regime.

Diego Córdoba: Global in time and mixing solutions for the Incompressible Porous Media equation (IPM).

Abstract: In this talk we will present two global existence results for IPM in two different settings and the existence of mixing solutions. First we prove global existence of smooth solutions with bounded density and finite energy in a confined scenario (joint work with Angel Castro and Daniel Lear). For the second setting we will consider solutions where the density takes two constant values at each side of a moving interface. This is known as the Muskat problem. We prove a global existence result of a unique strong solution for the 2D stable Muskat problem with arbitrary large finite slopes and finite energy (joint work with Omar Lazar). Furthermore, we show the existence of mixing solutions of the IPM equation for all Muskat type H5 initial data in the fully unstable regime (joint work with Angel Castro and Daniel Faraco).

Josephine Evans: Using Harris's theorem to show convergence to equilibrium for kinetic equations.

Abstract: I will discuss a joint work with Jose Canizo, Cao Chuqi and Havva Yolda. I will introduce Harris's theorem which is a classical theorem from the study of Markov Processes. Then I will discuss how to use this to show convergence to equilibrium for some spatially inhomogeneous kinetic equations involving jumps including jump processes which approximate diffusion or fractional diffusion in velocity. This is the situation in which the tools of 'Hypocoercivity' are used. I will discuss the connections to hypocoercivity theory and possible advantages and disadvantages of approaches via Harris's theorem.

François Golse: Linear Boltzmann Equation and Fractional DiffusionLinear Boltzmann Equation and Fractional Diffusion

Abstract: (Work in collaboration with C. Bardos and I. Moyano). Consider the linear Boltzmann equation of radiative transfer in a half-space, with constant scattering coefficient σ. Assume that, on the boundary of the half-space, the radiation intensity satisfies the Lambert (i.e. diffuse) reflection law with albedo coefficient α. Moreover, assume that there is a temperature gradient on the boundary of the half-space, which radiates energy in the half-space according to the Stefan-Boltzmann law. In the asymptotic regime where $\sigma \to +\infty$ and $1 - \alpha \sim C/\sigma$, we prove that the radiation pressure exerted on the boundary of the half-space is governed by a fractional diffusion equation. This result provides an example of fractional diffusion asymptotic limit of a kinetic model which is based on the harmonic extension definition of $\sqrt{-\Delta}$. This fractional diffusion limit therefore differs from most of other such limits for kinetic models reported in the literature, which are based on specific properties of the equilibrium distributions ("heavy tails") or of the scattering coefficient as in [U. Frisch-H. Frisch: Mon. Not. R. Astr. Not. **181** (1977), 273–280].

Frédéric Hérau: Cauchy theory and convergence to the equilibrium results about the inhomogeneous Boltzmann equation without cutoff

Abstract : In this talk, we present some results about the convergence to equilibrium, existence and uniquenes of solutions of the inhomogeneous Boltzmann equation without angular cutoff in a perturbative setting. We work in the torus and deal with the physical case of hard potentials type interactions (with a moderate angular singularity) but work in large spaces of Sobolev type with polynomial weights. We will explain the general strategy, in particular how the use of short time regularization estimates for the linearized problem help to overcome the difficulties linked with the non-linear perturbative term (this is a work with I. Tristani and D. Tonon).

Panki Kim: Generalized Time Fractional Poisson Equations: Representations and Estimates.

Abstract: In this talk, we discuss existence and uniqueness of strong/weak solutions for general time fractional Poisson equations. We show that there is an integral kernel representing solutions of time fractional Poisson equations with zero initial values, and that the integral kernel can be expressed in terms of the fundamental solution of the spatial infinitesimal generator and the corresponding subordinator associated with the time fractional derivative. We also show that this integral kernel, which we call the fundamental solution for the time tractional Poisson equation, can be expressed as a time fractional derivative of the fundamental solution for the homogenous time fractional equation under the assumption that the associated subordinator is a special subordinator and thus admits a conjugate subordinator. Furthermore, when the distribution of the corresponding subordinator is self-decomposable, we establish two-sided estimates for the fundamental solution to the time fractional Poisson equation through explicit estimates for transition density functions of subordinators. This is a joint work with Zhen-Qing Chen, Takashi Kumagai and Jian Wang.

Takashi Kumagai: Quenched invariance principle for random walks among random conductances with stable-like jumps.

Abstract: Consider random conductances that allow long range jumps. In particular we consider conductances $C_{xy} = w_{xy}|x - y|^{-d-\alpha}$ for distinct $x, y \in Z^d$ and $0 < \alpha < 2$, where $\{w_{xy} = w_{yx} : x, y \in Z^d\}$ are non-negative independent random variables with mean 1. We prove that under

some moment conditions for w, suitably rescaled Markov chains among the random conductances converge to a rotationally symmetric α-stable process almost surely w.r.t. the randomness of the environments. The proof is a combination of analytic and probabilistic methods based on the recently established de Giorgi-Nash-Moser theory for processes with long range jumps. If time permits, we also discuss quenched heat kernel estimates as well. This is a joint work with Xin Chen (Shanghai) and Jian Wang (Fuzhou).

Mateusz Kwaśnicki: Harmonic extension technique for functions of the Laplace operator.

Abstract: I will discuss the results of my joint work with Jacek Mucha, where we provide a representation of certain Lévy processes as traces left on the boundary by appropriate diffusions in a half-space. We thus extend the work of Molchanov and Ostrovski on stable processes. The main tool is Krein's theory of strings. The analytic counterpart of our result is a variant of Caffarelli-Silvestre extension technique for complete Bernstein functions of the Laplace operator.

Nicolas Lerner: Gelfand-Shilov smoothing effect for the Landau equation.

Abstract: We consider the inhomogenous non-linear Landau equation with Maxwellian molecules in the whole space $\mathbb{R}^3$. We prove that the Cauchy problem for the fluctuation around the equilibrium distribution, with small initial data in $H^r_x(L^2_v)$ with $r > 3/2$, enjoys some Gelfand-Shilov regularizing effect in the velocity variable and Gevrey regularizing effect in the position variable for all positive time. This is a joint work with K. Pravda-Starov, Y. Morimoto and C.-J. Xu.

Eulalia Nualart: Intermittency for some fractional stochastic heat equations on bounded domains.

Abstract: I will present an overview talk about recent results on moments, intermittency and chaotic behaviour of some fractional stochastic heat equation. I will also discuss some open problems.

Joaquim Serra: Stable phase transitions: from nonlocal to local.

Abstract: The talk will review the motivations, state of the art, recent results, and open questions on four very related PDE models related to phase transitions: Allen-Cahn, Peierls-Nabarro, Minimal surfaces, and Nonlocal Minimal surfaces. We will focus on the study of stable solutions (critical points of the corresponding energy functionals with nonnegative second variation). We will discuss new nonlocal results on stable phase transitions, explaining why the stability assumption gives stronger information in presence of nonlocal interactions. We will also comment on the open problems and obstructions in trying to make the nonlocal estimates robust as the long-range (or nonlocal) interactions become short-range (or local).

Stanley Snelson: Conditional regularity for the inhomogeneous Landau equation.

Absctract: This talk will describe some recent results guaranteeing solutions to the spatially inhomogeneous Landau equation are smooth and can be continued forward past a given time, under natural conditions on the corresponding hydrodynamic quantities. In particular, we will discuss conditional global upper bounds, C^∞ smoothing, and self-generating pointwise lower bounds, which together imply (in the case of moderately soft potentials) a solution can be continued as long as the mass and energy densities remain bounded above. This talk is based on joint work with Silvestre-Cameron, Henderson, and Henderson-Tarfulea.

Robert Strain: Global in time results for the Muskat problem.

Abstract: This talk is about the Muskat problem, modeling the filtration of two incompressible immiscible fluids in porous media. We consider the case in which the fluids have different constant densities together with different constant viscosities. In this situation the equations are non-local, not only in the evolution system, but also in the implicit relation between the amplitude of the vorticity and the free interface. Among other extra difficulties, no maximum principles are available for the amplitude and the slopes of the interface in L^∞. We prove global in time existence results for medium size initial stable data in critical spaces. This is joint work with Gancedo, Garcia-Juarez, and Patel.

Alexis Vasseur: The 3D Quasi-geostrophic equation: existence of solutions, lateral boundary conditions and regularity.

Abstract: The 3D Quasi-geostropic equation is a model used in climatology to model the evolution of the atmosphere for small Rossby numbers. It can be derived from the primitive equation. The surface quasi-geostrophic equation (SQG) is a special case where the atmosphere above the earth is at rest. The evolution then depends only on the boundary condition, and can be reduced to a 2D model. In this talk, we will show how we can derive the physical lateral boundary conditions for the inviscid 3D QG, and construct global in time weak solutions. Finally, we will discuss the global regularity of solutions to the QG equation with Ekman pumping.

Vlad Vicol: Intermittent weak solutions of the 3D Navier-Stokes equations.

Abstract: I will discuss recent developments concerning the non-uniqueness of distributional solutions to the Navier-Stokes equation.

Jian Wang: Homogenization of stable-like operators.

Abstract: In this talk, we show homogenization results for a large class of symmetric stable-like operators with degenerate coefficients, and we also consider cases of non-symmetic stable-like operators with periodic coefficients. This is based on joint work with X. Chen, Z.-Q. Chen and T. Kumagai.

Xicheng Zhang: Dirichlet problem for supercritical non-local operators.

Abstract: Let D be a bounded C^2-domain. Consider the following Dirichlet initial-boundary problem of nonlocal operators with a drift:

$$\partial_t u = \mathscr{L}_\kappa^{(\alpha)} u + b \cdot \nabla u + f \text{ in } \mathbb{R}_+ \times D, \quad u|_{\mathbb{R}_+ \times D^c} = 0, \ u(0, \cdot)|_D = \varphi,$$

where $\alpha \in (0, 2)$ and $\mathscr{L}_\kappa^{(\alpha)}$ is an α-stable-like nonlocal operator with kernel function $\kappa(x, z)$ bounded from above and below by positive constants, and $b : \mathbb{R}^d \to \mathbb{R}^d$ is a bounded C^β-function with $\alpha + \beta > 1$, $f : \mathbb{R}_+ \times D \to \mathbb{R}$ is a C^γ-function in D uniformly in t with $\gamma \in ((1 - \alpha) \vee 0, \beta]$, $\varphi \in C^{\alpha+\gamma}(D)$. Under some Hölder assumptions on κ, we show the existence of a unique classical solution $u \in L^\infty_{loc}(\mathbb{R}_+; C^{\alpha+\gamma}_{loc}(D)) \times C(\mathbb{R}_+; C_b(D))$ to the above problem. Moreover, we establish the following probabilistic representation for u

$$u(t, x) = \mathbb{E}_x \Big(\varphi(X_t) \mathbf{1}_{\tau_D > t} \Big) + \mathbb{E}_x \left(\int_0^{t \wedge \tau_D} f(t - s, X_s) \mathrm{d}s \right), \ t \geq 0, \ x \in D,$$

where $((X_t)_{t \geq 0}, \mathbb{P}_x; x \in \mathbb{R}^d)$ is the Markov process associated with the operator $\mathscr{L}_\kappa^{(\alpha)} + b \cdot \nabla$, and τ_D is the first exit time of X from D. In the sub and critical case $\alpha \in [1, 2)$, the kernel function κ can be rough in z. In the supercritical case $\alpha \in (0, 1)$, we classify the boundary points according to the sign of $b(z) \cdot \vec{n}(z)$, where $z \in \partial D$ and $\vec{n}(z)$ is the unit outward normal vector. Finally, we provide an example and simulate it by Monte-Carlo method to show our results. (This is a joint work with Guohuan Zhao).