ABSTRACTS

Pair correlations and equidistribution

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Consider an infinite sequence in the unit interval. The pair correlation statistic quantifies how many pairs of elements of the sequence (with index up to a fixed value) are within a certain short distance of each other. Here "short" means a distance proportional to the average spacing between the points. If the asymptotic distribution of pair correlations coincides with that of a random sequence, we say that the pair correlation behavior is Poissonian. Mathematical folklore presumed that having Poissonian pair correlations is a "finer" property than being equidistributed. In this talk we give a rigorous proof that Poissonian pair correlations imply equidistribution. For the proof we will use a blend of Fourier analysis and linear algebra.

On Distances to Lattice Points in Knapsack Polyhedra

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Abstract. Given $\boldsymbol{a} \in \mathbb{Z}^n$, $b \in \mathbb{Z}$, a *knapsack polyhedron* $P(\boldsymbol{a}, b)$ is defined as

$$P(\boldsymbol{a}, b) = \{\boldsymbol{x} \in \mathbb{R}^n_{\geq 0} : \boldsymbol{a}^T \boldsymbol{x} = b\}\,.$$

In this talk we will present some recent results on the ℓ_∞-distance from a vertex of a knapsack polyhedron to its nearest feasible lattice point (referred to as a *vertex distance*). We give a sharp upper bound for the vertex distance that only depends on the ℓ_∞-norm of the vector $\boldsymbol{a}$. In a randomised setting, we show that the vertex distance for a "typical" knapsack polyhedron is drastically smaller than the vertex distance that occurs in a worst case scenario.

6

Uniform distribution of modular signs

Mohammed Amin Amri

August 3, 2018

Abstract

Let $k \geq 2$, N be natural numbers, and let

$$f = \sum_{n \geq 1} a(n) q^n \qquad q := e^{2\pi i z}$$

be a normalised newenform of integral weight k for $\Gamma_0(N)$ with Dirichlet character $\varepsilon \pmod N$. In this talk, we will consider the sign changes and the distribution of signs of some sub-families of the sequence $(\Re e \{a(n) e^{-i\phi}\})_{n \in \mathbb{N}}$, with $\phi \in [0, \pi)$. Using the now-proven Sato-Tate Conjecture for modular forms we prove some equidistribuion results of the signs of these sequences.

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Basic properties of three-dimensional continued fractions

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Let the continued fraction representation of real α is a $[t_0; t_1, \dots, t_i, \dots]$, where $t_0 \in \mathbb{Z}$ and $t_i \in \mathbb{Z}_+ \quad (i = 1, 2, \dots)$ are called the coefficients or terms of the continued fraction and $p_i/q_i = [t_0; t_1, \dots, t_i] \quad (i = 1, 2, \dots)$ are called the convergens of the continued fraction.

In the continued fractions theory we know the Markov-Hurwitz theorem: $\forall i = 1, 2, \dots$

$$\min\left\{\left|\alpha - \frac{p_{i-1}}{q_{i-1}}\right| q_{i-1}^2, \left|\alpha - \frac{p_i}{q_i}\right| q_i^2, \left|\alpha - \frac{p_{i+1}}{q_{i+1}}\right| q_{i+1}^2\right\} \le \frac{1}{\sqrt{5}}.$$

The talk will be devoted to the analog of the theorem for three dimensional lattices.

This research is supported by the Russian Science Foundation (project 18-14-05001).

Random walks on the circle and Diophantine approximation

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Let (n_k) be an increasing sequence of positive integers and let α be an irrational number. The asymptotic behavior of the discrepancy of $\{n_k\alpha\}_{1\le k\le N}$ is well known for $n_k = k$ and for lacunary n_k, but otherwise no precise discrepancy estimates are known. Thus it is natural to consider random n_k and we investigate the simplest case when the gaps $n_{k+1} - n_k$ are independent and identically distributed random variables. Then $\{n_k\alpha\}$ is a random walk on the circle having interesting and unusual properties. By Markov chain theory, the distribution of $\{n_k\alpha\}$ converges to the uniform distribution on the circle, but the convergence speed $r_n = \sup_x |P(\{n_k\alpha\} < x) - x|$ is much slower than the exponential speed for finite Markov chains and depends strongly on the Diophantine approximation properties of α. We give a precise Berry-Esseen type estimate for r_n and describe the asymptotic properties of the discrepancy D_N of $\{n_k\alpha\}_{1\le k\le N}$ and $T_N = \sum_{k=1}^N f(n_k\alpha)$ for periodic measurable functions f. In particular, letting γ denote the Diophantine rank of α, we show that the order of magnitude of T_N and D_N is substantially different for $\gamma \le 2$ and $\gamma > 2$, a surprising critical phenomenon in the model.

Joint results with Bence Borda.

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Expansions in negative base, distribution modulo one and fractales de Rauzy.

September 5, 2018

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Abstract

Given a Pisot number with degree d we define three natural maps from the so-called $-\beta$ shift into $\mathbb{R}$ and $\mathbb{R}^d$ and onto the $d-$dimensional torus ; these maps allows us to define a "Pavé de Rauzy" and its multiple tiling of $\mathbb{R}^d$, a fractale analoguous to Rauzy fractale and a multiple autoaffine tiling of $\mathbb{R}^{d-1}$ with negative expansion value ; we also show that a number has a normal expansion in base $-\beta$ if and only if he has a normal expansion in base $+\beta$.

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Discrepancy and energy optimization on the sphere.

Abstract: Discrepancy and optimal energy are among the most standard ways to measure equidistribution of finite point sets. There first explicit connection between certain instances of these notions (L^2 spherical cap discrepancy and sum of distances) was provided by the Stolarsky invariance principal, which says that minimizing the former is equivalent to maximizing the latter. This principle has a experienced renewed attention in the recent years with numerous new proofs, different versions, extensions, and generalizations. We shall survey different manifestations of this principle and some of their applications.We also explore measures and discrete point distributions which minimize various energies on the sphere. Unlike classical examples, such as the Riesz energy, most of the potentials that we consider are minimized when points are orthogonal, thus minimizing energy imposes orthogonal-like structures (bases, frames etc). In particular, we consider the so-called p-frame energy: in the well-known case $p = 2$ the minimizers are precisely tight frames (or, more generally, isotropic measures) on the spheres, however, other values of $p > 0$ are much less understood (except even integers). We provide minimizers in some specific cases, as well as a number of candidates for other cases based on extensive numerical computations. We also present partial progress on the conjecture of Fejes Toth, which states that the pairwise sum of acute angles between n lines is maximized by the periodically repeated orthonormal basis.

Integer Multiplication of Continued Fractions Via Geometric Methods

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In recent years, the problem of deducing the continued fraction expansion of $\overline{p\alpha}$ from the continued fraction of $\overline{\alpha}$ has become a topic of increased interest. This is due in part to a reformulation of the p-adic Littlewood conjecture – an open problem in Diophantine approximation – which roughly states that the partial quotients of $\overline{p^k\alpha}$ become arbitrarily large as k tends to infinity (for p a fixed prime).

In this talk, we will discuss how one can interpret multiplication of a continued fraction by n some integer as a map between the Farey complex and the $1/n$-scaled Farey complex. In turn, this allows us to interpret the integer multiplication of continued fractions as a replacement of one triangulation of $\Gamma_0\backslash\mathbb{H}$ with another triangulation. We will then discuss how closed curves on $\Gamma_0\backslash\mathbb{H}$ directly correspond to periodic continued fractions, and how this correspondence allows us to deduce information about the divisibility of convergent denominators for $\overline{\alpha}$ as well as the growth rate of partial quotients of $\overline{p^k\alpha}$ for $\overline{\alpha}$ eventually periodic.

A SPECTRAL COCYCLE FOR SUBSTITUTION DYNAMICAL SYSTEMS (JOINT WITH BORIS SOLOMYAK)

Alexander Bufetov (CNRS Institut de Mathématiques de Marseille, alexander.bufetov@univ-amu.fr)

The talk, based on joint work with Boris Solomyak, is devoted to quantitative spectral estimates for suspension flows over substitution dynamical systems. In this situation, the spectrum is described by matrix analogues of Riesz products.We introduce a new cocycle over the skew-product whose base is our system and whose fibres are toral automorphisms. The Lyapunov exponent of the cocycle governs the spectral measure.

The sum-of-digits function, primes and uniform distribution modulo 1

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There is an intimate relation between the distribution of the q-ary sum-of-digits functions of primes $s_q(p)$, the uniform distribution of the sequence $\alpha s_q(p) \bmod 1$ and on special instances of the Sarnak conjecture. This kind of work war pioneered by the ground-breaking work by Mauduit und Rivat on the Gelfond-problems.

The purpose of this talk is to give a survey of recent results into this directions that have been obtained together with Mauduit and Rivat and also with Müllner and Spiegelhofer. In particular we show that the sequence $\alpha s_{q_1}(p) + \beta s_{q_2}(p) \bmod 1$ is uniformly distributed (for coprime q_1, q_2 and irrational α, β) and that the sequence $s_Z(n) \bmod 2$ satisfies the Sarnak conjecture, where $s_Z(n)$ denotes the Zeckendorf sum-of-digits function.

On the distance to the nearest square-free polynomial

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We investigate a variant of Turán's problem on the distance from an integer polynomial in $\mathbb{Z}[x]$ to the nearest irreducible polynomial in $\mathbb{Z}[x]$, where ,,irreducible" is replaced by ,,square-free".

In particular, we show that, for any polynomial $f \in \mathbb{Z}[x]$, there exist infinitely many square-free polynomials $g \in \mathbb{Z}[x]$ such that

$$L(f-g) \le 2,$$

where $L(h)$ denotes the sum of the absolute values of the coefficients of $h \in \mathbb{Z}[x]$. (We do not know if such a polynomial g exists if we require, in addition, $\deg g \le \deg f$, but we can bound $\deg g$ in terms of $d = \deg f$ and $L(f)$.) On the other hand, we show that this inequality cannot be replaced by

$$L(f-g) \le 1.$$

For this, for each integer $d \ge 15$ we construct infinitely many polynomials $f \in \mathbb{Z}[x]$ of degree d such that neither f itself nor any $f(x) \pm x^k$, where k is a non-negative integer, is square-free. One example of degree 15 polynomial is

$$\begin{aligned} f(x) =& 15552x^{15} + 5184x^{14} + 5616x^{13} + 8784x^{12} + 13908x^{11} \\ &+ 13756x^{10} + 96413x^9 - 18929x^8 - 57229x^7 + 6851x^6 \\ &+ 9435x^5 - 932x^4 - 346x^3 + 36x^2. \end{aligned}$$

Polynomials over prime fields and their distances to square-free polynomials are also considered.

These results are joint with Min Sha (Macquarie University, Sydney).

On the Markov equation and the outer automorphism of $\mathrm{PGL}(2,\mathbb{Z})$

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Abstract

The Markov numbers are the unions of the solutions $(x,y,z) \in \mathbb{Z}_+^3$ to the Markov equation $x^2+y^2+z^2 = 3xyz$. These numbers arise in many different contexts. Our interest is in the set of continued fraction expansions of Markov quadratic irrationals arised from Markov numbers. However, there is a fundamental involution of the real line called Jimm induced by the outer automorphism of the extended modular group $\mathrm{PGL}(2,\mathbb{Z})$. Its action on the real line, explicitly on continued fraction expansion, inspired us that this involution must play a role in Markov's theory. The main goal of this talk is to show the possible relation between Jimm involution and the Markov theory. More precisely, we will first present a method to find directly the quadratic form of the image of Markov irrationals under Jimm involution by using the fact coming from geometry of Markov numbers. Then we will show a general form of the image of the subset of Markov quadratic irrationals in relation with Fibonacci numbers.

ON STOCHASTICITY PARAMETER OF QUADRATIC RESIDUES

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Let A be an arbitrary subset of residue ring $\mathbb{Z}_m$. We can write $A = \{a_i\}_{i=1}^t$, where $0 \leqslant a_1 < a_2 < \ldots a_t < m$, and set $a_{t+1} := a_1 + n$ (thus we think of $\mathbb{Z}_m$ as of a circle). For studying the degree of randomness of points of A Arnold [1], §9, define a stochasticity parameter

$$S(A) := \sum_{i=1}^{t} (a_{i+1} - a_i)^2.$$

Too small or too large values of $S(A)$ testify to nonrandom behaviour of A: the points of A either "repel" or "attract" each other ($S(A)$ is minimal when the points are distributed equidistantly, and is maximal when all the points accumulate in the same place).

We investigate the quantity $S(R)$, where R is the set of quadratic residues modulo m. Let $S(k)$, $k \in \mathbb{N}$, be a mean value of S over all k-elements sets of $\mathbb{Z}_m$. A special case of a result of Konyagin, Garaev and Malykhin [2] is the following.

Theorem A. *Let $m = p$ be a prime number. Then*

$$S(R) = S(|R'|)(1 + o(1)), \quad p \to \infty$$

We present the following results.

Theorem 1. *Let $a \in \mathbb{N}$ be a fixed positive integer and $m = ap_1 \ldots p_t$, where $p_1 < \ldots < p_t$ are primes and p_1 is sufficiently large depending on a and t. Then*

$$S(R) = S(|R|)(1 + o(1)), \quad t \to \infty.$$

Theorem 2. *We have*

$$\varliminf_{m\to\infty} \frac{S(R)}{S(|R|)} < 1 < \varlimsup_{m\to\infty} \frac{S(R)}{S(|R|)}.$$

[1]The work is supported by the grant from Russian Science Foundation (Project 14-11-00702).

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Minkowski question-mark function: fixed points and the derivative

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The Minkowski question-mark function $?(x)$ is a continious and strictly increasing function, which maps the $[0,1]$ interval onto itself. If $[0; a_1, a_2, \ldots, a_n, \ldots]$ is the continued fraction representation of an irrational number x, then

$$?(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^{a_1+\ldots+a_k-1}} \tag{1}$$

If x is a rational number, then (1) is replaced by finite sum. One can easily see that the equation

$$?(x) = x \tag{2}$$

has at least 5 solutions, a folklore conjecture states that there are exactly 5 fixed points of $?(x)$. I will tell about some new results on Diophantine properties of fixed points of $?(x)$ (joint work with N. Shulga) and properties of the derivative of this function (joint work with I. Kan).

16

Multidimensional continued fractions and Diophantine exponents of lattices

Oleg N. German
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The talk is devoted to discussing multidimensional continued fractions and connected problems concerning Diophantine exponents of lattices.

Given a full rank lattice Λ in $\mathbb{R}^d$ we define its Diophantine exponent as

$$\omega(\Lambda) = \sup\left\{\gamma \in \mathbb{R} \;\middle|\; \Pi(\mathbf{x}) < |\mathbf{x}|^{-\gamma} \text{ admits } \infty \text{ solutions in } \mathbf{x} \in \Lambda\right\},$$

where $\Pi(\mathbf{x}) = |x_1 \cdot \ldots \cdot x_d|^{1/d}$ if $\mathbf{x} = (x_1, \ldots, x_d)$. This quantity generalizes to the case of lattices the concept of irrationality measure of a real number. It is well known that the measure of irrationality of θ can be expressed in terms of the growth of partial quotients of θ. Namely,

$$\mu(\theta) = 2 + \limsup_{n\to\infty} \frac{\ln(a_{n+1})}{\ln(q_n)}\,,$$

where a_n are the partial quotients of θ, and q_n are the denominators of its convergents.

We shall discuss possible multidimensional generalizations of this connection, considering Klein polyhedra as a multidimensional generalization of continued fractions.

Rényi $\alpha-$dimension of random variables with generalized Cantor distribution and Hausdorff dimension of generalized Cantor sets

Rita Giuliano*

Abstract

We discuss the notion of generalized Cantor set and generalized Cantor distribution; we introduce the notion of Rényi $\alpha-$dimension and prove that the Rényi lower $\alpha-$dimension of a random variable with generalized Cantor distribution coincides with the Hausdorff dimension of its support (generalized Cantor set). Under some assumptions, we also prove the holderianity of generalized Cantor distribution functions.

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Sets with fairly distributed sumsets

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This is a talk on a common work with Alain Faisant (Saint-Étienne), Ram Krishna Pandey (Roorkey) and Sai Teja Somu (Mumbai). Starting from a problem in Additive Number Theory (given $\alpha \in [0,1]$, find $A \subseteq \mathbf{N} := \{0,1,2,\dots\}$ such that $d(A+A) = \alpha$, where the "asymptotic density" d is defined as $d(X) := \lim_{n\to+\infty} n^{-1}|X \cap [1,n]|$), we found a manner to construct sets A of nonnegative integers such that *each* sumset jA, $1 \leq j \leq k$, is fairly distributed in $\mathbf{N}$. A sumset jA is defined by $jA := \{x_1 + \cdots + x_j; x_i \in A, 1 \leq i \leq j\}$. By "fair" we mean that the sumset has asymptotic density. Moreover the densities of the sumsets are regularly distributed in [0,1].

Asymptotic behaviour of the Sudler product of sines and a conjecture of Lubinsky.

Sigrid Grepstad

September 13, 2018

We study the asymptotic behaviour of the sequence of sine products

$$P_n(\alpha) = \prod_{r=1}^{n} |2 \sin \pi r \alpha|$$

for real quadratic irrationals α. In particular, we study the subsequence P_{q_n}, where q_n is the nth best approximation denominator of α, and show that $(P_{q_n})_{n \geq 1}$ converges to a periodic sequence whose period equals that of the continued fraction expansion of α. This verifies a conjecture recently posed by Mestel and Verschueren in [1]. Finally, we discuss a conjecture of Lubinsky [2] stating that

$$\liminf_{n \to \infty} P_n(\alpha) = 0,$$

and argue that this new information on the asymptotic behaviour of $P_n(\alpha)$ suggests otherwise.

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On the cross-combined measure of families of binary lattices and sequences

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The cross-combined measure (which is a natural extension of cross-correlation measure) is introduced and important constructions of large families of binary lattices with nearly optimal cross-combined measures are presented. These results are important in the study of large families of pseudorandom binary lattices but they are also strongly related to the one-dimensional case: An easy method is showed obtaining strong constructions of families of binary sequences with nearly otimal cross-correlation measures based on the previous constructions of lattices. The important feature of this result is that so far there exists only one type of constructions of *very large* families of binary sequences with small cross-correlation measure, and this only type of constructions was based on one-variable irreducible polynomials. Since it is very complicated to construct one-variable irreducible polynomials over $\mathbb{F}_p$, it beame necessary to show other types of constructions where the generation of sequences are much faster. Using binary lattices based on two-variable irreducible polynomials this problem can be avoided, however a slightly weaker upper bound is obtained for the cross-correlation measure than in the original construction. (But, contrary to one-variable polynomials, using Schöneman-Eisenstein criteria it is very easy to construct two-variable irreducible polynomials over $\mathbb{F}_p$.)

Statistical properties of Klein polyhedra

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Let Γ be an s-dimensional lattice. For each $\theta = (\theta_1, \dots, \theta_s)$, $\theta_i = \pm 1$, define the Klein polyhedron $K_\theta(\Gamma)$ as the covex hull of nonzero lattice points contained in the s-dimensional polyhedral cone

$$\left\{x \in \mathbb{R}^s : \quad x_i\theta_i \geq 0, \quad i = \overline{1,s}\right\}.$$

Klein's construction was motivated the following considerations. For each $\alpha \in (0,1)$ define the lattice Γ_α of points $\gamma = (Q, \alpha \cdot Q - P)$ with $Q, P \in \mathbb{Z}$. By the classical Lagrange approximation theorem, the set of vertices of the Klein polyhedra for the lattice Γ_α consists of the points

$$\pm(0,1), \quad \pm(Q_n, \alpha Q_n - P_n), \quad n = 0, 1, \dots,$$

where P_n/Q_n — is the nth convergent for α. Futhemore, let

$$a = (Q_{n-1}, \alpha Q_{n-1} - P_{n-1}) \qquad \tilde{a} = (Q_{n+1}, \alpha Q_{n+1} - P_{n+1})$$

be two adjacent vertices, that is, the segment $[a, \tilde{a}]$ is an edge of a Klein polyhedron for Γ_α. Then

$$q_{n+1}(\alpha) = \#(\Gamma \cap (a, \tilde{a}]). \tag{1}$$

Here $\#X$ denotes the cardinality of X.

We will discuss some statistical properties of multydimensional Klein polyhedra (see [1–4]).

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4. A. A. Illarionov, "Distribution of facets of higher-dimensional Klein polyhedra", *Sbornik: Mathematics*, 209:1 (2018).

Alexander Kalmynin

Large values of short character sums

Abstract. It is well-known that sums of values of non-principal Dirichlet characters over large enough intervals can be estimated non-trivially. Namely, for any $\varepsilon > 0$ there exists $c(\varepsilon) > 0$ such that any character sum of length $N \geqslant p^{1/4+\varepsilon}$ is $O(Np^{-c(\varepsilon)})$. However, it turns out that for any $A > 0$ there exist infinitely many primes p for which there is no non-trivial bound for the sum of values of quadratic character modulo p over the interval of length $(\log p)^A$. We will discuss the proof of this result, which relies on some classical facts about Siegel zeros and smooth numbers, and its possible generalizations.

On Bounded Remainder Sets and Strongly Non-Bounded Remainder Sets for Sequences $(\{a_n\alpha\})_{n\geq 1}$

Lisa Kaltenböck[*], Gerhard Larcher[†]

Abstract: Let $(x_n)_{n\geq 1}$ be an arbitrary sequence in $[0,1)$. An interval $[a,b) \subseteq [0,1)$ is called bounded remainder set (BRS) for $(x_n)_{n\geq 1}$ if

$$|\#\{1 \leq n \leq N : x_n \in [a,b)\} - (b-a)N| \leq c,$$

for all natural numbers N with a constant c independent of N. We give some results on the existence of BRS for sequences of the form $(\{a_n\alpha\})_{n\geq 1}$, where $(a_n)_{n\geq 1}$ - in most cases - is a given sequence of distinct integers. Further we introduce the concept of strongly non-bounded remainder sets (S-NBRS) and we show for a very general class of polynomial-type sequences that these sequences cannot have any S-NBRS, whereas for the sequence $(\{2^n\alpha\})_{n\geq 1}$ every interval is an S-NBRS.

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Construction of normal numbers

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Given a fixed integer $q \geq 2$, an irrational number ξ is said to be a *q-normal number* if any preassigned sequence of k digits occurs in the q-ary expansion of ξ with the expected frequency, that is $1/q^k$. In this talk, we expose new methods that allow for the construction of large families of normal numbers. This is joint work with Professor Jean-Marie De Koninck.

Automatic and q-multiplicative sequences through the lens of higher order Fourier analysis

Jakub Konieczny

Hebrew University of Jerusalem & Jagiellonian University

Automatic sequences are among the simplest models of computation. Intuitively speaking, an automatic sequence is one whose n-th term can be computed by a finite device given the digits on n as input. Perhaps the simplest example of an automatic sequence carries the name of Thue–Morse, and is given by $t(n) = (-1)^{s_2(n)}$, where $s_2(n)$ denotes the sum of digits of n base 2. It is also a 2-multiplicative sequence, meaning that $t(n)$ can be computed as a product of contributions which depend on consecutive binary digits of n.

Various uniformity properties of specific automatic sequences have long been studied. It was already in 1968 that Gelfond obtained quantitative estimates of the Fourier coefficients of the Thue–Morse sequence. As a consequence (observed later by Mauduit and Sarközy), approximately half of the terms in any sufficiently long arithmetic progression have an even sum of binary digits. Behaviour along more complicated sequences — such as polynomials, Piatetski–Shapiro sequences or the primes — has been extensively studied by many authors, including Drmota, Mauduit, Müllner, Rivat, Spiegelhofer and others.

With the advent of higher order Fourier analysis, new notions of uniformity have come to light. Speficically, a sequence f can be construed as uniform (of order $s \geq 2$) — or pseudorandom — if the Gowers uniformity norms $\|f\|_{U^s[N]}$ become small as $N \to \infty$. The usefulness of this notion stems in large part from the fact that the Gowers norms control the count of arithmetic progressions, as well as other linear patterns. We show that the Gowers norms of the Thue–Morse sequence are very small: $\|t\|_{U^s[N]} = O(N^{-c_s})$, where $c_s > 0$ is a constant. An analogous argument shows the same conclusion for another famous automatic sequence by the name of Rudin–Shapiro, and several other examples. Similar statements are also true for q-multiplicative sequences ($q \geq 2$): in joint work with A. Fan we show such sequences are either Gowers uniform of all orders, or correlate with a periodic sequence (hence are not Gowers uniform of order 2). The talk will be focused on these results, and — time permitting — some related results on ergodic theorems weighted by automatic sequences obtained jointly with T. Eisner.

ON SUBGRAPHS OF RANDOM CAYLEY SUM GRAPHS

S. V. KONYAGIN, I. D. SHKREDOV

Abstract.

We prove that asymptotically almost surely, the random Cayley sum graph over a finite abelian group $\mathbf{G}$ *has edge density close to the expected one on every induced subgraph of size at least* $\log^c |\mathbf{G}|$*, for any fixed* $c > 1$ *and* $|\mathbf{G}|$ *large enough. For details see [1].*

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Uncertainty in Finite Affine Planes

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The general uncertainty principle says that a function and its Fourier transform cannot both be highly concentrated. I will present a recent joint paper of András Biró and myself, where we establish a number of specific realizations of this principle for the additive group of the finite affine plane $\mathbb{F}_p^2$, with p prime.

As an application, I will give an estimate for the largest possible number of directions in $\mathbb{F}_p^2$ such that a (weighted) set $P \subset \mathbb{F}_p^2$ is perfectly uniformly distributed among the p lines in the direction in question.

30

Multifractal behaviour of the Brjuno function

Bruno Martin (Université du Littoral Côte d'Opale)

CIRM, October 2018

The Brjuno function is 1-periodic, nowhere locally bounded function, introduced by Yoccoz, because it encapsulates a key information concerning analytic small divisor problem in dimension 1. Its definition is directly related to the continued fraction expansion of a real number. With Stéphane Jaffard we show that this function is multifractal, which means that its pointwise Hölder regularity (in the L^1 sense) may widely change from point to point.

A DENSITY OF RAMIFIED PRIMES

CHRISTINE MCMEEKIN

Abstract. For "nice" number fields K, a certain spin [FIMR13] dependence relation occurs with density given [McMng] by the formula

$$\frac{2^{n-1} + (m_K n + 1)(n-1)}{2^n n}$$

where $n := [K : \mathbb{Q}]$ and m_K is a computable and bounded invariant of the number field K.

A consequence of this result is a conditional theorem giving a formula for the density of rational primes exhibiting a prescribed *ramified* factorization in a number field depending on the prime in question. This density is strictly between 0 and 1.

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Date: July 2018.

Distribution of short subsequences of the inversive generator

László Mérai

Abstract

For an integer $t \geq 3$, consider the map $\psi : \mathbb{Z}_{2^t}^* \to \mathbb{Z}_{2^t}^*$ of the form

$$\psi(u) = \frac{au + b}{cu + d} \quad \text{for } u \in \mathbb{Z}_{2^t}^*.$$

with $a \equiv d \equiv c + 1 \equiv b + 1 \mod 2$.

Starting from an initial value $u_0 \in \mathbb{Z}_{2^t}^*$ we study the distribution of short subsequences of (u_n) defined by the recurrence relation

$$u_{n+1} = \psi(u_n) \quad \text{for } n = 0, 1, \ldots$$

The main tool is to give bounds on short exponential sums of form

$$S_h(N) = \sum_{n=0}^{N-1} \exp\left(h \frac{2\pi i u_n}{2^t}\right), \quad \gcd(h, 2) = 1.$$

The Rudin-Shapiro sequence and similar sequences are normal along squares

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September 18, 2018

Recently, there have been discovered some interesting results for subsequences of automatic sequences and especially the Thue-Morse sequence. In 2013 Drmota, Mauduit and Rivat observed that the subsequence along the squares $(t(n^2))_{n\geq 0}$ of the Thue-Morse sequence $(t(n))_{n\geq 0}$ (that can be defined by $t(n) = s_2(n) \bmod 2$, where $s_2(n)$ denotes the binary sum-of-digits function) is a normal sequence on the alphabet $\{0, 1\}$, i.e. every block of length ℓ appears with asymptotic density $2^{-\ell}$.

Another well studied subsequence is $\lfloor n^c \rfloor$. Recently Spiegelhofer showed that the Thue-Morse sequence is uniformly distributed along $\lfloor n^c \rfloor$, where $1 < c < 2$) and in a joined work by Spiegelhofer and the author that the Thue-Morse sequence is normal for $1 < c < 3/2$.

In this talk, we discuss the first generalization of one of these results to more general sequences. One of the most well-known automatic sequences besides the Thue-Morse sequence is the Rudin-Shapiro sequence $(r(n))_{n\geq 0}$ that can be defined by $r(n) = f_{11}(n) \bmod 2$, where $f_{11}(n)$ counts the number of occurrences of the pattern 11 in the binary expansion of n. The Rudin-Shaprio sequence belongs to the class of block-additive functions mod$m$. Here, a block-additive function denotes a linear combination of pattern-counting functions f_w. We show that all block-additive functions mod$m$ (with obvious restrictions) are normal along the squares, which gives a whole class of, easy to compute, normal numbers.

Variations on a theme of K. Mahler

Abstract

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For an integer n denote $(n)_g$ the sequence of digits of the g-ary representation of n. Mahler [1] proved that the number $0.(1)_{10}(g)_{10}(g^2)_{10}\dots$ is irrational for any $g \geq 2$. It has many generalizations and refinements. Here we prove further generalizations. In the first direction we replace the sequence of powers by weighted sums of elements of a finitely generated multiplicative semigroup of a number field. In the second direction, the base g is replaced by an algebraic integer. As a byproduct, we prove a Mahler-type result replacing the sequence of powers by a fixed coordinate of solutions of a norm form equation.

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Tractability properties of the weighted star discrepancy

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Tractability properties of various notions of discrepancy have been intensively studied in the last two decades. In this presentation we consider the so-called weighted star discrepancy which was introduced by Sloan and Woźniakowski in 1998. The subject of tractability is concerned with the dependence of the weighted star discrepancy on the dimension. Roughly speaking, the weighted star discrepancy is said to be tractable if the inverse of weighted star discrepancy does not explode exponentially with the dimension.

We show that the Halton sequence achieves strong polynomial tractability for the weighted star discrepancy for product weights $(\gamma_j)_{j\geq 1}$ under the mildest condition on the weight sequence known so far for explicitly constructive sequences. However, there is also a hidden problem in this result which shall be discussed .

This is joint work with Aicke Hinrichs (JKU Linz) and Shu Tezuka (Kyushu University).

An extension of the digital method based on b-adic integers

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Keywords quasi-Monte Carlo methods, sequence construction, digital method, digit expansion, q-adic integers

We introduce a hybridization of digital sequences with uniformly distributed sequences in the domain of b-adic integers, $\mathbb{Z}_b, b \in \mathbb{N} \setminus \{1\}$, by using such sequences as input for generating matrices. The generating matrices are then naturally required to have finite row-lengths. We exhibit some relations of the 'classical' digital method to our extended version, and also give several examples of new constructions with their respective quality assessments in terms of $t, \mathbf{T}$ and discrepancy.

Uniform distribution for zeros of random polynomials

by

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Zeros of Kac polynomials (defined as linear combinations of monomials with i.i.d. random coefficients) are asymptotically uniformly distributed near the unit circumference under mild assumptions on the coefficients. We present several generalizations of such results for lacunary random polynomials, and for polynomials with random coefficients spanned by various bases, e.g., by orthogonal polynomials. In particular, we show almost sure convergence of the zero counting measures, and quantify this convergence via the expected discrepancy between the zero counting measures and the limiting measure.

Olivier Ramaré, CNRS / Aix-Marseille Université

Discrepancy estimates for generalized polynomials

Generalized polynomials have been the subject of several investigations in recent times. This talk will present my recent work with Rosewitha Hofer on one side and with Anirban Mukhopadhyay & G. Kasi Viswanadham on the other side where the discrepancy of the sequence $([p(n)\alpha]\beta)$ is bounded above, where $p(x)$ is a polynomial with real coefficients and α and β are irrational numbers satisfying certain conditions.

On the digits of primes and squares

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I will give a survey of our results on the digits of primes and squares (joint works with Michael Drmota and Christian Mauduit).

Quasi-random graphs and pseudo-random binary sequences

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The notion of quasi-random graphs was introduced in 1987 by F. R. K. Chung, R. L. Graham and R. M. Wilson, resp. A. Thomason. It has been shown that there is a strong connection between this notion and the pseudo-randomness of (finite) binary sequences. This connection can be utilized for constructing large families of quasi-random graphs by considering graphs defined by a circular adjacency matrix whose first column is a binary sequence with strong pseudo-random properties. Starting out from this construction principle one may extend, generalize and sharpen some definitions and results on quasi-randomness of graphs.

The Farey graph, continued fractions and SL_2-tilings

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In the 1970's, Coxeter studied certain arrays of integers that form friezes in the plane. He and Conway discovered an elegant way of classifying these friezes using triangulated polygons. Recently, there has been a good deal of interest in expanding Conway and Coxeter's ideas and relating them to other mathematical structures, such as SL_2-tilings. Our objective in this talk is to demonstrate how several significant theorems in this field can be explained in an elegant fashion by representing continued fractions geometrically as paths on an infinite graph known as the Farey graph. We will also put forward a programme for classifying integer friezes modulo n by working with continued fractions on certain quotients of the Farey graph.

On exponential sums and equations over multiplicative subgroups in finite field.

Iurii Shteinikov

Let $\mathbb{Z}_p$ be finite field with p elements and let $G \subset \mathbb{Z}_p^*$ be multiplicative subgroup. Consider the following exponential sums

$$S(a, G) = \sum_{g \in G} e^{2\pi i \frac{ag}{p}}$$

We are interested in non-trivial estimates of the form $|S(a, G)| = o(|G|)|$. Upper estimates for $|S(a, G)|$ can be obtained from upper estimates for $T_k(G)$, see [1] where

$$T_k(G) = |\{(x_1, \ldots, x_2) \in G^{2k} : x_1 + \ldots + x_k = x_{k+1} + \ldots + x_{2k}\}|$$

Several non-trivial estimates for T_k were obtained in [2] using method due to S.A. Stepanov [3]. In my talk I am planning to present the recent estimate for $T_3(G)$ ([4]) and some other applications.

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THE LEVEL OF DISTRIBUTION OF THE THUE–MORSE SEQUENCE

LUKAS SPIEGELHOFER

Abstract. The level of distribution of a complex valued sequence b measures how well b behaves on arithmetic progressions $nd + a$. Determining whether a given number θ is a level of distribution for b involves summing a certain error over $d \leq D$, where D depends on θ; this error is given by comparing a finite sum of b along $nd + a$ and the expected value of the sum. We prove that the Thue–Morse sequence has level of distribution 1, which is essentially best possible. More precisely, this sequence gives one of the first nontrivial examples of a sequence satisfying an analogue of the Elliott–Halberstam conjecture in prime number theory. In particular, this result improves on the level of distribution $2/3$ obtained by Müllner and the author.

Moreover, we show that the subsequence of the Thue–Morse sequence indexed by $\lfloor n^c \rfloor$, where $1 < c < 2$, is simply normal. That is, each of the two symbols appears with asymptotic frequency $1/2$ in this subsequence. This result improves on the range $1 < c < 3/2$ obtained by Müllner and the author and closes the gap that appeared when Mauduit and Rivat proved (in particular) that the Thue–Morse sequence along the squares is simply normal. In the proofs, we reduce both problems to an estimate of a certain Gowers uniformity norm of the Thue–Morse sequence similar to that given by Konieczny (2017).

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The sum of digits in two different bases

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Let $s_a(n)$ denote the sum of digits of an integer n in the base a expansion. We show that, provided a and b are multiplicatively independent integers, any positive real number is a limit point of the sequence $\{s_b(n)/s_a(n)\}_n$. We also provide upper and lower bounds for the counting functions of the corresponding subsequences. This is joint work with R. de la Bretèche and G. Tenenbaum.

Digital questions in finite fields

Cathy Swaenepoel
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The connection between the arithmetic properties of an integer and the properties of its digits in a given basis produces a lot of interesting questions and many papers have been devoted to this topic. In the context of finite fields, the algebraic structure permits to formulate and study new problems of interest which might be out of reach in $\mathbb{N}$. This study was initiated by C. Dartyge and A. Sárközy.

We will devote our interest to several new questions in this spirit:

(1) estimate precisely the number of elements of some special sequences of $\mathbb{F}_q$ whose sum of digits is fixed;
(2) given subsets $\mathcal{C}$ and $\mathcal{D}$ of $\mathbb{F}_q$, find conditions on $|\mathcal{C}|$ and $|\mathcal{D}|$ to ensure that there exists $(c,d) \in \mathcal{C} \times \mathcal{D}$ such that the sum of digits of cd belongs to a predefined subset of $\mathbb{F}_p$;
(3) estimate the number of elements of an interesting sequence of $\mathbb{F}_q$ with preassigned digits.

We notice that the notion of digits in $\mathbb{F}_q$ is directly related to the notion of trace which is crucial in the study of finite fields.

Discrepancy bounds for $\boldsymbol{\beta}$-adic Halton sequences

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Van der Corput and Halton sequences are well-known low-discrepancy sequences. In the 1990ies Ninomiya defined analogs of van der Corput sequences for β-numeration and proved that they also form low-discrepancy sequences provided that β is a Pisot number. Hofer, Iacó, and Tichy define $\boldsymbol{\beta}$-adic Halton sequences and show that they are equidistributed for certain parameters $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_s)$.
In this talk we give discrepancy estimates for $\boldsymbol{\beta}$-adic Halton sequences for which the components β_i are m-bonacci numbers. Our methods include dynamical and geometric properties of Rauzy fractals that allow to relate $\boldsymbol{\beta}$-adic Halton sequences to rotations on high dimensional tori. The discrepancies of these rotations can then be estimated by classical methods relying on W. M. Schmidt's Subspace Theorem.

Title of the talk: **NORMALITY and RANDOMNESS**

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We focus on various criteria introduced by Donald Knuth to describe (pseudo) random properties of sequences. As a starting point we consider the classical concept of normal numbers in q-ary digital expansion. We present probabilistic results as well as various constructions for normal numbers. Furthermore, we establish recent results on efficient constructions for absolutely normal numbers (i.e. numbers which are normal in any base q). We analyze the speed of convergence to normality as well as the computational complexity of the construction algorithms. In a final section, we obtain normality results with respect to more general numeration systems.

An elementary approach to Somos-4 sequences

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A sequence Somos-4 is defined by initial data s_0, s_1, s_2, s_3 and fourth-order recurrence

$$s_{n+2}s_{n-2} = \alpha s_{n+1}s_{n-1} + \beta s_n^2.$$

Usually properties of this sequence are studied by means of elliptic functions. The talk will be devoted to the new elementary approach to Somos-4 sequences. Hopefully it will be suitable for higher-rank Somos sequences corresponding to curves of higher genus.

Talk at UDT-2018, CIRM, Luminy, France, 1-5 October 2018.

GROWTH AND GEOMETRY IN $SL_2(\mathbb{Z})$ DYNAMICS

ALEXANDER P. VESELOV (LOUGHBOROUGH, UK)

Usual discrete dynamics can be considered as the action of the group of integers. What happens if we replace $\mathbb{Z}$ by $SL_2(\mathbb{Z})$?

There is a classical example of such dynamics goes back to remarkable work by Andrei A. Markov (1880), who described the solutions of the Diophantine equation

$$x^2 + y^2 + z^2 = 3xyz$$

(known now as Markov triples) as an orbit of $SL_2(\mathbb{Z})$. These triples surprisingly appeared in many areas of mathematics: initially in arithmetic, but more recently in hyperbolic and algebraic geometry, the theory of Teichmüller spaces, Frobenius manifolds and Painlevé equations.

Another example of such dynamics appears in the description of the values of a binary quadratic form $Q(x, y) = ax^2+bxy+cy^2$ with integer coefficients, the problem going back to Gauss. About 20 years ago John H. Conway proposed a "topographic" approach to this problem, using the planar trivalent tree, which can be considered as a discrete version of the hyperbolic plane.

The same approach can be applied to general $SL_2(\mathbb{Z})$ dynamics, and in particular to Markov dynamics as well. The growth of the corresponding numbers depends on the paths on such tree, which can be labelled by the points of real projective line.

I will discuss some results about the corresponding Lyapunov exponents found jointly with K. Spalding and A. Sorrentino, using the known links with the hyperbolic geometry.

On the maximum order complexity of subsequences of the Thue-Morse and Rudin-Shapiro sequence along squares

Arne Winterhof (Austrian Academy of Sciences, joint work with Z. Sun)

Automatic sequences such as the Thue-Morse sequence and the Rudin-Shapiro sequence are highly predictable and thus not suitable in cryptography. In particular, they have small expansion complexity. However, they still have a large maximum order complexity.

Certain subsequences of automatic sequences are not automatic anymore and may be attractive candidates for applications in cryptography. We show that subsequences along the squares of certain pattern sequences including the Thue-Morse sequence and the Rudin-Shapiro sequence have also large maximum order complexity but do not suffer a small expansion complexity anymore.

METRIC DISCREPANCY WITH RESPECT TO FRACTAL MEASURES

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Let $(n_k)_{k=1}^{\infty}$ be a lacunary sequence of integers. In joint work with N. Technau, we show that if μ is a probability measure on $[0,1)$ such that $|\widehat{\mu}(t)| \leq c|t|^{-\eta}$, then for μ-almost all x, the discrepancy $D_N(n_k x)$ satisfies

$$D_N(n_k x) \ll N^{-1/2}(\log\log N)^{1/2}, \quad N \to \infty.$$