

# ALGEBRAIC GEOMETRY AND COMPLEX GEOMETRY

CIRM, LUMINY, 17 - 21 DECEMBER, 2018

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## Mini-courses

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**Nicolas Addington**

*Cubic fourfolds*

*Abstract:* I will give three expository lectures on cubic fourfolds. I will discuss some classical geometry (rationality of cubics containing planes, quintic del Pezzo surfaces, or sextic elliptic ruled surfaces), some lattice theory (Torelli theorem, associated K3 surfaces), some hyperkähler geometry (the variety of lines, Lehn's 8-fold), and how Kuznetsov's K3 category clarifies these topics

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**Hélène Esnault**

*Fundamental groups in algebraic and arithmetic geometry*

*Abstract:* tba

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**Gerard Freixas i Montplet**

*On the Arakelov theory of Shimura varieties*

*Abstract:* In these lectures I will discuss a recent theorem of Bruinier-Howard-Kudla-Rapoport-Yang on the modularity of generating series of arithmetic special cycles on unitary Shimura varieties. This is the kind of prototype result we seek in the Arakelov geometric study of Shimura varieties. For this, I will first present basic definitions in arithmetic intersection theory. Then I will review the construction of the Shimura varieties in question, their toroidal compactifications and special cycles. Finally, I will state and explain the modularity theorem.

<https://webusers.imj-prg.fr/~gerard.freixas/Articles%20de%20Survol/Freixas-Fourier.pdf>

<https://webusers.imj-prg.fr/~gerard.freixas/Articles%20de%20Survol/Freixas-CIMPA.pdf>

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**Stefan Kebekus**

*Extension Theorems for differential forms and applications*

*Abstract:* We present new extension theorems for differential forms on singular complex spaces, which are particularly useful in the study of minimal varieties, with the singularities of the minimal model program. We sketch the proof and survey a number of applications, pertaining to classification and characterisation of special varieties, non-Abelian Hodge Theory in the singular setting, and quasi-étale uniformization.

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**Colleen Robles**

*On degenerations of Hodge structures*

*Abstract:* tba

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## Talks

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**Michele Ancona**

*Random section of line bundles over real Riemann surfaces*

*Abstract:* Given a line bundle  $L$  over a real Riemann surface, we study the number of real zeros of a random section of  $L$ . We prove a rarefaction result for sections whose number of real zeros deviates from the expected one.

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**Philippe Eyssidieux**

*Exemples de groupes Kählériens*

*Abstract:* Malgré les succès de la théorie de Hodge non abélienne de Corlette-Simpson pour exclusion que de nombreux groupes de présentation finie soient groupes fondamentaux de variétés projectives lisses (ou des groupes kählériens), les techniques de construction manquent. La construction de Campana du groupe fondamental orbifold d'une paire orbifolde permet de considérer le groupe fondamental des compactifications orbifolds d'une variété (ou champ) quasiprojective lisse donnée  $U$  qui, si quelques précautions sont prises et sous des hypothèses raisonnables - mais pas toujours faciles à vérifier, est un groupe kählérien. En choisissant bien la variété  $U$ , les groupes obtenus sont potentiellement intéressants et on utilise souvent des techniques inattendues pour établir les propriétés de leurs représentations linéaires. L'exposé fera un survey de cas particulièrement intrigants ou, par exemple,  $U$  est un complément d'arrangement de droites, une variété localement complexe hyperbolique non compacte ou un espace de modules de courbes pointées.

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**Andrea Fanelli**

*Fibrations de Fano en caractéristique positive*

*Abstract:* Dans cet exposé, à partir du point de vue de la caractéristique zéro, je discuterai les pathologies pour la fibre générique des fibrations de Fano en caractéristique  $p$ . La nouvelle approche du projet en cours avec Stefan Schröer a le double objectif de

- contrôler ces phénomènes bizarres; et
- décrire de nouveaux exemples.

Je vais me concentrer sur la dimension 3, motivé par les progrès récents en théorie de Mori en caractéristique positive.

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**Kevin Langlois**

*Intersection cohomology and torus actions of complexity one*

*Abstract:* Intersection cohomology is a tool that allows to describe the topology of singularities. In this talk, we focus on the calculation of the (rational) intersection cohomology Betti numbers of a complex complete normal algebraic varieties with a torus action of complexity one (i.e., an action of an algebraic torus whose general orbits are of codimension one). This class of algebraic varieties encompasses the complete toric varieties (by choosing a subtorus of codimension one) and the complete normal surfaces with a non-trivial  $\mathbb{C}^*$ -action. Intersection cohomology for the surface and toric cases was studied around the 90's by Stanley, Denef and Loeser, Fieseler, Bernstein and Lunts, Fieseler and Kaup, Braden and MacPherson, ..., and many others. We suggest a natural generalization using the geometric and combinatorial approach of Altmann, Hausen, and Süss for normal varieties with a torus action in terms of the language of divisorial fans. Roughly speaking, this description encodes for a normal variety with a complexity-one torus action, the data of an equivariant proper birational map (*the contraction map*), where the target space is our

initial variety, and the source space is a toric fibration over a smooth algebraic curve. Using recent results of de Cataldo, Migliorini, and Mustață, and looking at the decomposition theorem for the contraction map, we will explain how inductively describe the intersection cohomology Betti numbers in terms of the associated divisorial fan. (Joint work with Marta Agustín Vicente)

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## **Hsueh-Yung Lin**

### ***On the existence of algebraic approximations of compact Kähler manifolds***

*Abstract:* Let  $X$  be a compact Kähler manifold. The so-called Kodaira problem asks whether  $X$  has arbitrarily small deformations to some projective varieties. While Kodaira proved that such deformations always exist for surfaces. Starting from dimension 4, there are examples constructed by Voisin which answer the Kodaira problem in the negative. In this talk, we will focus on threefolds, as well as compact Kähler manifolds of algebraic dimension  $\alpha(X) = \dim(X) - 1$ . We will explain our positive solution to the Kodaira problem for these manifolds.

References:

- Algebraic approximations of compact Kähler threefolds of Kodaira dimension 0 or 1, arXiv:1704.08109.
  - Algebraic approximations of uniruled compact Kähler threefolds, arXiv:1710.01083.
  - Algebraic approximations of compact Kähler manifolds of algebraic codimension 1, à paraître sur arXiv.
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## **Frédéric Mangolte**

### ***Algebraic models of the line in the real affine plane***

*Abstract:* We study the following real version of the famous Abhyankar-Moh Theorem: Which real rational map from the affine line to the affine plane, whose real part is a non-singular real closed embedding of  $\mathbb{R}$  into  $\mathbb{R}^2$ , is equivalent, up to a birational diffeomorphism of the plane, to the linear one? We show that in contrast with the situation in the categories of smooth manifolds with smooth maps and of real algebraic varieties with regular maps where there is only one equivalence class up to isomorphism, there are plenty of non-equivalent smooth rational closed embeddings up to birational diffeomorphisms. Some of these are simply detected by the non-negativity of the real Kodaira dimension of the complement of their images. But we also introduce finer invariants derived from topological properties of suitable fake real planes associated to certain classes of such embeddings.

(Joint Work with Adrien Dubouloz). Fibrations de Fano en caractéristique positive

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## **Adrien Sauvaget**

### ***Mazur-Veech volumes and intersection theory on the Hodge bundle***

*Abstract:* The Hodge bundle is the moduli space of Riemann surfaces endowed with an holomorphic differential. This space is stratified according to the orders of the zeros of the differential. Each stratum has a natural volume form defined in period coordinates. The total volume of the stratum is called the Mazur-Veech volume. We will present a conjectural formula expressing these volumes in terms of intersection numbers on the compactified strata. Surprisingly, we prove that this conjectural formula would give a control on the number of geodesics of large length on a generic surface. (joint work with Dawei Chen and Martin Moller).

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## **Junyi Xie**

### ***The geometric Bogomolov conjecture***

*Abstract:* We prove the geometric Bogomolov conjecture over a function field of characteristic zero.

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