

ADVANCES IN STATISTICAL MECHANICS

Marseille, 27 - 31 August 2018

ABSTRACTS

Monday

- Marek Biskup, UCLA

Random walk driven by Gaussian Free Field

I will discuss random walk on the square lattice driven, in the sense akin to Langevin evolution, by samples of the Gaussian Free Field. This is one out of a whole class of random walks in logarithmically correlated environments for which physics arguments predict subdiffusive behavior with exponents depending on the “temperature” – which, in our case, is simply the overall scale of the logarithmic correlations. The main result of the talk is that many of these predictions can be validated for the specific case of the two-dimensional Gaussian Free Field. The method of proof is based on adapting techniques from two-dimensional critical percolation to the setting of effective resistivity. Based on joint work with J. Ding and S. Goswami.

- Lisa Hartung, NYU

From 1 to 3 and 3 to 6 in branching Brownian motion

Brownian motion is a classical process in probability theory belonging to the class of “Log-correlated random fields”. It is well known due to Bramson that the order of the maximum has a different logarithmic correction as the corresponding independent setting. In this talk we look at a version of branching Brownian motion where we slightly vary the diffusion parameter in a way that, when looking at the order of the maximum, we can smoothly interpolate between the logarithmic correction for independent random variables ($\frac{1}{2\sqrt{2}} \ln(t)$) and the logarithmic correction of BBM ($\frac{3}{2\sqrt{2}} \ln(t)$) and the logarithmic correction of 2-speed BBM with increasing variances ($\frac{6}{2\sqrt{2}} \ln(t)$). We also establish in all cases the asymptotic law of the maximum and characterise the extremal process, which turns out to coincide essentially with that of standard BBM. We will see that the key to the above results is a precise understanding of the entropic repulsion experienced by an extremal particle. (joint work with A. Bovier)

- Jiry Cerny, Basel

The maximal particle of branching random walk in random environment.

The behaviour of the maximal particle of branching random walk have been subject to intensive research recently. It is natural to ask how these properties change when a spatially dependent random branching rates are introduced to the process. In my presentation, I will

describe the first results in this direction, in particular a CLT for the position of the maximal particle, and explain their consequences for other models of interest: the randomized Fisher-KPP equation, and the parabolic Anderson model.

- Christoph Külske, Bochum

Gibbs-non Gibbs transitions in different geometries: The Widom-Rowlinson model under stochastic spin flip dynamics

The Widom-Rowlinson model is an equilibrium model for point particles in Euclidean space. It has a repulsive interaction between particles of different colors, and shows a phase-transition at high intensity. Natural versions of the model are moreover formulated in different geometries: in particular as a lattice system and a mean-field system. We will discuss recent results on dynamical Gibbs-non Gibbs transitions in this context. Main issues will be the possibility or impossibility of an immediate loss of the Gibbs property, and of full measure discontinuities of the time-evolved models.

(Collaborations with Benedikt Jahnel, Sascha Kissel)

- Dmitry Ioffe, Technion Haifa.

Level lines of low temperature interfaces

We shall discuss two families of 2+1 SOS interfaces with zero boundary conditions over lattice boxes of microscopic size N : interfaces with hard wall constraint (entropic repulsion) and interfaces coupled with bulk Bernoulli fields under canonical constraints (facet formation). Both models exhibit, at sufficiently low temperatures, nested families of macroscopic size level lines. These line ensembles have, along portions of flat boundaries of the confining box, an effective structure of ordered weakly interacting random walks under area tilts. In the model of facet formation values of area tilts do not depend on serial numbers of loops in the stack. In the case of entropic repulsion they do, and actually grow geometrically towards the boundary of the box. In both cases the appropriate scaling is the $(1/3, 2/3)$ -diffusive one. For the model of facet formation (with fixed number of facets for large N) the conjectured limits are Dyson Ferrari-Spohn diffusions. This was established for a simplified model of non-interacting ordered random walks. In the case of entropic repulsion, the determinantal structure is apparently lost and, furthermore, the number of macroscopic size loops grows with the linear size of the system N . Most recent results on the level of effective models imply tightness and convergence to countably infinite line ensembles, but the nature of the limit is still unclear.

Based on joint works with Pietro Caputo, Senya Shlosman, Yvan Velenik and Vitali Wachtel

- Aernout van Enter, Groningen.

One-sided versus two-sided

Stochastic systems can be parametrised by time (like Markov chains) or by one-dimensional space (like Markov fields). I will discuss some examples, generalising this to g -measures

versus Gibbs measures, when the two descriptions are the same and when they are different.

Joint work with R. Bissacot, E. Endo and A. Le Ny

Tuesday

- Francesco Guerra, Roma

The replica trick in the frame of replica interpolation

As it is very well known, the replica trick is based on the idea that for disordered models the annealed averages for replicated systems give some information on the quenched free energy (and the states) of the original system. We give a new interpretation of the replica trick in the general frame of interpolation on the number of replicas, extending on the traditional exploitation of the replica trick as connected with analytic continuation toward zero replicas. We give some applications concerning the so called Almeida-Thouless line and multispecies models.

- Erwin Bolthausen, Zurich

On the Gardner formula for the perceptron

The perceptron is particular neural net. In 1988 Gardner and Derrida derived a number of results on the memory capacity of such networks, using sophisticated versions of the replica method. Shortly afterwards, Mézard rederived them, using his cavity method. The first rigorous derivation of the so-called Gardner formula was given by Talagrand in a number of papers around 2000, based on a perturbative method, and a complicated approximation procedure. We propose an alternative, and we believe more transparent, derivation based on the TAP equations, and an approximation scheme of solutions of the TAP equations which is similar to my 2015 on the TAP equations for the Sherrington-Kirkpatrick model. The method can probably be extended to other models for which TAP type equations exist. This is joint work with Shuta Nakajima (Kyoto University).

- Gerard Ben Arous, NYU

Geometric description of the spherical Spin-Glass Gibbs measures and Temperature Chaos

The Gibbs measure of many disordered systems at low temperature may exhibit a very strong dependence on even tiny variations of temperature, usually called “temperature chaos”. I will discuss this question for Spin Glasses. I will report on a recent work with Eliran Subag (Courant) and Ofer Zeitouni (Weizmann and Courant), where we give a detailed geometric description of the Gibbs measure at low temperature, which in particular implies temperature chaos for a general class of spherical Spin Glasses at low temperature. This question has a very long past in the physics literature, and an interesting recent history in mathematics. Indeed, in 2015, Eliran Subag has given a very sharp description of the Gibbs measure for pure p-spin spherical Spin Glasses at low temperature, building on results on the complexity of these spin glasses by Auffinger-Cerny and myself, . This description (close to the so-called Thouless-Anderson-Palmer picture) excludes the existence of temperature chaos for the pure p-spin!! The recent work gives an extension of this very detailed geometric description of the Gibbs measure to the case of general mixed models, and shows that in fact the pure p-spin is very singular.

- Jennifer Chayes and Christian Borgs, Microsoft Research

Graphons and Graphexes as Limits of Sparse Graphs: Part I & II

Graphons and graphexes are limits of graphs which allow us to model and estimate properties of large-scale networks. In this pair of talks, we review the theory of dense graph limits, and give two alternative theories for limits of sparse graphs – one leading to unbounded graphons over probability spaces, and the other leading to bounded graphons (and graphexes) over sigma-finite measure spaces. Talk I, given by Jennifer, will review the general theory, highlight the unbounded graphons, and show how they can be used to consistently estimate properties of large sparse networks. This talk will also give an application of these sparse graphons to collaborative filtering on sparse bipartite networks. Talk II, given by Christian, will recast limits of dense graphs in terms of exchangeability and the Aldous Hoover Theorem, and generalize this to obtain sparse graphons and graphexes as limits of subgraph samples from sparse graph sequences. This will provide a dual view of sparse graph limits as processes and random measures, an approach which allows a generalization of many of the well-known results and techniques for dense graph sequences.

- David Belius, Basel

The TAP-Plefka variational principle for mean field spin glasses

In this talk I will recall the Thouless-Anderson-Palmer (TAP) approach to the Sherrington-Kirkpatrick model from one of the earliest papers on this model, and describe how it can be reinterpreted as a variational principle in the spirit of the Gibbs variational principle. Furthermore I will present a rigorous proof of this TAP-Plefka variational principle in the case of the spherical Sherrington-Kirkpatrick model.

Wednesday

- Loren Coquille, Grenoble

Crossing a fitness valley as a metastable transition in a stochastic population model

We consider a stochastic model of population dynamics where each individual is characterised by a trait in $\{0, 1, \dots, L\}$, and has a natural reproduction rate, a logistic death rate due to age or competition and a probability of mutation towards neighbouring traits at each reproduction event. We choose parameters such that the induced fitness landscape exhibits a valley: mutant individuals with negative fitness have to be created in order for the population to reach a trait with positive fitness. We focus on the limit of large population and rare mutations at several speeds. In particular, when the mutation rate is low enough, metastability occurs: the exit time of the valley is random, exponentially distributed.

This is a joint work with Anton Bovier and Charline Smadi.

- Sylvie Meleard, CMAP and Nicholas Champagnat IECL

Stochastic modeling and asymptotic analysis of a population of microorganisms with competition and horizontal transfer : Part I & II

We present a model for the dynamics of a population of bacteria with trait structure, who compete for resources and exchange genetic material by horizontal transfer. Phenotypic traits characterizing individuals may then be vertically inherited through mutations, and horizontally transmitted through transfer. Competition and horizontal transfer influence individual demographics and population size, which in turn feeds back on the dynamics of competitions and transfers. We consider a stochastic individual-based pure jump process taking values in the space of point measures. The jumps describe the individuals' reproductions (possibly with mutation), transfers and deaths. In this model there is a trade-off between the transfer and the effect of advantageous mutation: individuals with costly traits and hence smaller growth rates can transfer their traits to fitter individuals. Depending on the parameters of the model, different behaviors can be observed on simulations, including evolutionary suicides and cyclic phenomena. If the trait describes the resistance charge to some antibiotics carried by plasmids in a bacterium, this cyclic effect can for instance be associated to the appearance of resistant strains. In the limit of large populations and rare mutations, we explore mathematically these phenomena using either the standard asymptotics of adaptive dynamics or developing an approach introduced in a different context by Durrett, Mayberry and by Bovier, Coquille, Smadi. The idea is to consider population sizes in a log scale to keep track of the smaller subpopulations that have negligible sizes compared with the size of the dominant (so-called resident) population. The population dynamics observed in simulation can then be compared with the asymptotic behavior of the model in this log scale, relying in particular on coupling arguments with inhomogeneous branching processes with immigration.

This is joint work with Chi Tran Viet.

Thursday

- Michael Aizenman, Princeton

Power-law upper bound on the correlations in the $2D$ random field Ising model

As an example of the Imry-Ma phenomenon, the famed discontinuity of the magnetization in the two dimensional Ising model is unstable to the addition of quenched random magnetic field of uniform variance, even if that is small. The talk will focus on a quantitative version of the statement, proving a power-law upper bound on the decay rate of the effect of boundary conditions on the magnetization in finite systems, as function of the distance to the boundary. Unlike exponential decay which is only proven for strong disorder or high temperature, the power-law upper bound is established here for all field strengths and at all temperatures, including zero, for the case of independent Gaussian random field. Our analysis proceeds through a streamlined and quantified version of the Aizenman-Wehr derivation of the Imry-Ma rounding effect. The talk is based on a joint work with Ron Peled.

- Patrik Ferrari, Bonn

Space-time covariance of KPZ growth models

For some growth models in the Kardar-Parisi-Zhang universality class, the large time limit process of the interface profile is well established. Correlations in space-time are much less understood. Along special space-time lines, called characteristics, there is a sort of ageing. We study the covariance of the interface process along characteristic lines for generic initial conditions. Joint work with A. Occelli (arXiv:1807.02982).

- Margherita Disertori, Bonn

Plate-nematic phase in three dimensions

We consider a system of anisotropic plates in the three-dimensional continuum, interacting via purely hard core interactions. We assume that the particles have a finite number of allowed orientations. In a suitable range of densities, we prove the existence of a uniaxial nematic phase, characterized by long range orientational order and no translational order. The proof is based on a coarse graining procedure, which allows us to map the plate model into a contour model, and in a rigorous control of the resulting contour theory, via Pirogov-Sinai methods.

This is a joint work with A. Giuliani and I. Jauslin.

- Abel Klein, Irvine

Manifestations of localization in the random XXZ quantum spin chain

We study random XXZ quantum spin chains in the Ising phase. We prove droplet localization, a single cluster localization property that holds in an energy interval near the bottom of the spectrum. We establish dynamical manifestations of localization in the energy window of droplet localization, including non-spreading of information, zero-velocity

Lieb-Robinson bounds, and general dynamical clustering. A byproduct of our analysis is that this droplet localization can happen only inside the droplet spectrum. (Joint work with Alex Elgart and Gunter Stolz.)

- Charles Newman, NYU

The Riemann Hypothesis: Some History

One fairly standard version of the Riemann Hypothesis (RH) is that a specific probability density on the real line has a moment generating function (Laplace transform) that as an analytic function on the complex plane has all its zeros pure imaginary. We'll review a series of results that span the period from the 1920's to 2018 concerning a perturbed version of the RH. In that perturbed version, due to Polya, the log of the probability density is modified by a kind of mass term (in quantum field theory language). This gives rise to an implicitly defined real constant known as the de Bruijn-Newman Constant, Λ . The conjecture and now theorem (Newman 1976, Rodgers and Tau 2018) that Λ is greater than or equal to zero is complementary to the RH which is equivalent to Λ less than or equal to zero; The conjecture/theorem is a version of the dictum that the RH, if true, is only barely so.

We'll also briefly discuss some connections with quantum field theory and the Lee-Yang circle theorem.

- Vincent Beffara, Grenoble

Percolation methods for nodal lines

I will describe how methods from statistical mechanics, and more specifically Russo-Seymour-Welsh theory from percolation, can be applied to the setup of continuous models to obtain macroscopic informations about the topology of nodal domains. For positively correlated fields with fast decay of the two-point function (such as the Bargman-Fock model), the same qualitative behaviour as for Bernoulli percolation is obtained; I will also present recent progress towards the non-positively correlated cases. Joint work with Damien Gayet (Grenoble).

Friday

- Frank den Hollander, Leiden

Metastability for Glauber Dynamics on Random Graphs

We investigate the effect of percolation on the Curie-Weiss model with Glauber dynamics. In particular, we study metastability for spin-flip dynamics on the Erdos-Renyi random graph with N vertices and with edge retention probability $p \in (0, 1)$. Each vertex carries an Ising spin that can take the values -1 or $+1$. Single spins interact with an external magnetic field, while neighbouring spins interact with each other with a ferromagnetic interaction strength equal to $1/N$. Spins flip according to a Metropolis dynamics at a fixed subcritical temperature. We compute the average crossover time from the metastable state (with magnetization corresponding to the ‘minus-phase’) to the stable state (with magnetization corresponding to the ‘plus-phase’). We show that, to leading order, the average crossover time grows exponentially fast with N , with an exponent that is the same as for the Curie-Weiss model with the same external magnetic field but with a ferromagnetic interaction strength equal to p/N . We show that the correction term to the exponential asymptotics is a multiplicative error term that is at most polynomial in N .

Joint work with Oliver Jovanovski (Toronto)

- Alessandra Faggionato, Roma

Large deviations and uncertainty relations in periodically driven Markov chains with applications to stochastic thermodynamics

We will present large deviation principles for the empirical measure, flow and current of Markov chains with time-periodic jump rates. As an application we derive some Gallavotti-Cohen duality relations for the fluctuating entropy flux and we also derive trade-off relations between speed and precision for time-integrated currents. These theoretical results find applications in the thermodynamics of small systems, as biomolecular motors and molecular pumps.

The above results have been obtained in collaboration with A.C. Barato, L. Bertini, R. Chetrite and D. Gabrielli.

- Cristina Toninelli, LPSM Paris

Bootstrap percolation and kinetically constrained spin models: critical time scales

Recent years have seen a great deal of progress in understanding the behavior of bootstrap percolation models, a particular class of monotone cellular automata. In the two dimensional lattice there is now a quite complete understanding of their evolution starting from a random initial condition, with a universality picture for their critical behavior. Much less is known for their non-monotone stochastic counterpart, namely kinetically constrained models (KCM). In KCM each vertex is resampled (independently) at rate one by tossing a p -coin iff it can be infected in the next step by the bootstrap model. In particular infection can also heal, hence the non-monotonicity. Besides the connection with bootstrap

percolation, KCM have an interest in their own : when $p \rightarrow 0$ they display some of the most striking features of the liquid/glass transition, a major and still largely open problem in condensed matter physics. I will discuss some recent results on the characteristic time scales of KCM as $p \rightarrow 0$ and the connection with the critical behavior of the corresponding bootstrap models.