Approximation diophantienne et transcendance

CIRM, Luminy, du 10/09/2018 au 14/09/2018

Titres et résumés des exposés

1. B. Adamczewski : *Mahler's method in several variables.*
Abstract : Any algebraic (resp. linear) relation over the field of rational functions with algebraic coefficients between given analytic functions leads by specialization to algebraic (resp. linear) relations over the field of algebraic numbers between the values of these functions. Number theorists have long been interested in proving results going in the other direction. Though the converse result is known to be false in general, Mahler's method provides one of the few known instances where it essentially holds true. After the works of Nishioka, and more recently of Philippon, Faverjon and the speaker, the theory of Mahler functions in one variable is now rather well understood. In contrast, and despite the contributions of Mahler, Loxton and van der Poorten, Kubota, Masser, and Nishioka among others, the theory of Mahler functions in several variables remains much less developed. In this talk, I will discuss recent progresses concerning the case of regular singular systems, as well as possible applications of this theory. This is a joint work with Colin Faverjon.

2. S. Akhtari : *Lower bounds for the Mahler measures of polynomials that are sum of a bounded number of monomials.*
Abstract : I will talk about a joint work in progress with J. Vaaler, where we slightly improve a recent result of Dobrowolski and Smyth, to establish a sharp lower bound for the Mahler measures of polynomials in any number of variables. These bounds depend on the number of non-vanishing coefficients of the polynomials, and are independent of their degrees.

3. F. Amoroso : *Factorization of bivariate sparse polynomials.*
Abstract : We prove a function field analogue of a conjecture of Schinzel on the factorization of univariate polynomials over the rationals. The proof is based on Zannier "toric" Bertini Theorem. This is a joint work with M. Sombra.

4. M. Bennett : *Quartic forms : computations in search of a theorem.*
Abstract : I discuss joint work with Andrew Rechnitzer regarding algorithmic computation of binary quartic forms with bounded discriminants, highlighting some striking differences to analogous results for cubic forms and touching on results for corresponding Thue equations.

5. V. Beresnevich : *Simultaneous rational approximations to several functions of a real variable.*
Abstract : As is well known, simultaneous rational approximations to the values of smooth functions of real variables involve counting and/or understanding the distribution of rational points lying near the manifold parameterised by these functions. I will discuss recent results in this area regarding lower bounds for the Hausdorff dimension

of τ-approximable values, where $\tau \geq\geq 1/n$ is the exponent of approximations. In particular, I will describe a very recent development for non-degenerate maps as well as a recently introduced simple technique based on the so-called Mass Transference Principle that surprisingly requires no conditions on the functions except them being C^2.

6. L. Capuano : *An effective criterion for periodicity of p-adic continued fractions.*
Abstract : It goes back to Lagrange that a real quadratic irrational has always a periodic continued fraction. Starting from decades ago, several authors proposed different definitions of a p-adic continued fraction, and the definition depends on the chosen system of residues mod p. It turns out that the theory of p-adic continued fractions has many differences with respect to the real case; in particular, no analogue of Lagranges theorem holds, and the problem of deciding whether the continued fraction is periodic or not seemed to be not known until now. In recent work with F. Veneziano and U. Zannier we investigated the expansion of quadratic irrationals, for the p-adic continued fractions introduced by Ruban, giving an effective criterion to establish the possible periodicity of the expansion. This criterion, somewhat surprisingly, depends on the 'real' value of the p-adic continued fraction.

7. M. Carrizosa : *Counting polarisations on abelian varieties.*
Abstract : An abelian variety can be endowed with a polarization of fixed degree in only a finite number of essentially different ways. We want to bound this number. To do so, we use classical ideas from the theory of algebraic groups, and more precisely a theorem of Borel and Harish-Chandra.

8. M. Coons : *Two regular questions.*
Abstract : In this talk, I will discuss how two important questions in number theory – the finiteness conjecture and Lehmers Mahler measure question – can be found in the area of automatic and regular sequences.

9. L. DeMarco : *Height pairings, torsion points, and dynamics (Part 2).*
Abstract : We will present work in progress, joint with Hexi Ye, towards a conjecture of Bogomolov, Fu, and Tschinkel asserting uniform bounds for common torsion points of nonisomorphic elliptic curves. We introduce a general approach towards uniform unlikely intersection bounds based on an adelic height pairing, and discuss the utilization of this approach for uniform bounds on common preperiodic points of dynamical systems, including torsion points of elliptic curves.

10. J. Demeio : *Hilbert Property and Elliptic Fibrations.*
Abstract : .

11. J.-H. Evertse : *S-parts of values of polynomials.*
Abstract : Let S be a finite set of primes. Any non-zero integer a can be expressed as a product $a = a' \cdot a''$ where a' is a positive integer composed of primes from S and a'' is divisible only by primes outside S. We call a' the S-part of a, notation $[a]_S$. For polynomials $F \in \mathbf{Z}[X_1, \ldots, X_m]$ and reals $\varepsilon > 0$ one may study the inequality $[F(\mathbf{x})] \geq |F(\mathbf{x})|^{\epsilon}$ in $\mathbf{x} \in \mathbf{Z}^m$ and ask questions such as whether its set of solutions is finite or infinite, and if infinite, what can be said about its density. Such questions are as yet much too hard for arbitrary polynomials. In our talk we will focus on univariate

polynomials and decomposable forms (products of homogeneous linear forms) when more can be said. This is a joint work with Yann Bugeaud and Kálmán Győry.

12. S. Fischler : *Irrationality of odd zeta values, with (or without) Shidlovsky's lemma.*
Abstract : This lecture deals with irrationality of values at (positive) odd integers of the Riemann zeta function. Ball and Rivoal have proved that among the first s values, at least $\frac{\log s}{1+\log 2}$ (essentially) are irrational. In a joint work with Sprang and Zudilin, we improve on this lower bound. This result can be refined, and generalized to L-functions of Dirichlet characters, using a general version of Shidlovsky's lemma and a linear independence criterion.

13. K. Győry : *On the smallest number of terms of vanishing sums of units in number fields.*
Abstract : Let K be a number field. In the terminology of Nagell a unit ε of K is called *exceptional* if $1 - \varepsilon$ is also a unit. The existence of such a unit is equivalent to the fact that the unit equation $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0$ is solvable in units $\varepsilon_1, \varepsilon_2, \varepsilon_3$ of K. Numerous number fields have exceptional units. They have been investigated by many authors, and they have important applications. In my talk I deal with a generalization of exceptional units. Denote by $U(K)$ the smallest integer k with $k \geq 3$ such that the unit equation $\varepsilon_1 + \cdots + \varepsilon_k = 0$ is solvable in units $\varepsilon_1, \ldots, \varepsilon_k$ of K. If no such k exists, we set $U(K) = \infty$. Apart from trivial cases when $U(K) = \infty$, we gave an explicit upper bound for $U(K)$. We obtained several results for $U(K)$ in number fields of degree at most 4, cyclotomic fields and general number fields of given degree. We proved various properties of $U(K)$, including its magnitude, parity as well as the cardinality of number fields K with given degree and given odd resp. even value $U(K)$. Finally, an application will be presented to certain arithmetic graphs, namely to the representability of cycles. In the proofs, some diophantine and algebraic number-theoretic results and methods are combined. This is a joint work with Cs. Bertók, L. Hajdu and A. Schinzel.

14. N. Hirata-Kohno : *Linear forms in logarithms via Padé approximations and their applications.*
Abstract : We prove a new lowed bound for linear forms in logarithms of algebraic numbers which are supposed to be closed to 1 with algebraic coefficients. This is a joint work with Makoto Kawashima (Osaka University). Let K be an algebraic number field of finite degree over the field of rational numbers. Consider $\alpha_1, \cdots, \alpha_m \in K$ of sufficiently small absolute value. We present a refined lowed bound for linear form in $1, \log(1-\alpha_1), \cdots, \log(1-\alpha_m)$ with algebraic coefficients. The main integrant is based on the Hermite-Padé approximation shown by G. Rhin and P. Toffin ("Approximants de Padé simultanés de logarithmes", Journal of Number Theory, 24, (1986), p. 284–297). We give results both in the complex case and in the p-adic case. Our result improves, for example in the complex case, a statement of Yuri Nesterenko ("Linear forms in logarithms of rational numbers", in: Diophantine approxiation (Cetraro, 2000), LNM 1819, Springer, pp. 53–106) for sufficiently "small" rational numbers. We also explain applications to Diophantine equations.

15. H. Kaneko : *Diophantine approximation and base-b expansion of smooth numbers.*
Abstract : Many mathematicians have studied the uniformity of the digits in the base-b expansion of integers. In 1979, Erdős conjectured for each $m \geq 9$ that the digit 2 appears in the ternary expansion of 2^m. Moreover, Dupuy and Weirich conjectured for any sufficiently large m that each digit $0, 1, 2$ appears with average frequency tending

to 1/3 (as m tends to infinity) in the ternary expansion of 2^m. As a partial result for the conjectures above, Stewart gave lower bounds for the number of nonzero digits. As a natural generalization of Stewart's result, Bugeaud raised the question of the nonexistence of arbitrarily large smooth numbers having few nonzero digits in the base-b expansion. Giving lower bounds for the number of nonzero digits of smooth numbers, we introduce a kind of answer for the question. This is a joint work with Yann Bugeaud.

16. D. H. Kim : *The Lagrange and Markov Spectra of Pythagorean triples.*
Abstract : We discuss the Diophantine approximation for the Pythagorean triples - the rational points in the unit circle. We will introduce a dynamical system originally defined by Romik in 2008 and study its Lagrange and Markov spectra. This will provide a classical counterpart to our theory on Romiks dynamical system. This is joint work with Byungchul Cha.

17. H. Krieger : *Height pairings, torsion points, and dynamics (Part 1).*
Abstract : We will present work in progress, joint with Hexi Ye, towards a conjecture of Bogomolov, Fu, and Tschinkel asserting uniform bounds for common torsion points of nonisomorphic elliptic curves. We introduce a general approach towards uniform unlikely intersection bounds based on an adelic height pairing, and discuss the utilization of this approach for uniform bounds on common preperiodic points of dynamical systems, including torsion points of elliptic curves.

18. L. Kühne : *The Equidistribution Conjecture for Semiabelian Varieties.*
Abstract : The (Strong) Equidistribution Conjecture for semiabelian varieties yields substantial information on the points of small height on those varieties, including the Manin-Mumford and the Bogomolov Conjecture. Chambert-Loir has settled this conjecture affirmatively in the case of almost split semiabelian varieties. The general case, however, has remained intractable so far because the canonical height of a semiabelian variety is negative unless it is almost split. In fact, this places the conjecture outside the scope of Yuan's Equidistribution Theorem on algebraic dynamical systems. In my talk, I will outline my recent proof of the (Strong) Equidistribution Conjecture for general semiabelian varieties.

19. S. Le Fourn : *Around Baker's method for higher-dimensional varieties.*
Abstract : Baker's method, based on linear forms in logarithms, is one of the most successful tools to explicitly prove finiteness of integral points on algebraic curves . In this talk, I will recall its extension to higher-dimensional varieties by Levin, then explain a generalisation of Levin's theorem and compare it with previous results obtained by Runge's method. Finally, I will give perspectives of how further those results could reasonably be pushed to prove finiteness of integral points on varieties.

20. B. Matschke : *Solving Thue-Mahler equations via the Shimura-Taniyama conjecture.*
Abstract : In this talk we present a practical algorithm to solve cubic Thue–Mahler equations. Our algorithm relies on new height bounds, which we obtained using the method of Faltings (Arakelov, Parshin, Szpiro) combined with the Shimura–Taniyama conjecture (without relying on lower bounds on linear forms in logarithms), as well as several improved and new sieves. As an application, we computed the solutions of large classes of Thue–Mahler equations. We used our resulting data to motivate conjectures and questions on the number of solutions, also for associated diophantine problems. This is joint work with Rafael von Känel.

21. N. M. Mavraki : *Arithmetic equidistribution, dynamics and elliptic curves.*

Abstract : Bogomolov's conjecture (now a theorem following work of Zhang and Ullmo) concerns the geometry of points with small canonical height on abelian varieties. In joint work with Laura DeMarco, we prove an analogous result in the setting of families of products of elliptic curves. This extends theorems by Masser and Zannier in the theme of unlikely intersections. A key step in our proof is establishing an equidistribution result. In this talk, I will discuss this result as well as a generalization of it obtained recently. I will also describe some applications of our work towards a Bogomolov-type extension of a recent result by Barroero and Capuano. This is joint work with Laura DeMarco.

22. N. Moshchevitin : *Irrationality measure functions and Diophantine spectra.*

Abstract : We discuss the relationships between some new and relatively old results on the behavior of different irrationality measure functions. In the simplest setting for real ξ we consider irrationality measure function

$$\psi_\xi(t) = \min_{1\le q\le t,\, q\in\mathbf{Z}} ||q\xi||$$

and compare the theorem by Kan and Moshchevitin (2010) which states that the difference

$$\psi_\alpha(t) - \psi_\beta(t)$$

oscillates when $t \to \infty$ provided $\alpha \pm \beta \notin \mathbf{Z}$ with a new result which states that that there exist arbitrary large values of t with

$$|\psi_\alpha(t) - \psi_\beta(t)| \ge \left(\sqrt{\frac{\sqrt{5}+1}{2}} - 1\right) \min(\psi_\alpha(t), \psi_\beta(t)),$$

under the same conditions. We will speak about other types of irrationality measure functions as well as about multidimensional generalizations which are more complicated. Some related problems consecring Diophantine spectra will be also considered.

23. F. Pazuki : *Regulators of elliptic curves.*

Abstract: In a recent collaboration with Pascal Autissier and Marc Hindry, we prove that up to isomorphisms, there are at most finitely many elliptic curves defined over a fixed number field, with Mordell-Weil rank and regulator bounded from above, and rank at least 4.

24. G. Rémond : *Nouveaux théorèmes d'isogénie.*

Abstract : Dans un travail en commun avec Éric Gaudron, nous démontrons de nouvelles bornes pour le degré minimal d'une isogénie entre deux variétés abéliennes sur un corps de nombres. La méthode repose comme dans les versions antérieures (dues initialement à Masser et Wüstholz) sur un énoncé appelé le théorème des périodes et établi par des méthodes transcendantes. La différence principale se situe lors de l'application du théorème des périodes, plus directe, et qui repose sur de nouveaux objets associés canoniquement à une variété abélienne. L'objet de l'exposé est d'introduire ces objets puis d'esquisser leur utilité pour les théorèmes d'isogénie.

25. J. Thunder : *A Height Heuristic for a-Numbers.*
Abstract : We discuss how some counting results for heights over function fields yield interesting heuristics for arithmetic statistics involving hyperelliptic curves; such statistics are reminiscent of Cohen-Lenstra type conjectures for real quadratic number fields. In particular, in characteristic 3 we give a heuristic for the density of such curves with a given a-number as the genus g tends to infinity. We will briefly describe what the a-number of a curve is and the connection with height counting arguments.

26. S. Velani : *Inhomogeneous Diophantine Approximation on M_0-sets with restricted denominators.*
Abstract : Let $F \subseteq [0,1]$ be a set that supports a probability measure μ with the property that $|\widehat{\mu}(t)| \ll (\log|t|)^{-A}$ for some constant $A > 0$. Let $\mathcal{A} = (q_n)_{n\in\mathbf{N}}$ be a sequence of natural numbers. If $\mathcal{A}$ is lacunary and $A > 2$, we establish a quantitative inhomogeneous Khintchine-type theorem in which (i) the points of interest are restricted to F and (ii) the denominators of the 'shifted' rationals are restricted to $\mathcal{A}$. The theorem can be viewed as a natural strengthening of the fact that sequence $(q_n x \bmod 1)_{n\in\mathbf{N}}$ is uniformly distributed for μ almost all $x \in F$. Beyond lacunary, our main theorem implies the analogous quantitative result for sequences $\mathcal{A}$ for which the prime divisors are restricted to a finite set of k primes and $A > 2k$. This is joint work with Andrew D. Pollington (NSF, Washington), Agamemnon Zafeiropoulos (TU Graz) and Evgeniy Zorin (York).

27. C. Viola : *Linear independence of* $1, \mathrm{Li}_1$ *and* Li_2.
Abstract : In a forthcoming joint paper with G. Rhin (to appear in Moscow Journal of Combinatorics and Number Theory) we improve and extend the irrationality results, which we proved in 2005 for dilogarithms of positive rational numbers, to qualitative and quantitative results of linear independence over $\mathbf{Q}$ of $1, \mathrm{Li}_1(x)$ and $\mathrm{Li}_2(x)$ for suitable x in $\mathbf{Q}$, both for $x > 0$ and for $x < 0$.

28. M. Widmer : *Averages for the ℓ-torsion in class groups.*
Abstract : We present new upper bounds for the average size of the ℓ-torsion $\mathrm{Cl}_K[\ell]$ of the class group of K, as K runs through certain natural families of number fields and ℓ is a positive integer. Our results use a refinement of a key argument, used in most results of this type, which links upper bounds for $\mathrm{Cl}_K[\ell]$ to the existence of many primes splitting completely in K, and that are small compared to the discriminant of K. The improvements are achieved by introducing a new family of specialised invariants of number fields to replace the discriminant in this key argument, in conjunction with new counting results for these invariants. This is joint work with Christopher Frei.