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Introduction to pharmacokinetics and pharmacodynamics of anticancer drugs

Florence Hubert Aix-Marseille University France

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What is a cancer?

Cancer is a very old disease, already reported by Hippocrate (460-377 b. JC). He compared cancer to a crab, karkinoma in greek.

Definition. Cancer is a group of diseases involving abnormal cell growth with the potential to invade or spread to other parts of the body.

Characteristics.

- Limitless replicative potential
- Self-sufficiency in growth signals
- Insensitivity in anti-growth signals
- Sustained angiogenesis
- Tissue invasion and metastases
- Dedifferentiation
- Genome instability and mutation
- Deregulation of metabolism
- Deregulation of immune system
- Promoting inflammation







What is a cancer?

Main steps of the disease

1 Avascular growth

- Cancer cells start dividing
- Nutrients are transported by blood vessel and diffuse in the tissues
- \rightsquigarrow 3-4mm of diameter.

2 Vascular growth or angiogenesis

- Hypoxic cells secrete Endothetial Growth Factors inducing the creation of new vessels.
- The new vascularization provide the nutrients necessary to the tumor growth.

6 Metastatic invasion

- Tumor cells can leave the primary tumor through the blood vessels or the lymphatic network and colonize a distant place.
- \rightsquigarrow Development of secondary tumors or metastases.







Cancer Treatments

- Surgery
- 2 Radiation therapy
- 3 Anti-cancer drug
 - Chemotherapy
 - Targeted therapy (ex anti angiogenic drugs)
 - Immune therapy
 - Hormone therapy
- Stem cells transplant



Some medical issues

- Is it possible to improve the efficiency of existing treatments?
 - There might exist a metastatic boost after surgery. Can it be anticipated ?
 - Chemotherapies induce high toxicities requiring delay between two administrations. Is it possible to reduce this delay?
 - Chemotherapies induce resistance. Is there an administration protocole that could lead to less resistance or postpone it?
 - Chemotherapy are more and more used in combination with anti-angiogenic or immune therapies. Is it possible to optimize the efficiency of such treatment?
 - Treatments efficiency and toxicity is patient dependant. Is it possible to individualize the drug delivery?
 - In radiotherapy, can it be possible to improve the radiated zone?

Some medical issues

How to evaluate the metastatic state of the patients?

- Metastases are the major cause of death in cancer. But metastatic state of the patient is often difficult to evaluate, as micro-tumors are hardly detectable from imagery.
- Among patients with a breast cancer detected at early stage that followed an adjuvant chemotherapy, only few of them presented probably a metastatic risk. Elias (2006), Spielmann & al (2006)

Is it possible to better understand the action of some existing drug?

• Some chemotherapy agents (MTAs) reveal to have an anti-angiogenic affect at lower dose. Is it possible to understand the mechanism?

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Mathematical issues

- How build adequate tumor growth models?
- **2** How model the efficiency and toxicity of the treatments?

Choose the right scale (Genes-proteins-enzymes-tissus-organs)



Intra-cellular scale



Macroscopique scale



Microscopic scale



Multi-scale interactions

Mathematical issues

- How build adequate tumor growth models?
- **2** How model the efficiency and toxicity of the treatments?

Choose the right mathematical tools

- Deterministics models
 - ODE models (population models)
 - PDE models (population structured models, diffusion models, transport models,...)
- Stochastic models

Common issues

- Calibration or estimation of the parameters' s model from the biological data
- Validation of the model in term of reproductability, predictability
- Use the system within an optimization problem (eg. optimize in silico the treatment's protocole)

Outline of the talk

- 1 The simplest tumor growth models
- 2 Pharmacokinetics and pharmacodynamics (PK/PD) of a drug
- 3 A phase I/II clinical trial driven by a mathematical model
- 4 Low grade glioma : prolonged action of TMZ
- **5** One example of model of drug resistance
- 6 One example of interaction with the immune system
- 7 PDE system : meningioma
- 8 Radiotherapy driven by imagery
- 9 Structured models
- **10** Microtubule targeting agent

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The simplest tumor growth models

- A tumor can be seen as a population of cancer cells
 - Y number of individuals

 $Y'(t) = \underbrace{\text{birth number}}_{N_n} - \underbrace{\text{death number}}_{N_m}$

Malthus model - end of $18^{th}\ {\rm century}$

$$Y'(t) = \lambda Y(t) - \mu Y(t) \Rightarrow \frac{Y'}{Y} = \lambda - \mu := a$$

Logistic or Verhultz model (1838)▶ Populations are able to regulate their natality !

$$\frac{Y'(t)}{Y(t)} = a\left(1 - \frac{Y(t)}{K}\right) \Rightarrow \frac{Y'(t)}{Y(t)} \sim_{t \to \infty} C e^{-at}$$

Gompertz model (1825) ► Exponential decay of the growth rate

$$\frac{Y'(t)}{Y(t)} = \mu_0 e^{-at} \Rightarrow \left(\frac{Y'}{Y}\right)' = -a(\ln(Y))' \Rightarrow \frac{Y'(t)}{Y(t)} = a\ln\left(\frac{b}{Y(t)}\right)$$



1766-1834



1804-1849



1779-1865

The simplest tumor growth models

Logistic model (1838)

$$Y'(t) = a\left(1 - \frac{Y(t)}{K}\right)Y(t)$$

Gompertz model (1825)

$$Y'(t) = a_g \ln\left(\frac{b}{Y(t)}\right) Y(t)$$

Von Bertalanffy model (1949)

$$Y'(t) = a\left(\left(\frac{Y(t)}{K}\right)^{-\frac{1}{3}} - 1\right)Y(t)$$

West model (1997)

$$Y'(t) = a\left(\left(\frac{Y(t)}{K}\right)^{-\frac{1}{4}} - 1\right)Y(t)$$



▶ Sigmoid shape

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Pharmacokinetics (PK)/Pharmacodynamics (PD) of a drug



• PK : How the organism affects the drug

2 PD : How the drug affects the organism

How the organism affects the drug One compartment model - infusion - one administration

• Example of a cytotoxic : Etoposide oral

Jong et al 1997

• Small cell lung cancer

 $\underbrace{\begin{matrix} u(t) \\ \downarrow \\ V, c(t) \\ \downarrow Cl \end{matrix}}^{u(t)}$

$$\frac{dc}{dt} = -\frac{Cl}{V}c + \frac{u(t)}{V}, c(t_{inj}) = 0$$

- $t \mapsto u(t)$ infusion protocol
- V Specific volume
- *Cl* Clearance

How the organism affects the drug One compartment model - Oral administration - one tablet

• Example of targeted therapy (kinase inibitors) : Imatinib

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Widmer et al 2006
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• Chronic myelogenous leukemia or Gastro Intestinal Stromal Tumors



$$\begin{aligned} \frac{dq_a}{dt} &= -k_a q_a, \, q_a(t_{abs}) = D \\ \frac{dc}{dt} &= -\frac{Cl}{V}c + \frac{k_a}{V}q_a, \, c(t_{abs}) = 0 \end{aligned}$$

- D dose
- k_a Absorption rate
- V Specific volume

•
$$Cl = \frac{k_{10}}{V}$$
 Clearance

How the organism affects the drug

Informations on Imatinib PK found in the Vidal



La pharmacocinétique de l'imatinib a été évaluée à des doses comprises entre 25 et 1000 mg. Les profils pharmacocinétiques plasmatiques ont été analysés à J1, puis à J7 ou J28, au moment où les concentrations plasmatiques ont atteint un état d'équilibre.

Absorption : La biodisponibilité absolue moyenne de l'imatinib est de 98 %. Il existe une forte variabilité interpatient de l'ASC de l'imatinib plasmatique après une prise orale. Lorsqu'il est pris au cours d'un repas riche en lipides, le taux d'absorption de l'imatinib est peu réduit (diminution de 11 % de la Cmax et prolongation de 1,5 h de Tmax), avec une légère diminution de l'ASC (7.4 %) comparée à une prise à jeun. L'effet d'une chirurgie gastro-intestinale antérieure sur l'absorption du produit n'a pas été étudiée. Distribution : A des concentrations d'imatinib cliniquement significatives, la fraction liée aux protéines plasmatiques est approximativement de 95 %, sur la base des études in vitro; il s'agit principalement d'une liaison à l'albumine et aux alphaglycoprotéines acides et, dans une faible mesure, aux lipoprotéines.

How the organism affects the drug

Informations on Imatinib PK found in the Vidal



Elimination : Après administration d'une dose orale d'imatinib marqué au ${}^{14}C$, environ 81 % de la dose est éliminée au bout de 7 jours (68 % dans les fèces et 13 % dans les urines). La forme inchangée représente 25 % de la dose (5 % dans les urines, 20 % dans les fèces), le reste étant composé de métabolites.

Pharmacocinétique plasmatique : Après administration par voie orale chez le volontaire sain, la demi-vie, d'environ 18 h, est compatible avec une prise quotidienne unique. L'augmentation de l'ASC moyenne de l'imatinib est linéaire et proportionnelle à la dose administrée à des doses orales allant de 25 à 1000 mg. Lors d'administrations répétées en prise quotidienne unique, la cinétique de l'imatinib n'est pas modifiée, mais son accumulation, à l'état d'équilibre, est augmentée d'un facteur de 1,5 à 2,5.

How the organism affects the drug One compartment model - Oral administration - one week of treatment

• Example of targeted therapy (kinase inibitors) : Imatinib

Widmer et al 2006

• Chronic myelogenous leukemia or Gastro Intestinal Stromal Tumors



How the organism affects the drug Two compartments model - injection

• Example of anti-angiogenic drug : Bevacizumab

Bruno et al 1996

• Lung cancer, kidney cancer, glioblastoma,...

$$\begin{array}{c|c} u(t) \\ \downarrow \\ \hline V_1, c_1(t) \\ \hline Cl_1 \\ \hline k_{21} \\ A(t) = c_2(t) \end{array} \end{array} \begin{array}{c|c} \frac{dc_1}{dt} &= -\left(\frac{Cl_1}{V_1} + k_{12}\right)c_1 + k_{21}\frac{V_2}{V_1}c_2(t) + \frac{U(t)}{V_1} \\ \frac{dc_2}{dt} &= k_{12}\frac{V_1}{V_2}c_1 - \left(\frac{Cl_2}{V_2} + k_{21}\right)c_2 \end{array}$$

How the organism affects the drug Three compartment model - injection

- Example of a chemotherapy agent : Doxetacel Meille et al 2008
 - Breast cancer



$$\begin{array}{lcl} \displaystyle \frac{dc_1}{dt} & = & \displaystyle -\left(\frac{Cl_1}{V_1}+k_{12}+k_{13}\right)c_1+k_{21}\frac{V_2}{V_1}c_2(t) \\ & & \displaystyle +k_{31}\frac{V_3}{V_1}c_3(t)+\frac{U(t)}{V_1} \\ \\ \displaystyle \frac{dc_2}{dt} & = & \displaystyle k_{12}\frac{V_1}{V_2}c_1-\left(\frac{Cl_2}{V_2}+k_{21}\right)c_2 \\ \\ \displaystyle \frac{dc_3}{dt} & = & \displaystyle k_{13}\frac{V_1}{V_3}c_1-\left(\frac{Cl_3}{V_3}+k_{31}\right)c_3 \end{array}$$

How the organism affects the drug $% f(x)=\int dx \, dx \, dx$

Compartment models



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How the organism affects the drug

How to choose the best model?

• Population studies using parameter estimation with non-linear mixed effect models (SAEM, see eg Monolix software)



Marc Lavielle, Chapman et al 2014

Pharmacodynamics of the drug

How drug affects the organism A non quantitative approach : the minimal concentration

Demetri et al, 2009

• Imatinib - Gastro Intestinal Stromal Tumors



Efficiency is deeply patient dependant!

Pharmacodynamics of the drug

How drug affects the organism A non quantitative approach : the minimal concentration

Demetri et al, 2009

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The dose 600mg is classical administrated, it corresponds to the minimal dose ensured the efficiency of the drug for a "mean"

Pharmacodynamics of the drug

How drug affects the organism A non quantitative approach : the minimal concentration

Honoré, Hubert, 2016

• Imatinib - Gastro Intestinal Stromal Tumors



The efficiency is deeply related to the regularity of the uptake!

Pharmacodynamics of a cytotoxic drug

How drug affects the organism

A quantitative approach. First models

Action proportional to

• The drug concentration in the plasma. Example for a cytotoxic drug :

$$Y'(t) = Y(t) \ln\left(\frac{b}{Y(t)}\right) - C(t)Y(t)$$

• The concentration above a threshold C_{thres}

$$Y'(t) = Y(t) \ln\left(\frac{b}{Y(t)}\right) - (C(t) - C_{thres})^+ Y(t)$$

Pharmacodynamics of a cytotoxic drug

How drug affects the organism

A quantitative approach. Interface model Meille et al 2008 Drug efficiency is linked to its exposition c_e

$$\frac{dc_e}{dt} = -Ac_e e^{-Bc_e} + (C(t) - C_{thres})^+$$

• Case A = 0. c_e corresponds to $\int_0^t (C(s) - C_{thres})^+ ds$: AUC

• General case. Model saturation effects in the effect compartment.

$$Y'(t) = Y(t) \ln\left(\frac{b}{Y(t)}\right) - c_e(t)Y(t)$$

Extension of the Gompertz model

- The tumor size Y follows a Gompertz law.
- The maximal size θ of the tumor changes with its vascularization = the carrying capacity.

$$\frac{dY}{dt} = aY \ln\left(\frac{\theta}{Y}\right)$$
$$\frac{d\theta}{dt} = \underbrace{cY}_{(\text{VEGF})} \underbrace{-d\theta Y^{\frac{2}{3}}}_{\text{Vasculature inhibition}}$$

M. J. Folkmann (1933-2008)

Extension of the Gompertz model

Hahnfeldt, Folkman & al model

Effect of a combined anti-angiogenic therapy

$$\begin{aligned} \frac{dx}{dt} &= ax \ln\left(\frac{\theta}{x}\right) \\ \frac{d\theta}{dt} &= \mathcal{R}_2(c_{angio}(t))x - d\theta x^{\frac{2}{3}} - \gamma \theta \mathcal{R}(c_{angio}(t)) \end{aligned}$$

• Antiangiogenic drugs acts on the carrying capacity :

- Reduction
- Possible stimulation at the beginning reflecting the normalization of its vascularization.

Ebos & al. Cancer cell (2009)

Extension of the Gompertz model

Hahnfeldt, Folkman & al model

Effect of a combined anti-angiogenic/chemotherapy

$$\begin{aligned} \frac{dx}{dt} &= ax \ln\left(\frac{\theta}{x}\right) - \mathcal{F}(x) \mathcal{R}_1(c_{chemo}(t)) \\ \frac{d\theta}{dt} &= \mathcal{R}_2(c_{angio}(t)) x - d\theta x^{\frac{2}{3}} - \gamma \theta \mathcal{R}(c_{angio}(t)) \end{aligned}$$

- Chemotherapy acts on the tumor size.
- Antiangiogenic drugs acts on the carrying capacity.
 - Reduction.
 - Possible stimulation at the beginning reflecting the normalization of its vascularization.

Ebos & al. Cancer cell (2009)

Extension of the Gompertz model

Hahnfeldt, Folkman & al model

Benzekry, Chapuisat, Ciccolini, Erlinger, H. (2011) Feature :

- Chemotherapy is distributed through vessels.
- Antiangiogenic drugs on one hand destroy vessels, on the other hand normalize vasculature.
- $\Rightarrow {\rm Take \ into \ account \ stable \ (functional, \ mature, ...) \ endothelial \ cells} \\ {\rm and \ unstable \ (non \ functional, \ new...) \ ones \ !}$

Biological assumptions :

- Only stable ECs are able to distribute nutrients, drugs.... with a rate depending on their quality.
- Only unstable ECs are perturbed by stimulation or inhibitor growth factors.
- The quality of vasculature depends on the proportion of stable ECs amount the global amount of ECs.

Extension of the Gompertz model

Hahnfeldt, Folkman & al model

Benzekry, Chapuisat, Ciccolini, Erlinger, H. (2011)

Equations :

Variables

- Y(t): tumor size
 - s(t): density of stable ECs
- u(t): density of unstable ECs
- q(t): quality of the vasculature



Optimal delay between the two drugs : One week



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Clinical trial of phase I/II (2005-2009) : MODEL I

CRO2 Marseille/ Hôpitaux Lyon Sud The group

Le groupe de chercheurs

- Hôpitaux Lyon Sud (Group of Pr Freyer)
- Mathematicians and pharmacokinetician of Marseille (D. Barbolosi and A. Iliadis)

Clin Pharmacokinet DOI 10.1007/s40262-016-0874-7	CrossMark
ORIGINAL RESEARCH ARTICLE	
Revisiting Dosing Regimen Using Pharmacokinetic/ Pharmacodynamic Mathematical Modeling: Densification and Intensification of Combination Cancer Therapy Christophe Melle ^{1,2} , Dominique Barbolou ⁴ , Joseph Ciccolind ⁴ , Giltes Freyer ^{3,M+} , Athanassies Iliadis ⁴	
Brand Canoer Res Trant	
DOI 10.1007/s10549-016-3760-9	CrossMark
DDI 10.1007/s10549-016-3760-9 CLINICAL TRIAL	CrossMark

Clinical trial of phase I/II (2005-2009) : MODEL I

CRO2 Marseille/ Hôpitaux Lyon Sud

Densification of a chemotherapy driven by a mathematical model with a control of hematological toxicity.

Clinical description

- 20 patients with a metastatic breast cancer (HER2-, hormon resistant),
- Classical protocol in the 2000's : 6 cycles of chemo for each patients, cocktail of two chemotherapeutic agents DTX + EPI with 21-days cycle
- Question : is it possible to administrate such a cocktail on 15-days cycles, while controling toxicities ?

The chemotherapeutical drugs

- Kill indifferently all proliferating cells.
- Cause severe toxicities (hematologic, ...)
- Necessity to space treatments, to let patient recover their immunity 21-days cycles
Clinical trial of phase I/II (2005-2009) : MODEL I

CRO2 Marseille/ Hôpitaux Lyon Sud

Mathematical description

• Tumor growth model

Gompertz model adapted to take into

account the treatment. Let

 $eff_{trait}(t, u)$ be death rate due to

drugs evaluated through PK-PD model.

- Toxicities constraints → hematological constraints.
 W_D WBC should not fall down below a concentration W_D
 - W_U The patient should not stay too long aplasia.
 - W_a WBC should recover a concentration W_a before a new cycle.
 - $\Rightarrow \text{ Results : Modeling the hematoxicity model leads to}$ $t \mapsto W(t; u), \text{ contraints becomes } F(t, u) \leq W \text{ where}$ $t \mapsto u(t) \in \mathbb{R}^n \text{ stands for doses of the drugs.}$

$$x'_{u} = ax_{u}\ln\left(\frac{b}{x_{u}}\right) - x_{u}\operatorname{eff}_{trait}(t, u)$$



Clinical trial of phase I/II (2005-2009) : MODEL I

CRO2 Marseille/ Hôpitaux Lyon Sud

Mathematical description

• Tumor growth

$$x'_u = ax_u \ln\left(\frac{b}{x_u}\right) - x_u \operatorname{eff}_{treat}(t, u)$$

with $eff_{treat}(t, u)$ obtained through an ODE system

• Toxicity constraints. $t \mapsto W(t)$ solves an ODE system involving $tox_{treat}(t, u)$. So that toxicities can be reduced into

 $K = \{u/F(t, u) \le C\}$

with F(t, u) obtained through an ODE system.

• Optimisation of the protocol

 $\min_{u \in K} \min_{t \in [0,T]} x_u(t)$

Find an "admissible" protocol leading to the best tumoral recession.

Clinical trial of phase I/II (2005-2009) : MODEL I

CRO2 Marseille/ Hôpitaux Lyon Sud

Mathematical results

• Existence of an optimal protocol requiring an injection of G-CSF.

Methodology

- Protocol of the first cycle based on average parameters.
- Assay of ANC et wafer on 3 blood specimen during the first cycle.
- Estimation of individual PK/PD parameters thanks to Bayesian methods.
- Optimisation of the protocol for the following cycles.

Conclusion

- Densification possible.
- Optimized protocols require injection of hematopoietic factors G-csf. Necessity to reverse the order of administration of the two drugs.
- •
- Response rate lightly lower than in previous study (31.5% vs 49-88 %) .
- But Weeker progression (6% vs 5-18 %) and a better survival median (54.6 vs 19.5-34 month) .

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Low grade glioma

ENS Lyon group/ CHU Lyon Prolonged action of TMZ



Low grade glioma

ENS Lyon group/ CHU Lyon

Observations



Low grade glioma

ENS Lyon group/ CHU Lyon

Model

$$\frac{d}{dt}C = -KDEC$$

$$\frac{d}{dt}P = \lambda_{p}P\left(1 - \frac{P + Q + Q_{p}}{K}\right)$$

$$+kQ_{p}PQ_{p} - kPQP$$

$$-\gamma e^{-\gamma est}KDECP$$

$$\frac{d}{dt}Q = k_{PQ}P - \gamma KDECQ$$

$$\frac{d}{dt}Q_{p} = -kQ_{p}PQ_{p} - \delta Q_{p} + \gamma KDECQ$$



Results

- Predition of the amplitude and the duration of the response to TMZ.
- A tool to optimize the drug scheduling leading optimise to the best response.

More results

• See also works of Víctor M Pérez-García group at Universidad de Castilla-La Mancha, SPAIN

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Tumor heterogeneity

- Only part of the cells are sensible to the drug.
- Sensible cells may control the resistant cells.



Tumor heterogeneity

- Only part of the cells are sensible to the drug.
- Sensible cells may control the resistant cells.

$$S'(t) = \rho S(t) \left(1 - \frac{S(t) + mR(t)}{K} \right) \underbrace{-\alpha C(t)S(t)}_{\text{Drug effect}}$$
$$R'(t) = \rho R(t) \left(1 - \frac{S(t) + mR(t)}{K} \right) \underbrace{-\beta R(t)S(t)}_{\text{Control by sensible cells}}$$

 \rightsquigarrow Optimize protocoles studied by C. Carrère.

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Example in the case of the pancreatic cancer



- C PCC Pancreatic Cancer Cells
- P PSC Pancreatic Stellate Cells
- R Fraction of pro-inflammatory macrophage amoung the macrophage
- T CD8+ Tcells (immune cells cytotoxic)

Example in the case of the pancreatic cancer



- \blacksquare PCC promote growth and activity of PSC through the secretion of TGF_β
- **2** PSC promote PCC growth and metastase
- **③** TCells are cytotoxic
- 0 R is the fraction of pro-inflammatory macrophage so promote Tcells
- PSC and PCC inhibites pro-inflammatory macrophages recruitment

Example in the case of the pancreatic cancer



The dynamical system

$$\frac{dC}{dt} = \underbrace{\left(k_{c} + \mu_{c}P\right)}_{2} C^{\frac{3}{4}} \left(1 - \left(\frac{C}{C_{0}}\right)^{\frac{1}{4}}\right) - \underbrace{\frac{\lambda_{c}}{K_{c} + (1 - R)}}_{\frac{K_{c}}{2}} CT$$

$$\frac{dP}{dt} = \underbrace{\left(k_{p} + \frac{\mu_{p}C}{K_{p} + C}\right)}_{\frac{K_{p}}{2}} P\left(1 - \frac{P}{P_{0}}\right) - \lambda_{p}P$$

$$\frac{dR}{dt} = k_{r} - \underbrace{\left(\lambda_{r} + \gamma_{p}P + \gamma_{c}C\right)}_{\frac{K_{c}}{2}} R$$

$$\frac{dT}{dt} = \underbrace{\frac{k_{t}R}{K_{t} + (1 - R)}}_{\frac{K_{t}}{2}} - \lambda_{t}T$$

Action of a drug treatment

$$\begin{array}{lll} \frac{dC}{dt} & = & \left(k_c + \mu_c P\right) C^{\frac{3}{4}} \left(1 - \left(\frac{C}{C_0}\right)^{\frac{1}{4}}\right) - \frac{\lambda_c}{K_c + (1 - R)} CT \\ \\ \frac{dP}{dt} & = & \left(k_p + \frac{\mu_p C}{K_p + C}\right) P \left(1 - \frac{P}{P_0}\right) - \lambda_p P \\ \\ \frac{dR}{dt} & = & k_r - (\lambda_r + \gamma_p P + \gamma_c C) R \\ \\ \frac{dT}{dt} & = & \frac{k_t R}{K_t + (1 - R)} - \lambda_t T \end{array}$$

- TGF_{β} silencing : $\gamma_c, \gamma_p, \mu_p$ reduced of 10%
- Immune activation EGFR silencing : k_t multiply by a factor 2



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Meningioma

Group INRIA MONC / CHU Bordeaux

Issues How to model an predict the tumour growth in such a complex geometry ?



Properties. Slow and relatively homogeneous growth Difficulty. The 3D geometry plays an important role



Meningioma

A PDE model with 4 equations

• Tumor growth P(t, x)

 $\partial_t P + \operatorname{div}(vP) = MP$

• Host system S(t, x)

 $\partial_t S + \operatorname{div}(vS) = 0$

• Vascularization M(t, x)

 $\partial_t M = -\alpha M$

• Velocity induced by the growth v(t, x)

Group INRIA MONC / CHU Bordeaux





 $S+P=1 \Rightarrow \operatorname{div}(v) = MP$ with $v = \nabla \pi$ and $\nabla \pi \cdot n = 0$ on an archnoid matter

Approach already used in

• Ribba et al JTB 2006

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Radiotherapy driven by imagery

Many works!

Swanson Lab, Group INRIA Asclepios Sophia Antipolis

Magnetic Resonance in Medicine 54:616-624 (2005)

Simulation of Anisotropic Growth of Low-Grade Gliomas Using Diffusion Tensor Imaging

Saâd Jbabdi,¹ Emmanuel Mandonnet,^{2*} Hugues Duffau,^{1,2} Laurent Capelle,^{1,2} Kristin Rae Swanson,³ Mélanie Pélégrini-Issac,¹ Rémy Guillevin,⁴ and Habib Benali^{1,5}

Physics in Medicine and Biology

PAPER

Radiotherapy planning for glioblastoma based on a tumor growth model: improving target volume delineation

Jan Unkelbach¹, Bjoern H Menze^{2.3}, Ender Konukoglu⁴, Florian Dittmann⁺, Matthleu Le^{1.2}, Nicholas Ayache² and Helen A Shih¹ Published 20 January 2014 • 2014 Institute of Physics and Engineering In Medicine

Physics in Medicine and Biology, Volume 59, Number 3

Physics in Medicine and Biology

PAPER

Radiotherapy planning for glioblastoma based on a tumor growth model: implications for spatial dose redistribution

Jan Unkelbach¹, Bjoern H Menze^{2,3}, Ender Konukoglu⁴, Florian Dittmann¹, Nicholas Ayache² and Helen A Shih¹ Published 20 January 2014 • 2014 Institute of Physics and Engineering in Medicine

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 $Group \ INRIA \ Asclepios \ Sophia \ Antipolis/Centre \ Lacassagne/MGH \ Boston$

One issue :

- Irradiated zone not always covers the infiltrated zone
- \Rightarrow How to optimize the radiated zone?



Swanson group

Tumor model

• The Fisher Kolmogorov equation :

Spatio-temporal evolution of the density of tumor cells

$$\partial_t u = \underbrace{\operatorname{div}(D\nabla u)}_{\text{Anisotropic diffusion}} \underbrace{+\rho u(1-u)}_{\text{Logistic growth}}$$

Parameters

- Anisotropic diffusion D(x) in the white matter, but isotropic in the grey matter.
- Growth rate ρ .

Radiotherapy driven by imagery

Towards personnalized therapy Original studies

Swanson group

- Anisotropy in the white matter D_0 (obtained with a diffusion MRI)
- Infiltration index : d/ρ (obtained with one MRI T2 flair + MRI T1 Gd)
- Propagation speed : $2\sqrt{d\rho}$ (obtained with two acquisitions of MRI T1 Gd)





Radiotherapy driven by imagery

Towards personnalized therapy A more complex strategy

Group INRIA Asclepios Sophia Antipolis/Centre Lacassagne/MGH Boston

• Radiotherapy optimization

 \rightsquigarrow Estimation for each voxels of the dose d_i minimizing the number of survival cells for a total dose imposed.



Outline of the talk

- 1 The simplest tumor growth models
- 2 Pharmacokinetics and pharmacodynamics (PK/PD) of a drug
- **3** A phase I/II clinical trial driven by a mathematical model
- 4 Low grade glioma : prolonged action of TMZ
- 5 One example of model of drug resistance
- 6 One example of interaction with the immune system
- 7 PDE system : meningioma
- 8 Radiotherapy driven by imagery
- 9 Structured models
- **10** Microtubule targeting agent

First example : A population balance equation for mitosis

Perthame, 2007

To take into account that cell division may depend on their age A tumor cell of size a can divide into two cells of age 0.

$$\partial_t \rho + \partial_a \rho = -B(a)\rho(t,a), x > 0, t > 0$$

 $\rho(t,0) = 2 \int_0^\infty B(a)\rho(t,a) \, da, \, \rho(0,x) = \rho_0(x)$

- $\rho(t, a)$ density of tumor cells at time t of age a.
- B(a) division rate
- $-B(a)\rho(t, a)da dt$ number of cell of age between a and a + da that divide between time t and t + dt.

McKendrick-vonFoerster equation or renewal equation

First example : A population balance equation for mitosis

Perthame, 2007

To take intot account that cell division may depend on their size A tumor cell of size x can divide into two cells of equal size x/2.

 $\partial_t \rho + \partial_x (g(x)\rho) = -B(x)\rho(t,x) + 4B(2x)\rho(t,2x), \ x > 0, \ t > 0$ $\rho(t,0) = 0, \ \rho(0,x) = \rho_0(x)$

- $\rho(t, x)$ density of tumor cells at time t of size x.
- B(x) division rate
- $-B(x)\rho(t,x)dx dt$ number of cell of size between x and x + dx that divide between time t and t + dt.
- $2B(2x)\rho(t,2x)d(2x) dt$ number of cell of size between 2x and 2(x + dx) that divide between time t and t + dt.

 \rightsquigarrow Extensions to bacteria proliferation (Doumic, Gabriel, Martin 2018).

 \rightsquigarrow Extensions to account the immune system (Atsou, Goudon 2018).

The original model of metastases

Iwata & al (2000) Verga, PhD Marseille (2010) A tumor growth = ODE system : gompertz's law

$$x'(t) = g_{a,b}(x) := ax \ln\left(\frac{b}{x}\right)$$

Metastases = renewal equation $\rho(t, x)$ density of metastases at time t of size x.



$$\partial_t \rho + \partial_x \left(g_{a,b}(x)\rho \right) = 0, \, t > 0, \, x \ge 1$$

Emission of metastases = a boundary layer : $\beta(x) = mx^{\alpha}$

 $g_{a,b}(1)\rho(t,1) =$

$$\underbrace{\beta(x_p(t))}_{\text{Emission by the primary tumor}}$$

 $+ \int_{1}$

 $\beta(x)\rho(t,x)\,dx \quad t > 0$

emission by the metastases

 \rightsquigarrow McKendrick-vonFoerster equation

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Metastases = renewal equation $\rho(t, x)$ density of metastases at time t of size x.



 $\partial_t \rho + \partial_x \left(g_{a,b}(x) \rho \right) = 0, \ t > 0, \ x \ge 1$

thus

$$\rho(t+\delta t, x+g(x)\delta t)(\delta x+(g(x+\delta x)-g(x))\delta t)=\rho(t,x)\delta x$$

and then

 $(\rho(t,x) + \delta t(g(x)\partial_x\rho(x,t) + \partial_t\rho(x,t)) + o(\delta x,\delta t))(\delta x + \delta t\delta xg'(x) + o(\delta x)) = \rho(t,x)\delta x$

Koscienly & al (1984)

Calibration of Iwata & al. model

Breast cancer: Relationship between the size of the primary tumour and the probability of metastatic dissemination

S. Koscielny¹, M. Tubiana², M.G. L^{&2}, A.J. Valleron¹, H. Mouriesse², G. Contesso² & D. Sarrazin²

¹Unité de Recherches Biomathématiques et Bostiatistiques Inserne U 263 and Université Paris 7-3, Place Justien 7253 Paris Colex BJ, Hautus Gustave Rouzy, Department of radiation therapy, pathology and modical statisticis, Rev Constitu Demonstration 24600 Vidigiaf, France.

Verga PhD (2010)

Tubbiana's study

- 2648 patients treated for breast cancer at Institut Gustave Roussy, Paris between 1954 and 1972.
- Proportion of patients that present metastases at detection.

In silico study

• 800 virtual patients. Parameters a, b fixed. Parameters m and α following a log-normal distribution.

Results



Identifiability of the parameters

 $\partial_t \rho + \partial_x (g_{\boldsymbol{a},\boldsymbol{b}}(x)\rho) = 0, \ g(1)\rho(t,1) = \beta_{\boldsymbol{m},\boldsymbol{\alpha}}(x_p) + \int_1^b \beta_{\boldsymbol{m},\boldsymbol{\alpha}}(x)\rho(t,x) \ dx$



a real patient.

A set of parameter $a, b, m, \alpha, x_0, \cdots$

Parameters of the primary tumor spreading

• Parameters (a, b, x_0) can be identify from 3 observations from the primary tumor.

Parameters of the metastases spreading : link to Volterra equation

Hartung (2013)

• Observables $F_f(t) = \int_1^b f(x)\rho(t,x) dt$ are solution of a Volterra equation

$$F_f(t) = [f(x_p) * \beta(x_p)](t) + [F_f * \beta(x_p)](t)$$

 \rightsquigarrow if $F_f \in \mathcal{C}^1, F_f(0) = 0$ and $F_f + f(x_p) \in \mathcal{C}^1, F_f + f(x_p)(0) \neq 0$, β can be identified from $F_f(t)$ and x_p .

Preclinical validation

J. Ciccolini, S. Mollard (CRO2)

Animal experiments

• Animals : 16 Female NOD Scid mice (8 weeks old)

→ very immunodefficient.

• Graft : orthoptic xenograft (Mammary glands).

 \rightsquigarrow human tumor Luciferase transvected cells

• Cells (human) MDA-MB-231-LUC (Caliper)

 \rightsquigarrow cells that emits photons in presence of Luciferin.

- Injection at d=0 150 000 cells/ 50μ L Matrigel
- Follow up by bioluminescence twice a week.

 \rightsquigarrow 3D reconstruction of the main tumor and the metastases (IVIS Sectrum. Living Image 4.2).



J. Ciccolini, S. Mollard (CRO2)

Comparison with the model

• 16 mice with few data per mouse. Total amount of observations : 166.





Strategy to identify a, b, x_0, m, α

• Use of Stochastic algorithm of Expectation-Maximization proposed Monolix tools (SAEM algorithm).



Extension of the Iwata & al. model

$$\begin{cases} \frac{\partial}{\partial t}\rho(t,x) + \frac{\partial}{\partial x}[g_m(x)\rho(t,x)] = 0, \ x \in [1,b), \ t \ge 0\\ g_m(1)\rho(t,1) = \int_1^b \beta(x)\rho(t,x)dx + \beta(x_p(t))\\ \rho(0,x) = 0, \end{cases}$$

with

$$x'_p = g_p(x_p)$$

where g_p and g_m are one of the classical growth speed :

Gompertz model (1825)	$g(x) = ax \ln\left(\frac{b}{x}\right)$
Logistic model (1838)	$g(x) = ax\left(1 - \frac{x}{K}\right)$
Von Bertalanffy (1949)	$g(x) = ax\left(x^{-\frac{1}{3}} - c\right)$
West& al (1997)	$g(x) = ax\left(x^{-\frac{1}{4}} - d\right)$

N. Hartung (I2M)

Conclusions

- The logical model is rejected by statistical tests.
- Overestimation the value of x_0 due to a poor estimate of the initial growth speed.
- \Rightarrow An hybrid model is necessary!
 - Gomp-exp model $g_p(x) = \min\left(a_{invitro}, ax \ln\left(\frac{b}{x}\right)\right)$
 - West-exp model $g_p(x) = \min\left(a_{invitro}, ax\left(\left(\frac{x}{b}\right)^{-\frac{1}{4}} 1\right)\right)$
 - ▶ The parameter $a_{invitro}$ is evaluated in vitro!
 - ▶ The new estimated sizes x_0 correspond to a 40-50% loss of cells after the graft that sounds reasonnable.
 - ▶ In peritoneum, for most the mice, we observed two secondary tumoral mass. We proved that their growth can not be explain by the classical ODE models. That enforced the utility of such a metastases model.
 - ▶ The estimated growth rate a_m in metastases differs from a_p .
Metastases and chemotherapy

Verga PhD 2010

A tumor growth = ODE system : gompertz's law extended

$$x'(t) = G(t, x) := ax \ln\left(\frac{b}{x}\right) - xC_{chemo}(t)$$

where C_{chemo} resumes the PK/PD of the chemotherapeutic agent. Metastases = a new transport equation $\rho(t, x)$ density of metastases at time t of size x.

 $\partial_t \rho + \partial_x \left(G(t, x) \rho \right) \, t > 0, \, x \ge 1$

Emission of metastases seen as a boundary layer : Birth law



 \rightsquigarrow Individualization of protocoles, taking into account metastases

Second example : metastase spreading

Combined anti-angiogenesis/chemotherapy I $_{\tt Hahnfeldt \& al}$ (1999), Benzekry & al (2012)

 $Tumor\ growth: Hahnfeldt\ \&\ al\ model$

$$\begin{aligned} \frac{dx}{dt} &= ax \ln\left(\frac{\theta}{x}\right) - \mathcal{F}(x) \mathcal{R}_1(c_{chemo}(t)) \\ \frac{d\theta}{dt} &= \mathcal{R}_2(c_{angio}(t)) x - d\theta x^{\frac{2}{3}} - \gamma \theta \mathcal{R}(c_{angio}(t)) \end{aligned}$$

- Chemotherapy acts on the tumor size.
- Antiangiogenic drugs acts on the carrying capacity.
 - Reduction of the growth velocity.
 - Possible stimulation at the beginning reflecting the normalization of the tumor.

Ebos & al. Cancer cell (2009)

Metastases growth = A transport equation

 $\rho(t, x, \theta)$ density of metastases at time t with a feature $X = (x, \theta)$.

 $\partial_t \rho + \operatorname{div} \left(G(t, X) \rho \right), \, t > 0, \, X \in \Omega$

Second example : metastase spreading

Extension to general emission

$$\begin{split} \frac{\partial}{\partial t}\rho(t,x) &+ & \frac{\partial}{\partial x}[g_m(x)\rho(t,x)] = k(x,x_p(t)) \\ &+ \int_x^{+\infty} k(x,y)\rho(t,y)\,dy - \rho(t,x)\int_0^x k(y,x)\,dy \\ \rho(t,1) &= & 0 \\ \rho(0,x) &= & 0, \\ &x'_p(t) &= & g_p(x_p(t)) \end{split}$$

▶ k(x, y) probability for a tumor of size x to emmit a metastase of size y.

Schlicke, 2018. Hubert, Tournus 2018

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- **10** Microtubule targeting agent

- A therapeutic target in oncology
 - MTs play a crucial role in
 - cell division
 - cell migration
 - intracellular transport



- MTs are a favorite target of Microtubule Targeting Agents (MTAs)
- MTAs (taxanes, vinca alkaloids) are successfully used as antimitotic and antiangiogenic agent in cancer treatments but also in neurodegenerative diseases.
- MTs are highly dynamic.
 - The dynamics is complex
 - The dynamics is mandatory to cell division and cell migration.



MT in the cell

- MTs are part of the cytosqueleton.
- MTs are caracterized by their instabilities.

Protein structure



- Each MT is a long (up to 50μ m) hollow cylinder of 25nm diameter built from about 13 protofilaments.
- Each protofilament is composed by an assembly of $\alpha|\beta$ tubulin dimers.
- The assembly is polarized with different dynamics at the + end or end.





MT structure

- Each MT is a long (up to 50μ m) hollow cylinder of 25nm diameter built from about 13 protofilaments.
- Each protofilament is composed by an assembly of $\alpha | \beta$ tubulin dimers.
- The assembly is polarized with different dynamics at the + end or end.
 - + End (tubulin β) : highly dynamic
 - - End (tubulin α) : link to centrosome in cells

Energetic structure of the dimers

- Dimers can be in two energy states :
 - GTP : Guanosine triphosphate active form
 - GDP : Guanosine diphosphate inactive form



Different state of the dimers

	Polymerized	Non polymerized
Active form	GTP polymerized in MTs	Free GTP
Inactive form	GDP polymerized in MTs	Free GDP

Stabilizing GTP -cap

Thanks to EB-GFP fluorescent proteins that bind to GTP-tubulin, are observed

- A GTP-stabilizing cap
- The disparition of the cap at the catastrophe

Main reactions







MTs in polymerization

• u(t, a, z, x) density of MT in polymerization

• t time, a age, x length, z length of the cap.

- 2 v(t, a, x) density of the population of MT in depolymerization
 - t time, a age, x length.
- **3** p = p(t) Free GTP tubulin

 $\rightsquigarrow t$ time.

• q = q(t) Free GDP tubulin $\rightarrow t$ time.

A. Barlukova PhD 2016

Balance equation for MT in Polymerization u



Boundary conditions for u

• Nucleation,

 $u(t, a, x, x) = \psi(x)\Psi(a)\mathcal{N}(p(t)).$

• Rescue event, if $\gamma_{pol}(p(t)) - \gamma_{hydro}(a) > 0$

$$v \Rightarrow u$$



A. Barlukova PhD 2016

Equation for MT in depolymerization \boldsymbol{v}

$$\partial_t v \underbrace{-\gamma_{depol}\partial_x v}_{\text{Depolymerization}} + \partial_a u = I_{u \to v} - I_{v \to u}$$

where

 $I_{v \to u}$: Rescue event

 $I_{u \to v}$: Catastrophe event



A. Barlukova PhD 2016

Equation for free GTP p



Equation for free GDP q

$$\frac{d}{dt}q = \underbrace{\gamma_{depol} \int_0^\infty \int_0^\infty v(t, a, x) \, da \, dx}_{\text{Depolymerization}} \quad \underbrace{-\kappa q}_{\text{Recycling}}$$

Output of the model concerning MTA

• Small delay in the hydrolysis may explain the comportment at low doses !

MTAs and migration

- MTAs reduce endothelial migration even at non-cytotoxic concentration.
- \Rightarrow Antiangiogenic effect at low dose.



For more informations on MT and migration - see R.Tesson



Conclusions

Winter 2020. Residential month on mathematical issues in biology Coordinators : F.Hamel, F. Hubert, E. Pardoux, P. Pudlo (I2M)

- Week 1 : February 3-7 Winter School PDE and probability for biology
 - OC. G. Chapuisat, B. Cloez C. Henderson, P. Pudlo, G. Raoul
- Week 2 : February 10-14 Workshop on Mathematical issues of evolutionary biology
 - **OC.** N. Champagnat, J. Coville, R. Gomulkiewicz, F. Hamel, L. Roques
- Week 3 : February 17-21 Workshop on Mathematical modeling and statistical analysis of infectious disease outbreaks
 - OC. T. Britton, E. Pardoux
- Week 4 : February 24-28 Workshop on Mathematical issues of complex systems in biology and medecine
 - OC. M. Cristofol, J.- M Freyermuth, C. Gomez, F. Hubert, S. Ryan, M. Tournus
- Week 5 : March 2-6 Winter school on Networks and molecular biology
 - OC. A. Baudot, B. Mossé, E. Remy, L. Tichit, M. Vignes

Website : https ://mathsbiomonth.sciencesconf.org

Conclusions



Thank you for your attention!