Fluid-structure interaction in the cardiovascular system

Part 2 -
Inverse problems

CEMRACS
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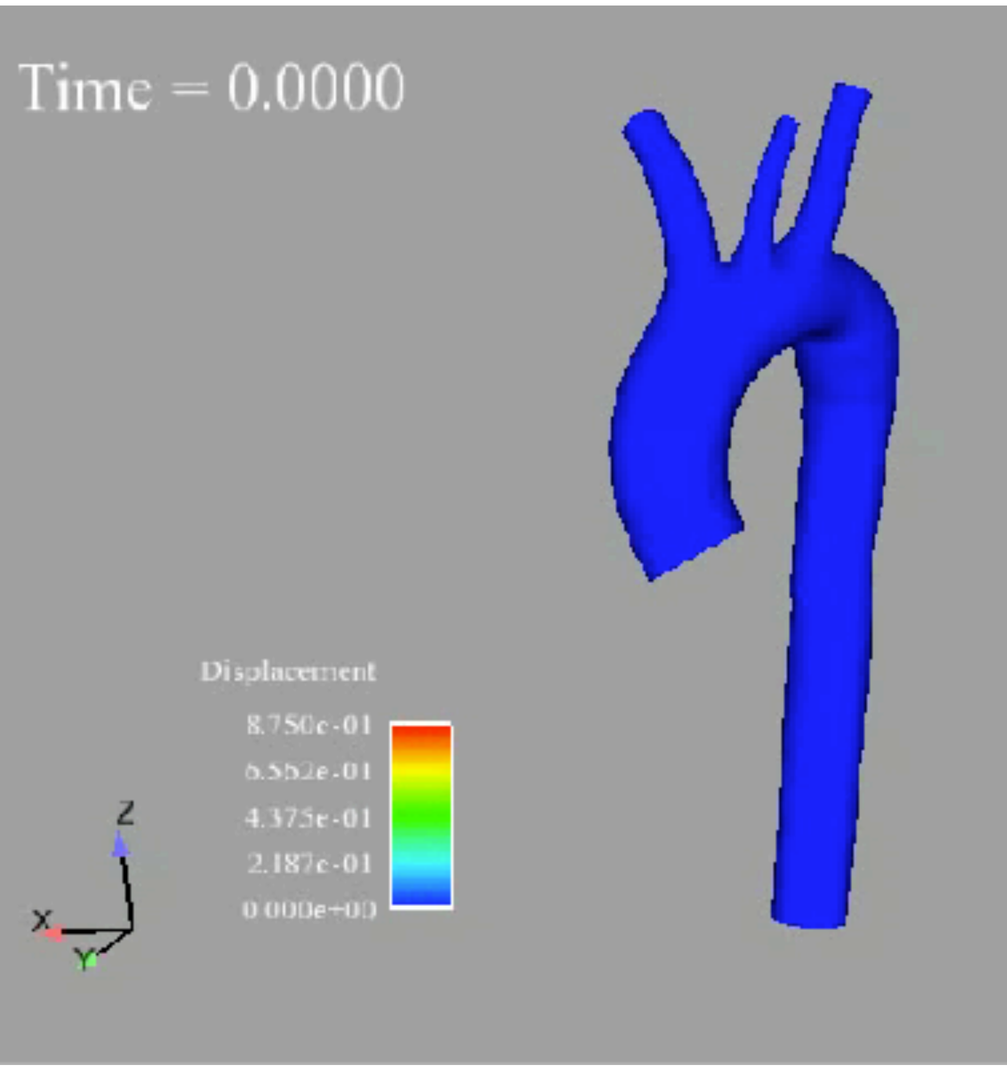
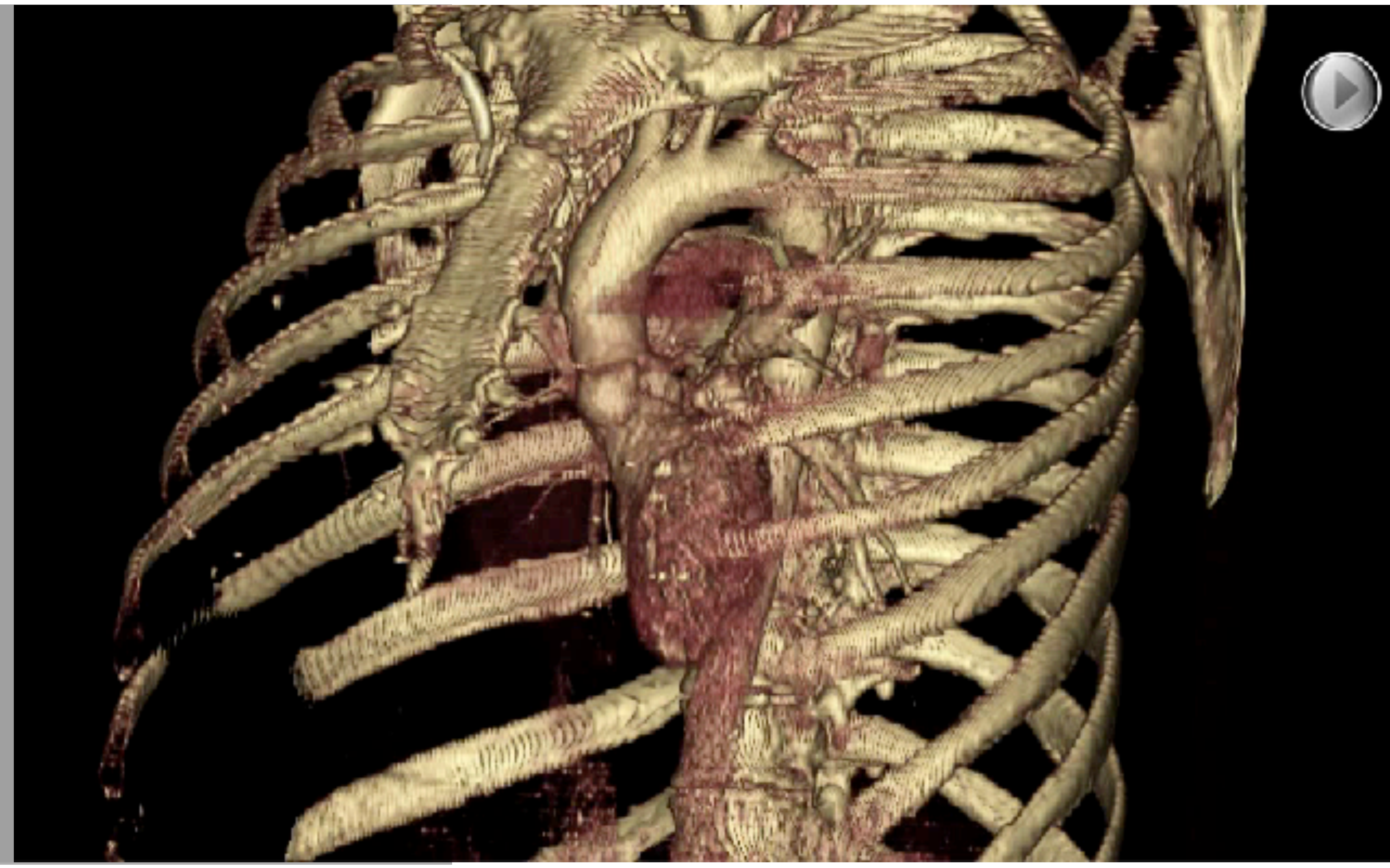
Medical data assimilation



FSI simulation

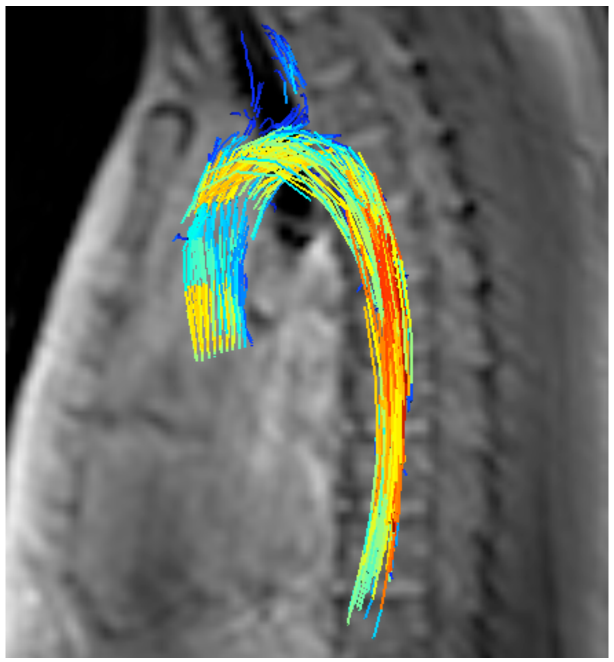


CT scan

P Moireau, C Bertoglio, N Xiao, CA Figueroa, CA Taylor, D Chapelle, JF Gerbeau (2013). "Sequential identification of boundary support parameters in a fluid-structure vascular model using patient image data". Biomechanics and modeling in mechanobiology, 12(3), 475-496.

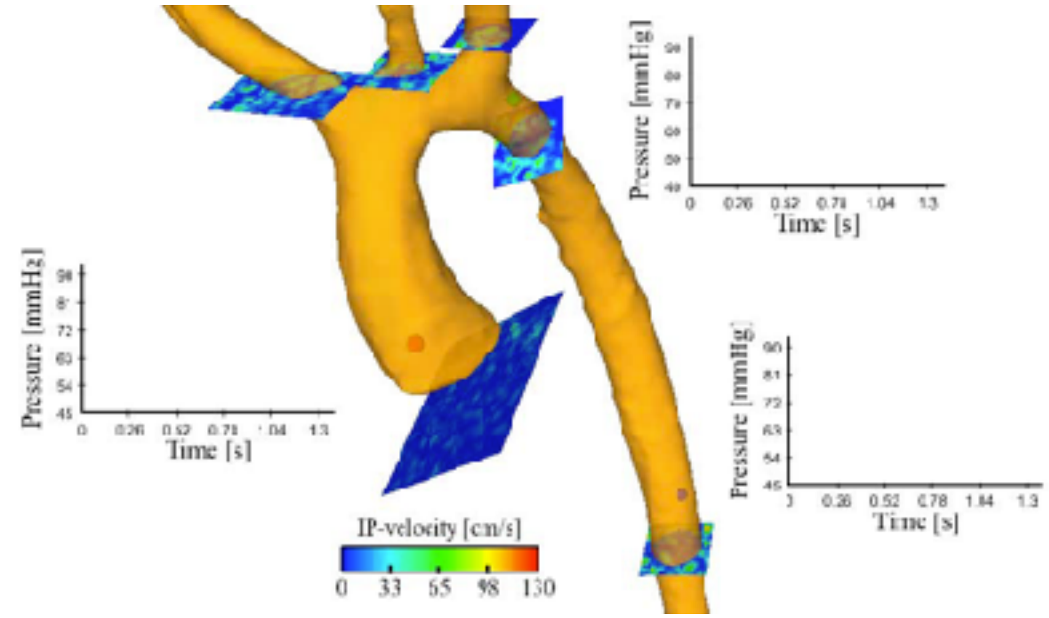
Medical data assimilation

Examples of data:



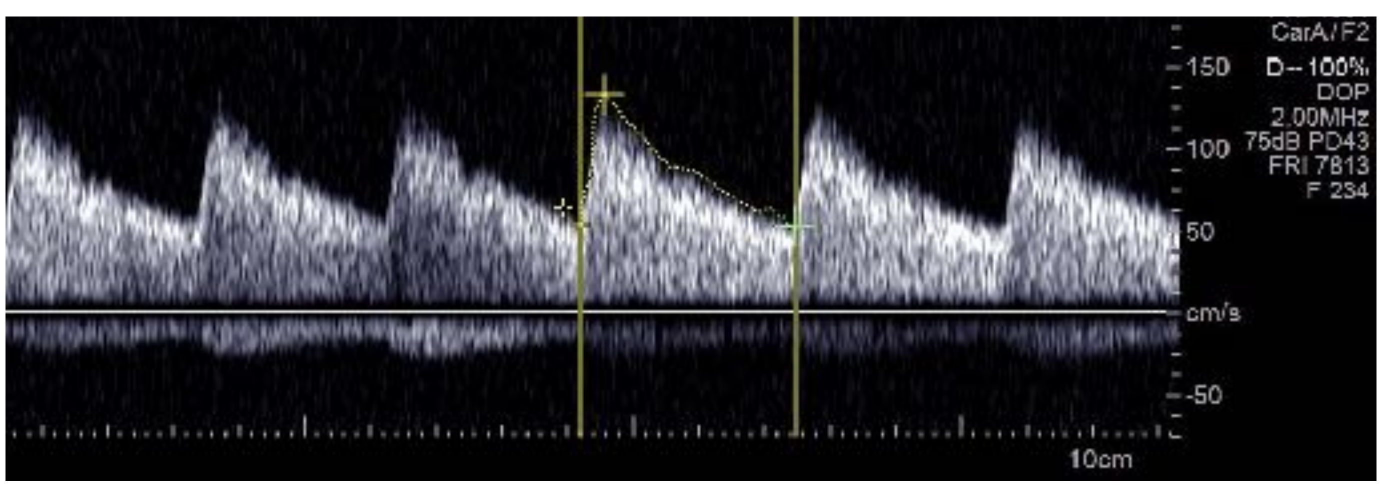
MRI: velocity field

Data: I. Valverde, P. Beerbaum (KCL)



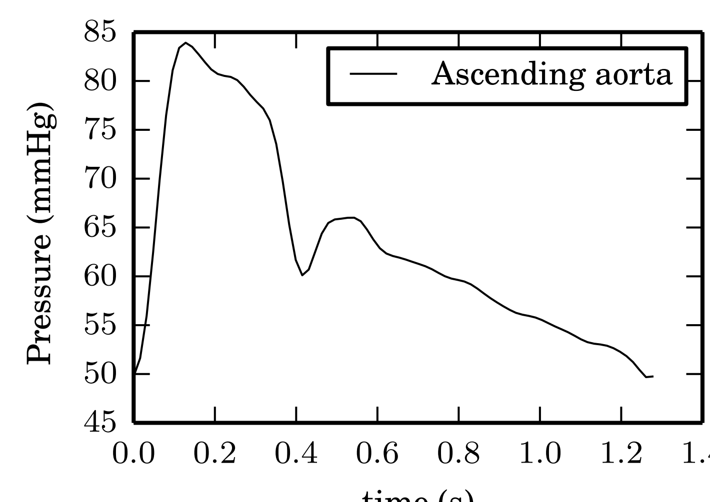
MRI: wall movement
+ velocity in a few planes

Data: I. Valverde, P. Beerbaum (KCL)



Ultrasound: velocity in the carotid

Data: S. Verlhac (Créteil)



Catheter: pressure in the aorta

Data: Stacom 2013 challenge

Medical data assimilation

Models
- Navier-Stokes equations
- Solid mechanics
- ...

Measurements
- medical imaging (CT, MRI, ...)
- blood flow (US, PC-MRI,...)
- pressure (catheter)
- ...

Estimate parameters
- artery wall stiffness
- boundary condition
- ...

Access to hidden quantities
- pressure
- wall stress
- ...

Improve measurements
- regularization
- interpolation
- ...

Outline

- Reminders on Data Assimilation
- Luenberger filters and FSI
- Applications to blood flow

Data assimilation

- General dynamical system: $\left\{ \begin{array}{rcl} \dfrac{dx}{dt} & = & A(x, \theta, t) \\ x(0) & = & x_0 \end{array} \right.$
  - The operator A represents the physical model
  - $x(t)$ denotes the state variable
  - θ denotes the physical parameters

- Example in FSI:
  - State variable: $x = [\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{v}^{\mathrm{s}}]$ (fluid velocity, solid displacement and velocity)
  - Parameters: $\theta = [\rho^{\mathrm{f}}, \mu, E, \ldots]$ (density, viscosity, Young modulus, ...)

Data assimilation

- General dynamical system: $$\left\{\begin{array}{rcl} \dfrac{dx}{dt} & = & A(x,\theta,t) \\ x(0) & = & x_0 \end{array}\right. \quad (1)$$

Imperfect knowledge of x_0 and θ

- Decomposition into a known part $(x_{0,\diamond},\theta_\diamond)$ and an unknown part (ζ_x,ζ_θ):

$$\left\{\begin{array}{rcl} x_0 & = & x_{0,\diamond}+\zeta_x \\ \theta_0 & = & \theta_\diamond+\zeta_\theta \end{array}\right. \quad (2)$$

with $(x_{0,\diamond},\theta_\diamond)$ is the *a priori* knowledge.

- **Goal:** estimate ζ_x and ζ_θ using measurements.
- **Notation:** $x_{[\zeta_x,\zeta_\theta]}$ denotes the solution to (1)-(2)
- **State estimation:** estimate ζ_x
- **Parameter identification:** estimate ζ_θ

Data assimilation

- Assume we have measurements $z^\dagger$ and an observation operator H:

$$\boxed{z^\dagger = H(x^\dagger, t) + \xi}$$

where $x^\dagger$ denotes the "true" trajectory.
and ξ denotes the measurement (observational) noise.

- A possible strategy in **data assimilation** is to minimize:

$$J(\zeta_x, \zeta_\theta) = \frac{1}{2}\int_0^T \underbrace{\|z^\dagger - H(x_{[\zeta_x,\zeta_\theta]}(t))\|^2_{W^{-1}}}_{\text{data misfit}} \, dt + \underbrace{\frac{1}{2}\|\zeta_x\|^2_{P^{-1}_{x,\diamond}} + \frac{1}{2}\|\zeta_\theta\|^2_{P^{-1}_{\theta,\diamond}}}_{\text{discrepancy with the } a\ priori \text{ knowledge}}$$

where $\|\cdot\|_{W^{-1}}, \|\cdot\|_{P^{-1}_{x,\diamond}}, \|\cdot\|_{P^{-1}_{\theta,\diamond}}$ are given norms.

- The norms weight differently the different terms, reflecting our confidence.

- For example $\|\theta\|^2_{P^{-1}_{\theta,\diamond}} = \frac{1}{\sigma^2_\theta}|\theta|^2,$ where σ_θ quantifies the lack of confidence in $\theta_\diamond$

Data assimilation

- **Formally**, state and parameter estimation can be reformulated as the state estimation of an **augmented system**:

$$\begin{cases} \dfrac{dx}{dt} &= A(x, \theta_\diamond + \zeta_\theta, t) \\ x(0) &= x_{0,\diamond} + \zeta_x \end{cases} \longrightarrow \begin{cases} \dfrac{d\tilde{x}}{dt} &= \tilde{A}(\tilde{x}, t) \\ \tilde{x}(0) &= \tilde{x}_{0,\diamond} + \tilde{\zeta} \end{cases}$$

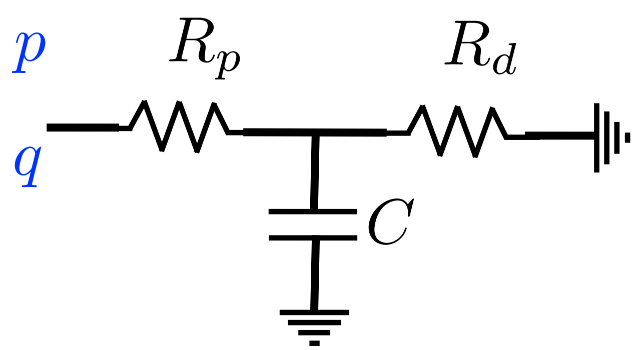
with $\tilde{x} = \begin{pmatrix} x \\ \theta \end{pmatrix}$, $\tilde{A}(\tilde{x}, t) = \begin{pmatrix} A(x, \theta, t) \\ 0 \end{pmatrix}$, and $\tilde{x}_{0,\diamond} = \begin{pmatrix} x_{0,\diamond} \\ \theta_\diamond \end{pmatrix}$.

- **Minimize**:

$$J(\tilde{\zeta}) = \frac{1}{2} \int_0^T \|z^\dagger - H(\tilde{x}_{[\tilde{\zeta}]}(t))\|^2_{W^{-1}}\, dt + \frac{1}{2} \|\tilde{\zeta}\|^2_{P_\diamond^{-1}}$$

Data assimilation

- The linear assumption is *really* too restrictive !
- **Example:**
  - the **linear** RCR equation:



$$C\frac{dp}{dt} + \frac{p}{R_d} = \left(1 + \frac{R_p}{R_d}\right) q + R_p C \frac{dq}{dt}$$

  - becomes **nonlinear** when augmented to estimate the **parameters**:

$$\left\{ \begin{array}{rcl} C\dfrac{dp}{dt} + \dfrac{p}{R_d} & = & \left(1 + \dfrac{R_p}{R_d}\right) q + R_p C \dfrac{dq}{dt} \\ \dfrac{dR_p}{dt} & = & 0 \\ \dfrac{dR_d}{dt} & = & 0 \\ \dfrac{dC}{dt} & = & 0 \end{array} \right.$$

$$p(0) = p_{0,\diamond},\ R_p(0) = R_{p,\diamond},\ R_d(0) = R_{d,\diamond},\ C(0) = C_\diamond$$

Data assimilation

- **Variational methods**

  Optimization algorithm to solve the (constrained) minimization problem

$$J(\tilde{\zeta}) = \frac{1}{2}\int_0^T \|z^\dagger - H(\tilde{x}_{[\tilde{\zeta}]}(t))\|^2_{W^{-1}}\, dt + \frac{1}{2}\|\tilde{\zeta}\|^2_{P_\diamond^{-1}}$$

  - Usually based on gradients (adjoint equations)
  - Needs several direct and backward resolutions (the problem and its adjoint)

- **Sequential methods (filtering)**

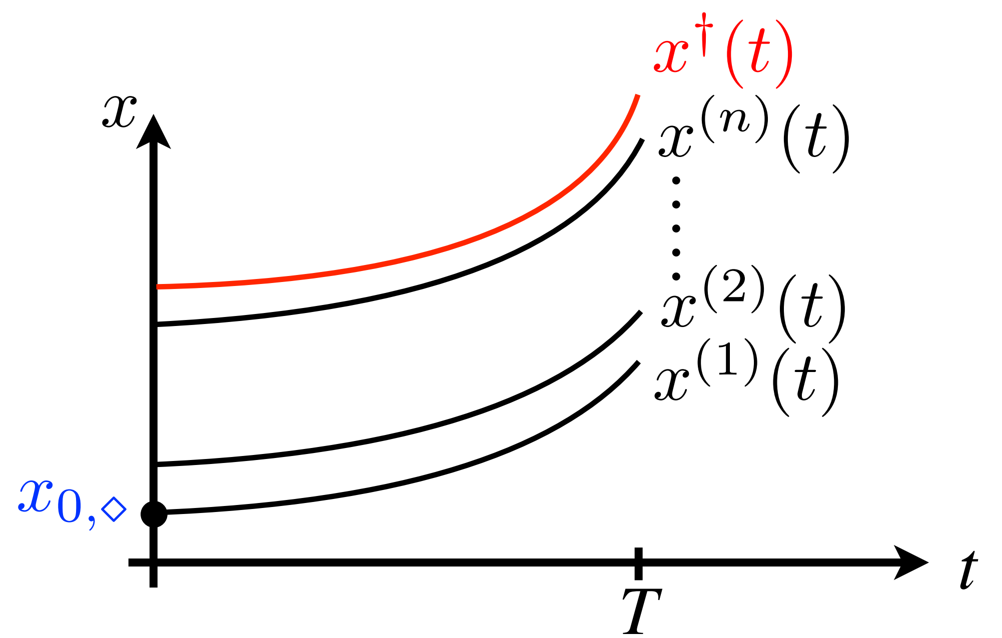
  Introduce an "**observer**", i.e. a modified dynamical system:

$$\frac{d\hat{x}}{dt} = A(\hat{x}, t) + G(z^\dagger - H(\hat{x}))$$

  where G is an operator called the *observer gain* or *filter*.

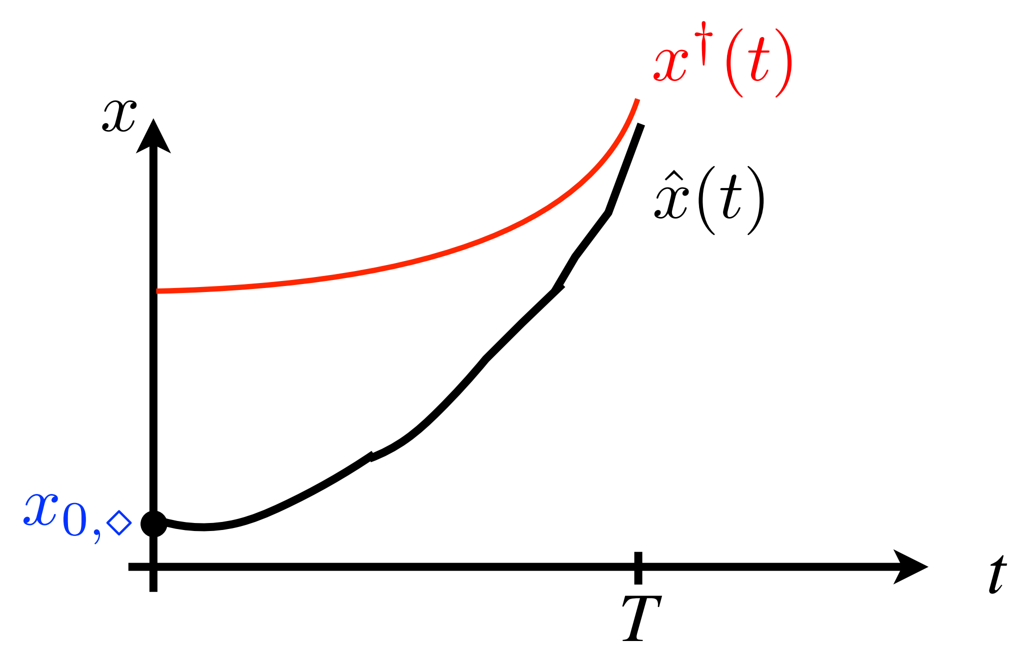
Data assimilation

- **Variational methods:**



$x^{\dagger}(t)$: true trajectory
$x^{(k)}(t)$: successive iterations

- **Sequential methods:**



$x^{\dagger}(t)$: true trajectory
$\hat{x}(t)$: filtered trajectory

Data assimilation

- **How to construct operator G in sequential methods ?**

$$\frac{d\hat{x}}{dt} = A(\hat{x}, t) + G(z - H(\hat{x}))$$

- **We will use :**
  - Nonlinear extension of Kalman filter
  - Luenberger observers

- **But we should ask before:**
  - Is it reasonable to perform the identification ? (identifiability...)

On Identifiability of Nonlinear ODE Models and Applications in Viral Dynamics,
H Miao, X Xia, A Perelson, H Wu, SIAM Review, Vol. 53, num 1, p. 3-39

K Thomaseth, C Cobelli, Generalized sensitivity functions in physiological
system identification. Annals of biomed engng 27.5 (1999): 607-616.

S Pant, B Fabrèges, JF Gerbeau, I Vignon-Clementel, A methodological paradigm for patient-
specific multi-scale CFD simulations. *Int. J. Num. Meth. Biomed. Engng*, 30(12) :1614–1648, 2014.

Data assimilation

Extension of Kalman to nonlinear problems

Unscented Kalman Filter (UKF):

- In Kalman and EKF, the covariances were obtained by recursive formulas
- In UKF, they are obtained by empirical averaging
- Propagation of the N_p particles: $x_{i,-}^{k+1} = F^{k+1}(x_{i,+}^{k})$ and $z_{i,-}^{k+1} = H(x_{i,-}^{k})$

Empirical means: $\bar{x} = \sum_{i=1}^{N_p} \omega_i x_i, \;\; \bar{z} = \sum_{i=1}^{N_p} \omega_i z_i$

Empirical covariances: $P_z = \sum_{i=1}^{N_p} \omega_i' (z_i - \bar{z})(z_i - \bar{z})^T, \; P_{xz} = \sum_{i=1}^{N_p} \omega_i' (x_i - \bar{x})(z_i - \bar{z})^T$

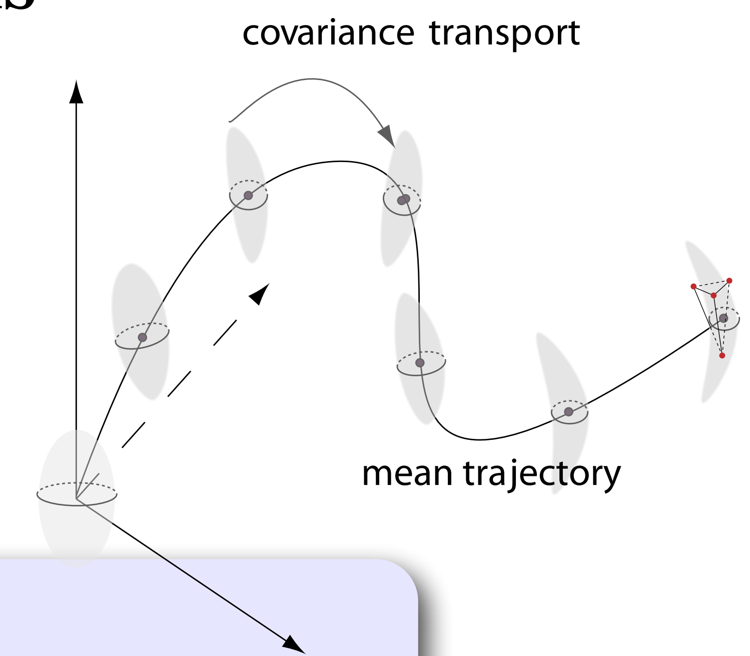
- UKF gain: $\boxed{K = P_{xz} P_z^{-1}}$

By analogy with the Kalman gain: $K = \underbrace{PH^T}_{\mathbb{E}[(x-x^\dagger)(z-z^\dagger)^T]} \big(\underbrace{R + HPH^T}_{\mathbb{E}[(z-z^\dagger)(z-z^\dagger)^T]}\big)^{-1}$

SJ Julier, JK Uhlmann, New extension of the Kalman filter to nonlinear systems, *AeroSense'97*, p. 182-193, 1997

Data assimilation

Extension to nonlinear problems



Unscented Kalman Filter (UKF)

- Nonlinear propagation of a "particles"
- **No tangent operator:** Empirical mean and covariance
- **Embarrassingly parallel: 1 particle = 1 run**

- Rationale in 1D:
  - Let $\bar{X} = \mathbb{E}(X)$ and $\sigma^2 = \mathrm{Var}(X) = \mathbb{E}((X - \bar{X})^2)$
  - Let be 3 "particles": $X_1 = \bar{X}, \quad X_2 = \bar{X} + \sqrt{\frac{3}{2}}\sigma, \quad X_3 = \bar{X} - \sqrt{\frac{3}{2}}\sigma$
  - By construction: $\displaystyle\sum_{i=1}^{3} \frac{1}{3} F(X_i) = F(\bar{X}) + \frac{\sigma^2}{2} F''(\bar{X}) + \ldots$

which is an approximation of $\mathbb{E}(F(X))$ **better** than $F(\bar{X})$.

- *N* dimensions: *2N+1* particles (but a strategy with only *N* particles exists)
- Cholesky factorisation to compute the square root of the covariance matrices

SJ Julier, JK Uhlmann, New extension of the Kalman filter to nonlinear systems, *AeroSense'97*, p. 182-193, 1997

Data assimilation

- Reminder: state-parameter augmented system:

$$\begin{cases} \dfrac{dx}{dt} & = & A(x, \theta, t) \\ x(0) & = & x_0 \end{cases} \longrightarrow \begin{cases} \dfrac{d\tilde{x}}{dt} & = & \tilde{A}(\tilde{x}, t) \\ \tilde{x}(0) & = & \tilde{x}_0 \end{cases}$$

- Example in **FSI**:
  - State variable: $x = [\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{v}^{\mathrm{s}}] \in \mathbb{R}^n$ (fluid velocity, solid disp. and velocity)
  - Parameters: $\theta = [\rho^{\mathrm{f}}, \mu, E, \ldots] \in \mathbb{R}^p$ (density, viscosity, Young modulus, $\ldots$]
  - Measurements: $z \in \mathbb{R}^m$ (wall displacement, flow rate, velocity field, $\ldots$]

- **Major concern**: matrix K is full and its size is $(n + p) \times m$

Intractable for large systems (PDE) !

Remarks about implementation

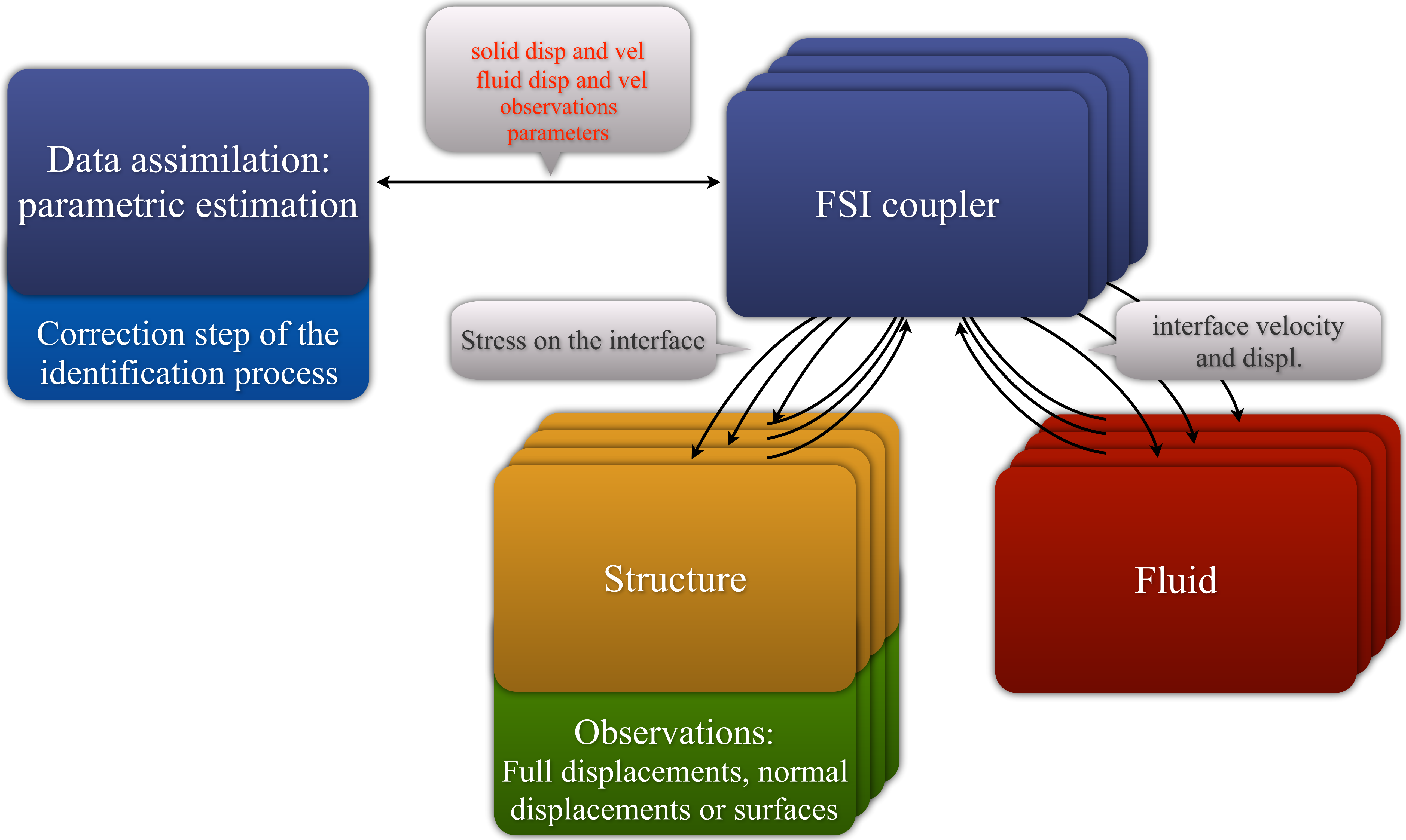
- **Strategy:**
  - **UKF for the parameters:** external minimally-intrusive algorithm
  - **Luenberger for the state:** implemented in the finite element solvers

P Moireau, D Chapelle. Reduced-order "Unscented Kalman Filtering with application to parameter identification in large-dimensional systems", *ESAIM: Control, Optimisation and Calculus of Variations*, 17:2 (2011) p. 380-405.

Automatic control : *Zhang-02*
Oceanography: *Pham-Verron-Roubeaud-97*

- **Remarks about implementation:**
  - Implementation much simpler than adjoint based algorithms
  - Parallelism is trivial
  - Very easy to estimate new parameters

Remarks about implementation



Outline

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P Moireau, D Chapelle, P Le Tallec, Joint state and parameter estimation for distributed mechanical systems, *Comput. Methods Appl. Mech. Eng.* 197 (2008) p. 659–677.

C Bertoglio, D Chapelle, MA Fernández, JF Gerbeau, P Moireau, State observers of a vascular fluid–structure interaction model through measurements in the solid, *Comput. Methods Appl. Mech. Eng.* 256 (2013) (2011) 149–168

Luenberger observers in a nutshell

Luenberger filters: may look simple but...

- there are some pitfalls
- there is room for creativity!

The case of a linear dynamics:

- "Real" dynamics (without noise): $\dfrac{dx^\dagger}{dt} = Ax^\dagger + G\underbrace{(z^\dagger - Hx^\dagger)}_{=0}$
- Observer *(Luenberger)*: $\dfrac{d\hat{x}}{dt} = A\hat{x} + G(z^\dagger - H\hat{x})$

- Dynamics of the error $e = x^\dagger - \hat{x}$:

$$\boxed{\frac{\mathrm{d}e}{\mathrm{d}t} = (A - GH)e}$$

Luenberger observers in a nutshell

- Spectral properties of the error dynamics:

$$(A - GH)\Phi_k = \lambda_k \Phi_k$$

≤ 0

Goal: Devise an operator G to reduce $\max\left(Re(\lambda_k)\right)$

- **Remark**: In dissipative system, error in initial condition is "forgotten".... but, in view of **joint state-parameter** estimation, we want to forget it as quickly as possible !

- Typically, to decrease the initial error by a factor β in a time T_c:

$$\max\left(Re(\lambda_k)\right) \leq \frac{\log \beta}{T_c}$$

- Ex: to have $\beta = 10$ in $T_c = 0.1s$, $\max\left(Re(\lambda_k)\right) \approx -25$

Luenberger observers in elastodynamics

- Elastodynamics equations $x = [\boldsymbol{d}, \boldsymbol{v}]$
- Velocity filtering: *Direct Velocity Feedback* (**DVF**)

$$\begin{cases} M_s \dfrac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t} + K_s \hat{\boldsymbol{d}} &= R + \gamma_v \overbrace{H^T M_H (Z - H\hat{\boldsymbol{v}})}^{\int_{\Sigma_0} (\boldsymbol{z} - \hat{\boldsymbol{v}}) \cdot \boldsymbol{\phi}_i} \\ \dfrac{\mathrm{d}\hat{\boldsymbol{d}}}{\mathrm{d}t} &= \hat{\boldsymbol{v}} \end{cases}$$

- Equation of the error: $e_{\boldsymbol{v}} = \boldsymbol{v} - \hat{\boldsymbol{v}}$, $e_{\boldsymbol{d}} = \boldsymbol{d} - \hat{\boldsymbol{d}}$

$$\boxed{M_s \frac{\mathrm{d}e_{\boldsymbol{v}}}{\mathrm{d}t} + K_s e_{\boldsymbol{d}} = -\gamma_v H^T M_H H e_{\boldsymbol{v}}}$$

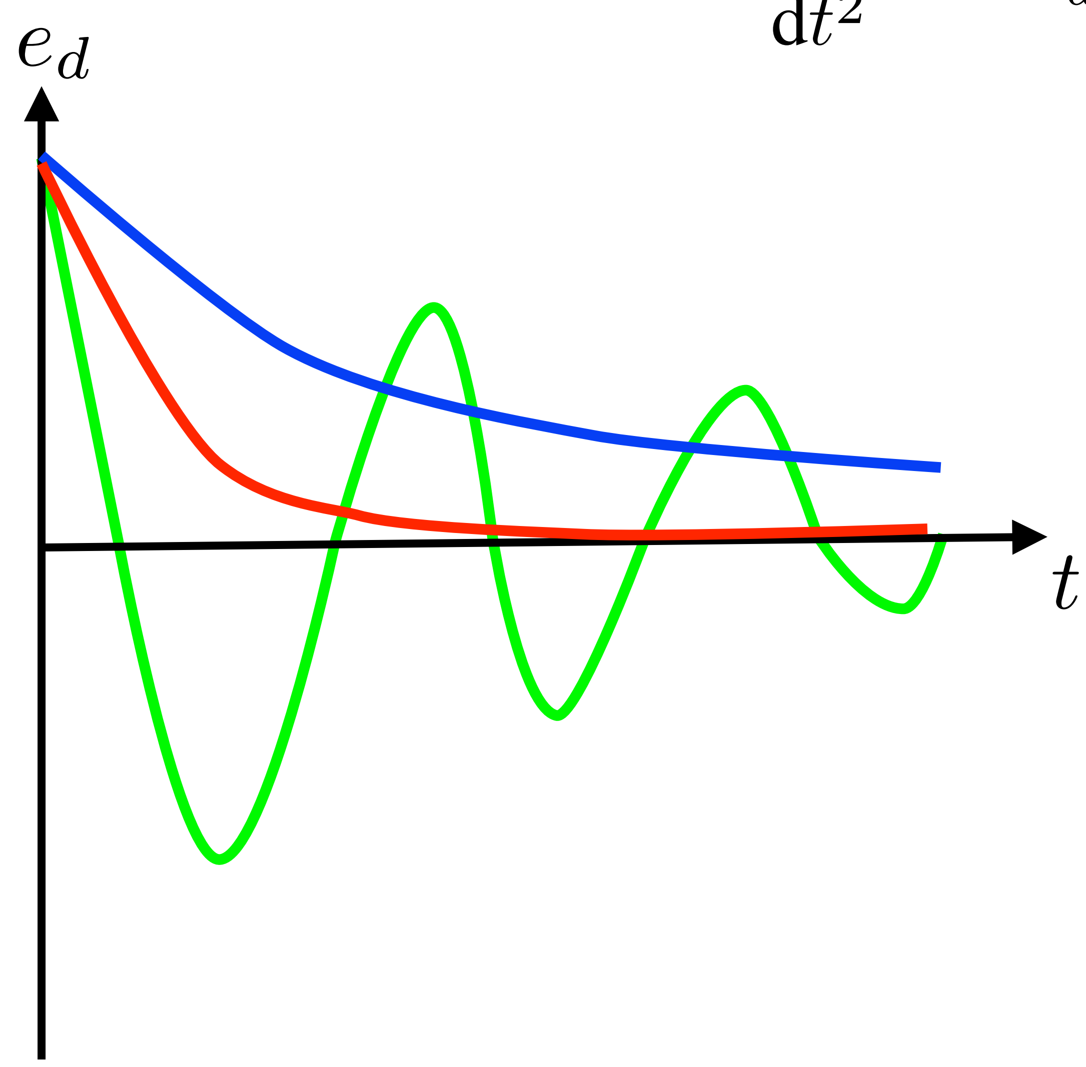
- Energy equation of the error: $e_{\boldsymbol{v}} = \boldsymbol{v} - \hat{\boldsymbol{v}}$, $e_{\boldsymbol{d}} = \boldsymbol{d} - \hat{\boldsymbol{d}}$

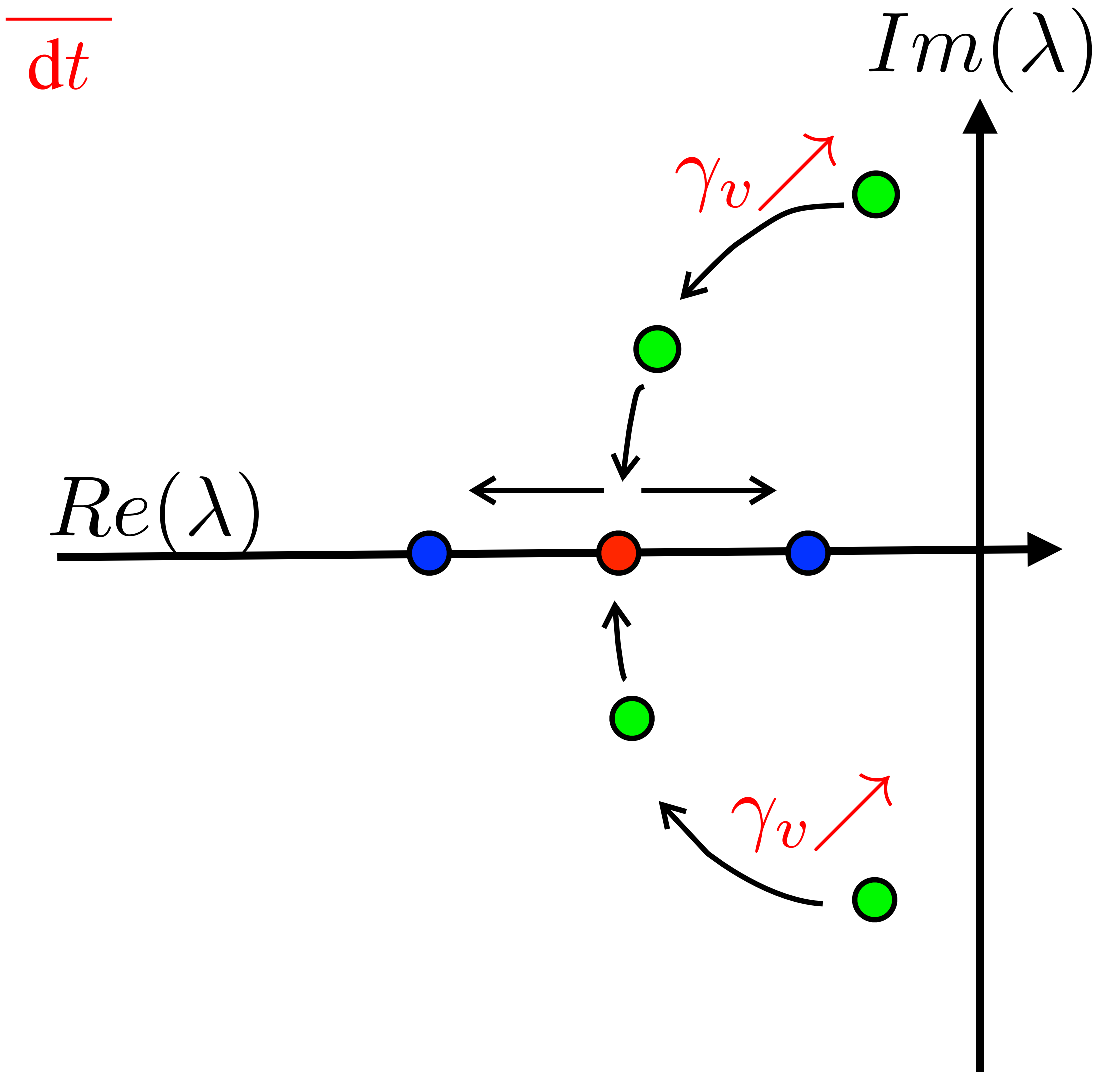
$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\Big[(M_s e_{\boldsymbol{v}}, e_{\boldsymbol{v}}) + (K_s e_{\boldsymbol{d}}, e_{\boldsymbol{d}})\Big] = -\gamma_v (M_H H e_{\boldsymbol{v}}, H e_{\boldsymbol{v}})$$

P Moireau, D Chapelle, P Le Tallec, Joint state and parameter estimation for distributed mechanical systems, *Comput. Methods Appl. Mech. Eng.* 197 (2008) p. 659–677.

- A trivial example: linear oscillator

$$m\frac{\mathrm{d}^2 e_d}{\mathrm{d}t^2} + k e_d = -\gamma_v \frac{\mathrm{d}e_d}{\mathrm{d}t}$$





- Let $\omega_0 = \sqrt{\dfrac{k}{m}}$ and $\beta = \dfrac{\gamma_v}{2m}$

  - If $\beta < \omega_0$: underdamped
  - If $\beta = \omega_0$: critically damped
  - If $\beta > \omega_0$: overdamped

Luenberger observers in elastodynamics

- In practice, it is often more convenient to work with **displacements**

- First option:

$$\begin{cases} M_s \dfrac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t} + K_s \hat{\boldsymbol{d}} & = & R + \gamma_d H^T M_H (Z - H\hat{\boldsymbol{d}}) \\ \dfrac{\mathrm{d}\hat{\boldsymbol{d}}}{\mathrm{d}t} & = & \hat{\boldsymbol{v}} \end{cases}$$

- **Remarks:**
  - Related to the "**Image Force Method**" used in the medical imaging community
  - Poor behavior (except maybe for systems with very large dissipation)

Luenberger observers in elastodynamics

- Displacement filtering: *Schur Displacement Feedback* **(SDF)**

$$\begin{cases} M_s \dfrac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t} + K_s \hat{\boldsymbol{d}} &= R \\ \dfrac{\mathrm{d}\hat{\boldsymbol{d}}}{\mathrm{d}t} &= \hat{\boldsymbol{v}} + \gamma_d K_\mu^{-1} H^T M_H (Z - H(\hat{\boldsymbol{d}})) \end{cases}$$

with $K_\mu = K_s + \mu H^T M_\Gamma H$.

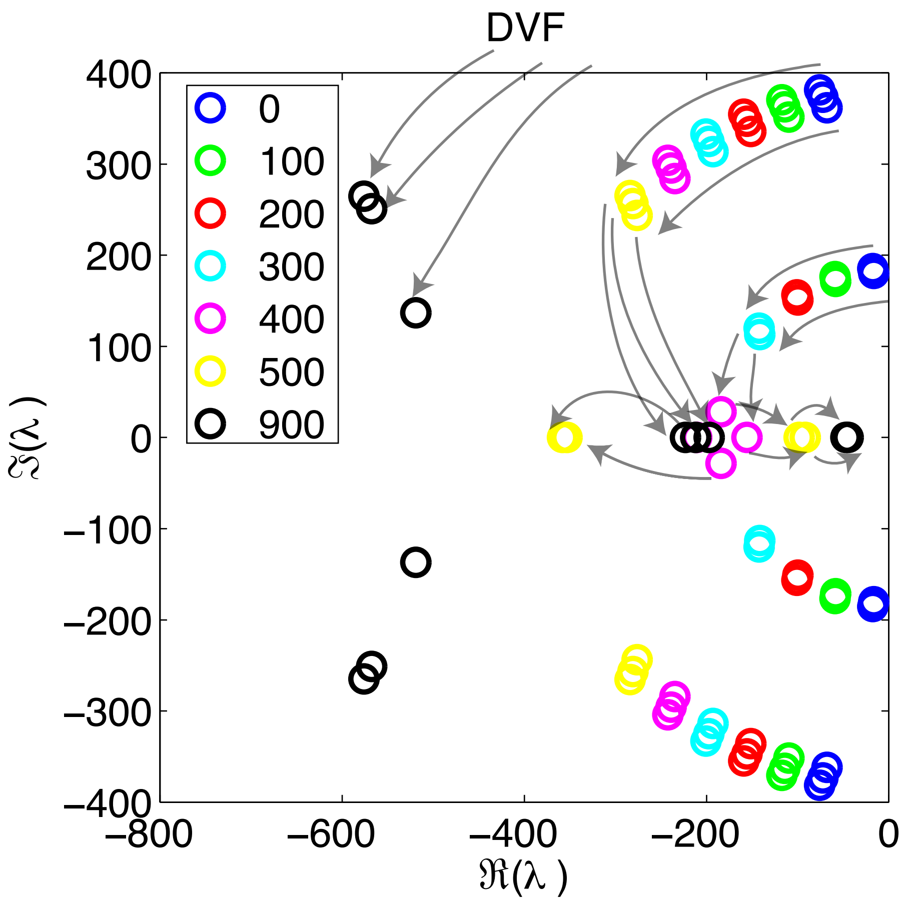
- **Remarks:**
  - Velocity is no longer the derivative of displacement

$$\frac{\partial \boldsymbol{d}}{\partial t} = \boldsymbol{v} + \gamma_d \mathrm{Ext}(\boldsymbol{z} - \boldsymbol{d})$$

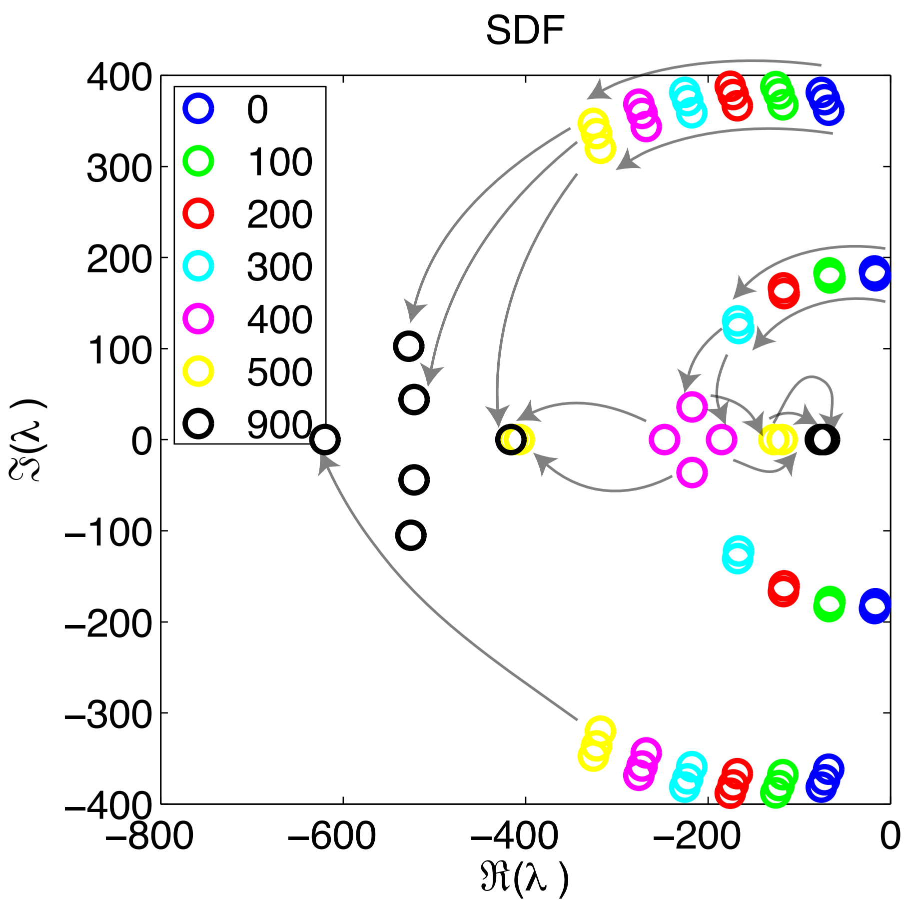
  - **The norm matters!**

P Moireau, D Chapelle, P Le Tallec, Filtering for distributed mechanical systems using position measurements: perspectives in medical imaging, *Inverse Problems* 25 (3) (2009)

Luenberger observers in elastodynamics



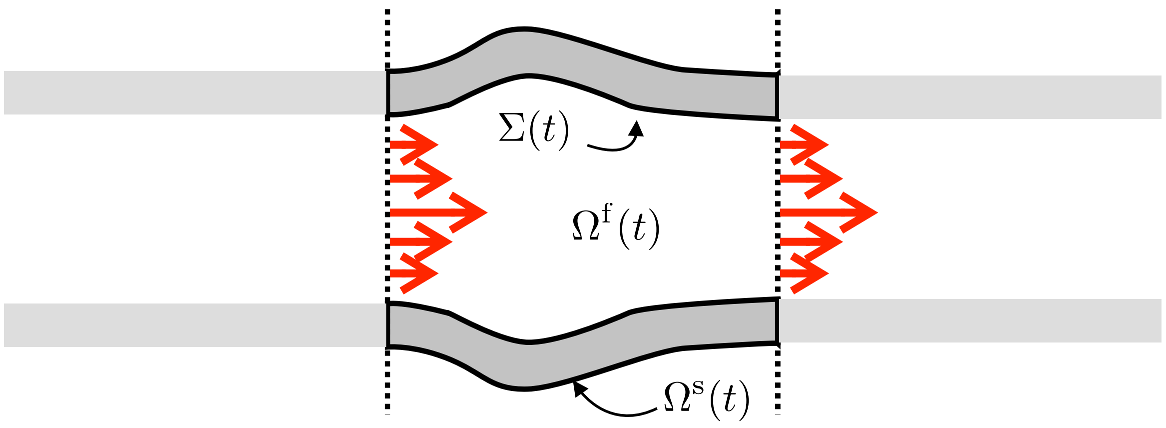
Velocity feedback



Displacement feedback

➡ **DVF and SDF have a similar behavior in elastodynamics**

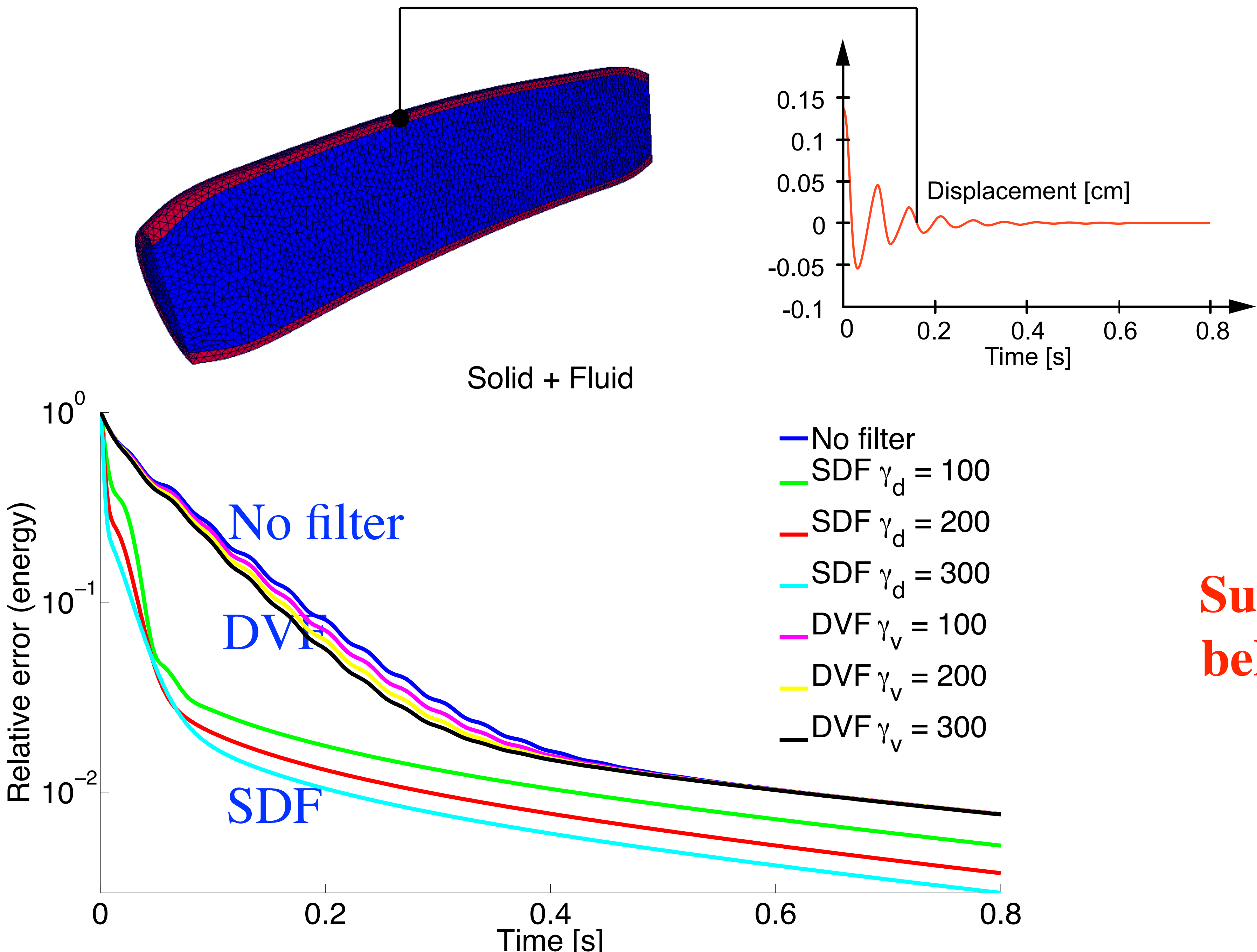
Luenberger observers in FSI



- We limit ourselves to ***solid measurements***
- We are interested in:
  - The effect of the FSI coupling
  - The effect of boundary conditions
  - The effect of fluid dissipation

1st nonlinear test: stabilization to equilibrium

- Fluid initially at rest
- Initial perturbation in the solid

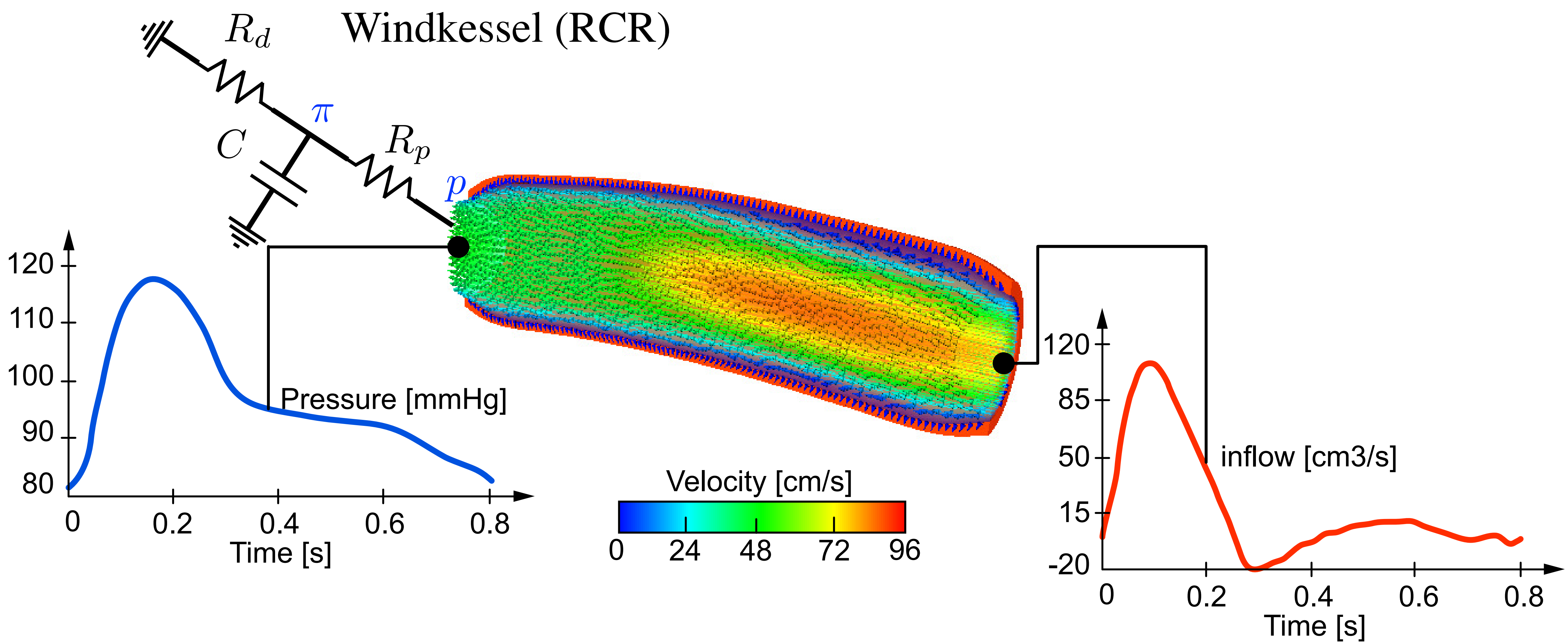


Energy norm of the error

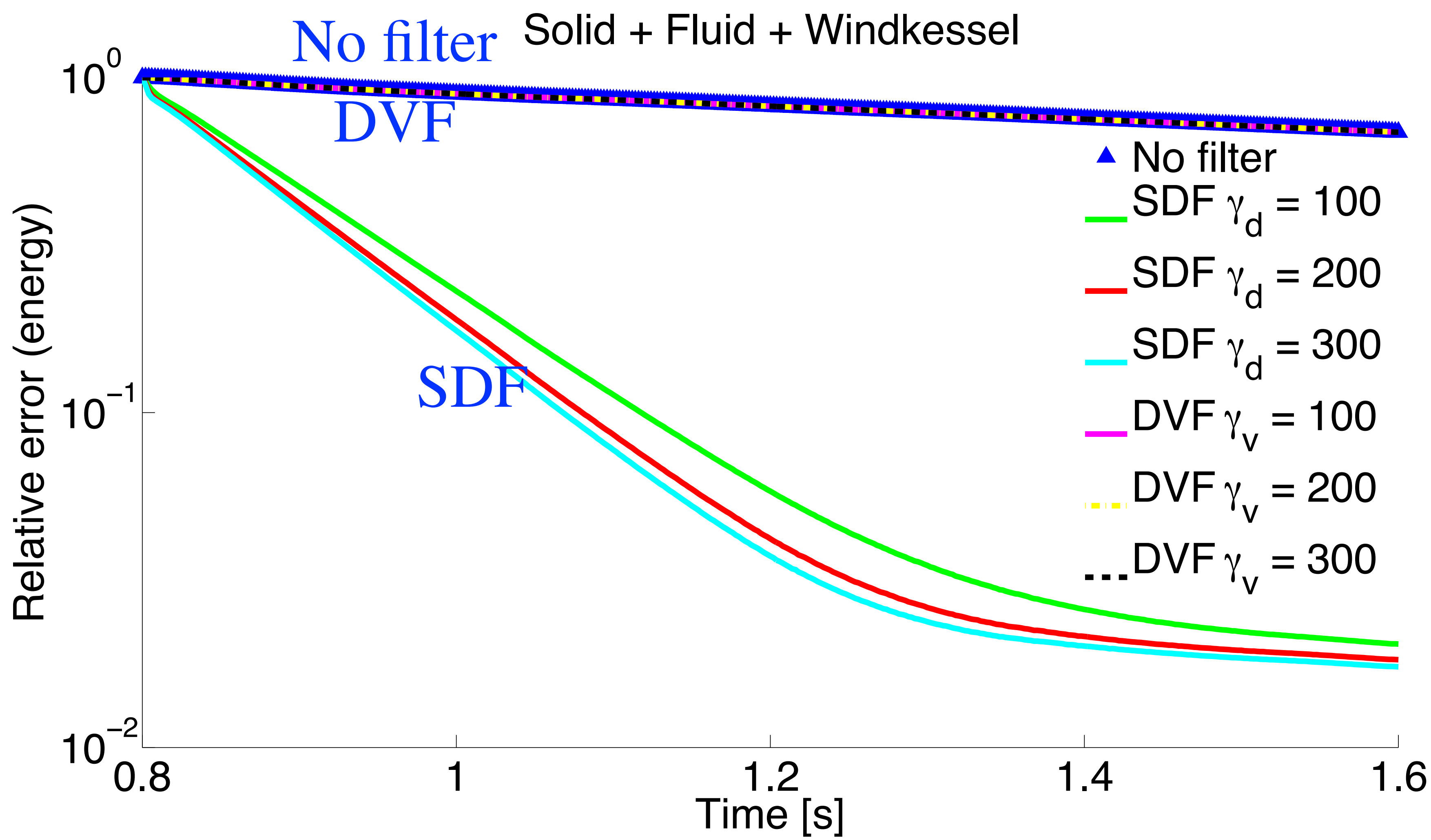
Surprisingly poor behavior of DVF !

2d nonlinear test: hemodynamics

$$p = \pi + R_p Q \quad \text{with} \quad \begin{cases} C\dfrac{d\pi}{dt} + \dfrac{\pi}{R_d} & = & Q \\ \pi\big|_{t=0} & = & \pi_0 \end{cases} \quad \text{and} \quad Q = \int_{\Gamma_{\text{out}}} \boldsymbol{u}\cdot\boldsymbol{n}\, dS$$



2d nonlinear test: hemodynamics



Energy norm of the error

Why DVF behaves so poorly in FSI compared to elastodynamics ?

SDF and DVF in FSI

Analysis of a toy model

- Toy model for the fluid:

$$\left\{\begin{aligned} \rho^{\mathrm{f}}\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{\nabla} p &= 0, \text{ in } \Omega^{\mathrm{f}} \\ \operatorname{div} \boldsymbol{u} &= 0, \text{ in } \Omega^{\mathrm{f}} \\ \boldsymbol{u}\cdot\boldsymbol{n} &= \dot{\boldsymbol{d}}, \text{ on } \Sigma \end{aligned}\right. \quad \underset{\text{div}}{\Longrightarrow} \quad \left\{\begin{aligned} -\Delta p &= 0, \ \text{ in } \Omega^{\mathrm{f}} \\ \frac{\partial p}{\partial \boldsymbol{n}} &= -\rho^{\mathrm{f}}\frac{\partial \boldsymbol{u}}{\partial t}\cdot\boldsymbol{n} = -\rho^{\mathrm{f}}\ddot{\boldsymbol{d}}\cdot\boldsymbol{n}, \ \text{ on } \Sigma \end{aligned}\right.$$

- Let $\mathcal{M}_{\mathrm{A}}$ be the "Neumann-to-Dirichlet" operator: $p_{|\Sigma} = -\rho^{\mathrm{f}}\mathcal{M}_{\mathrm{A}}\ddot{\boldsymbol{d}}\cdot\boldsymbol{n}$

- Linear elasticity:

$$\left\{\begin{aligned} \rho^{\mathrm{s}}\ddot{\boldsymbol{d}} - \operatorname{div}\sigma(\boldsymbol{d}) &= 0, \ \text{ in } \Omega^{\mathrm{s}} \\ \sigma(\boldsymbol{d})\cdot\boldsymbol{n} = p|_{\Sigma}\boldsymbol{n} &= -\rho^{f}\mathcal{M}_{A}\ddot{\boldsymbol{d}}\cdot\boldsymbol{n}\,\boldsymbol{n}, \text{ on } \Sigma \end{aligned}\right.$$

SDF and DVF in FSI

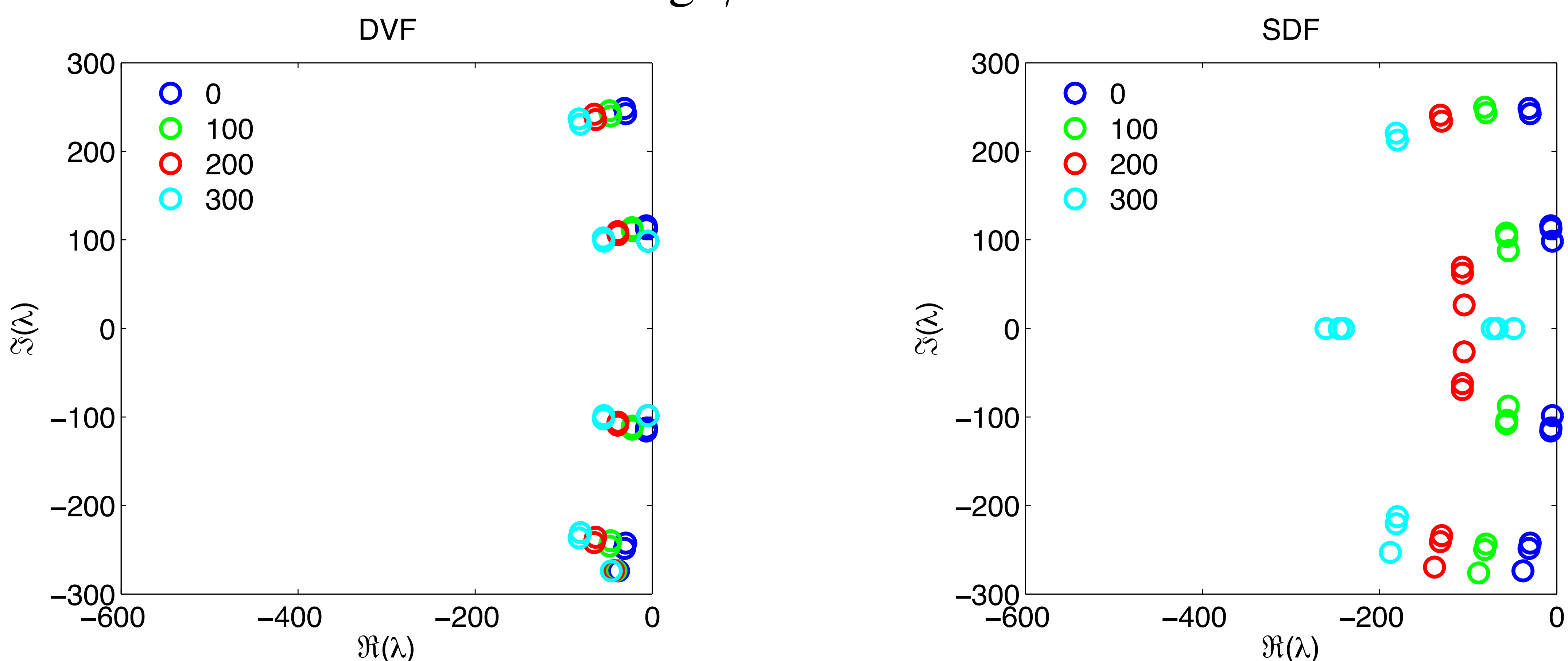
Analysis of a toy model

Added mass (FSI)

- Simplified FSI problem, with SDF or DVF

$$\begin{cases} (M_s + M_A)\dfrac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t} + K_s\hat{\boldsymbol{d}} = R + \gamma_v H_v^T M_\Gamma (Z_v - H_v(\hat{\boldsymbol{v}})) \\ K_\mu \dfrac{\mathrm{d}\hat{\boldsymbol{d}}}{\mathrm{d}t} = K_\mu \hat{\boldsymbol{v}} + \gamma_d H_d^T M_\Gamma (Z_d - H_d(\hat{\boldsymbol{d}})) \end{cases}$$

- Evolution of λ for increasing γ:



N.B.: viscoelastic structure

SDF and DVF in FSI

Analysis of a toy model

Sensitivity

- Let $(\lambda(\gamma), \Phi(\gamma))$ an eigenmode. Assuming full observation:
  - Velocity filter: $\displaystyle \frac{\partial \lambda}{\partial \gamma_v}\Big|_{\gamma_v=0} = -\frac{1-\Phi^T M_A \Phi}{2}$
  - Displacement filter: $\displaystyle \frac{\partial \lambda}{\partial \gamma_d}\Big|_{\gamma_d=0} = -\frac{1}{2}$

Remark: In blood flows $\Phi^T M_A \Phi$ is close to 1

C Bertoglio, D Chapelle, MA Fernández, JF Gerbeau, P Moireau, State observers of a vascular fluid–structure interaction model through measurements in the solid, *Comput. Methods Appl. Mech. Eng.* 256 (2013) 149–168

How to improve DVF in FSI ?

➡ Change norm used to measure the data misfit

"DVFam" filter for fluid structure problems

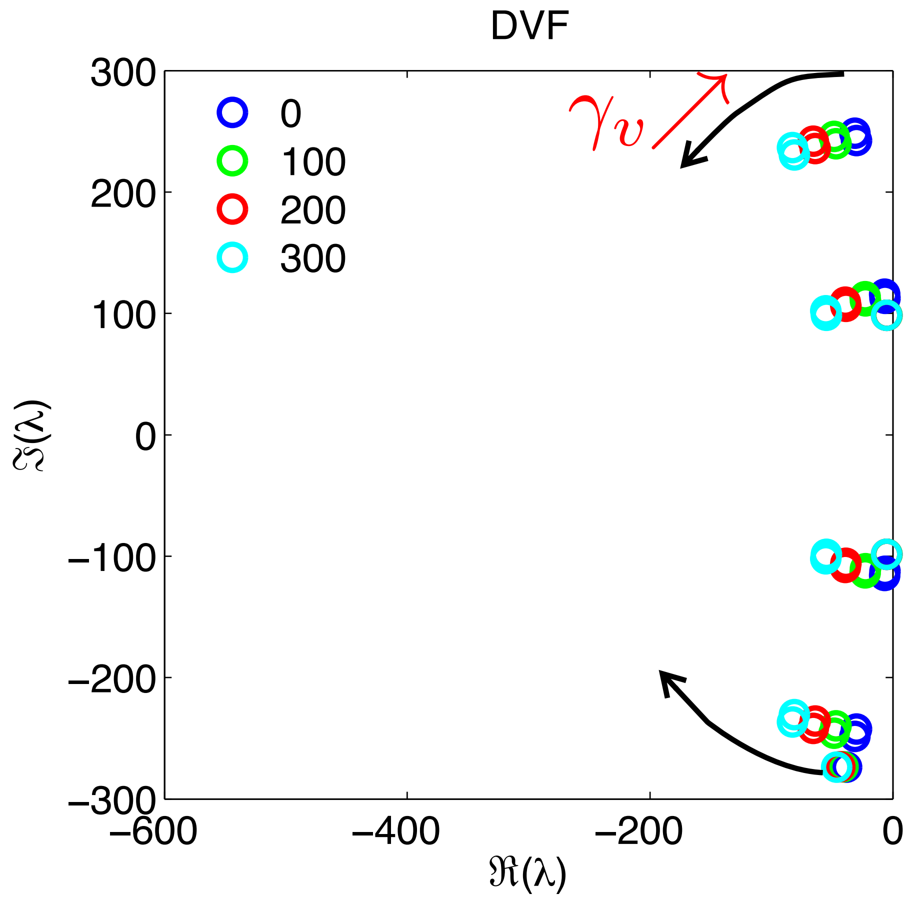
$$\begin{cases} M_s \dfrac{\mathrm{d}\hat{\boldsymbol{v}}}{\mathrm{d}t} + K_s \hat{\boldsymbol{d}} & = & R + \gamma_v H^T M_\Gamma (Z - H\hat{\boldsymbol{v}}) \\ \dfrac{\mathrm{d}\hat{\boldsymbol{d}}}{\mathrm{d}t} & = & \hat{\boldsymbol{v}} \end{cases}$$

with $M_\Gamma = M_{s,\Gamma} + M_A$, where M_A is the added-mass operator.

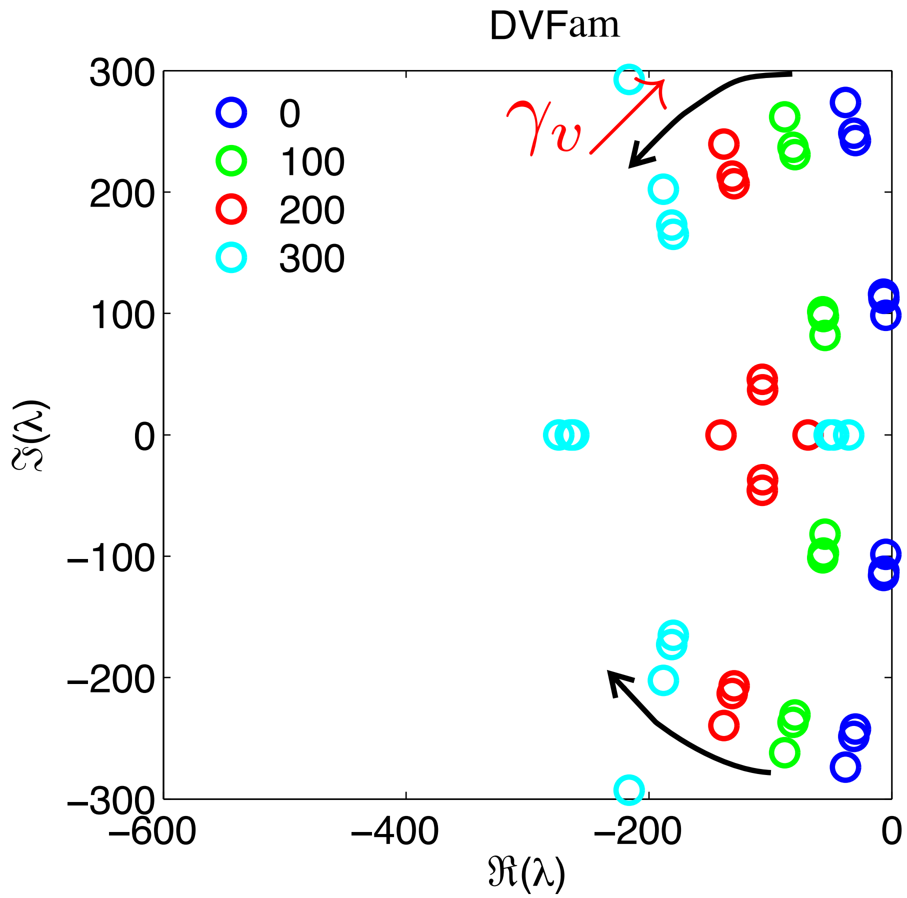
➡ Then we recover $\displaystyle \frac{\partial \lambda}{\partial \gamma_v}\Big|_{\gamma_v=0} = -\frac{1}{2}$

C Bertoglio, D Chapelle, MA Fernández, JF Gerbeau, P Moireau, State observers of a vascular fluid–structure interaction model through measurements in the solid, *Comput. Methods Appl. Mech. Eng.* 256 (2013) (2011) 149–168

Improved DVF in FSI

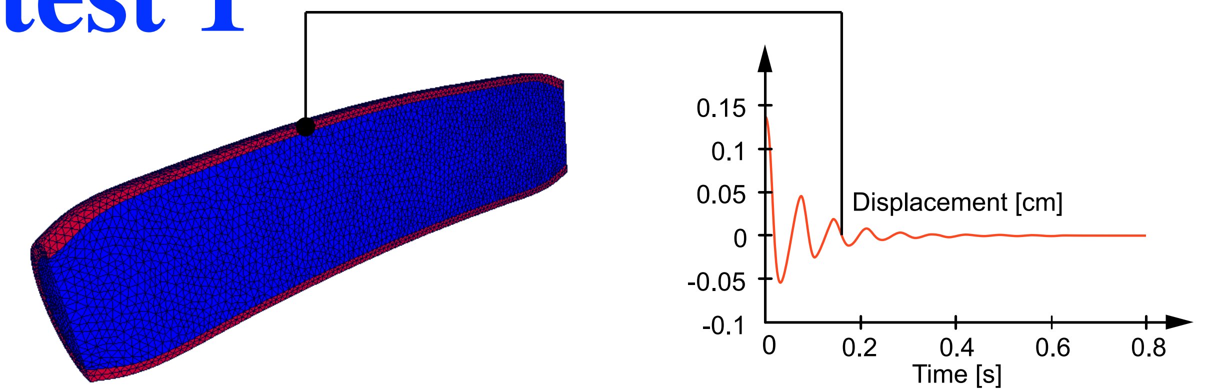


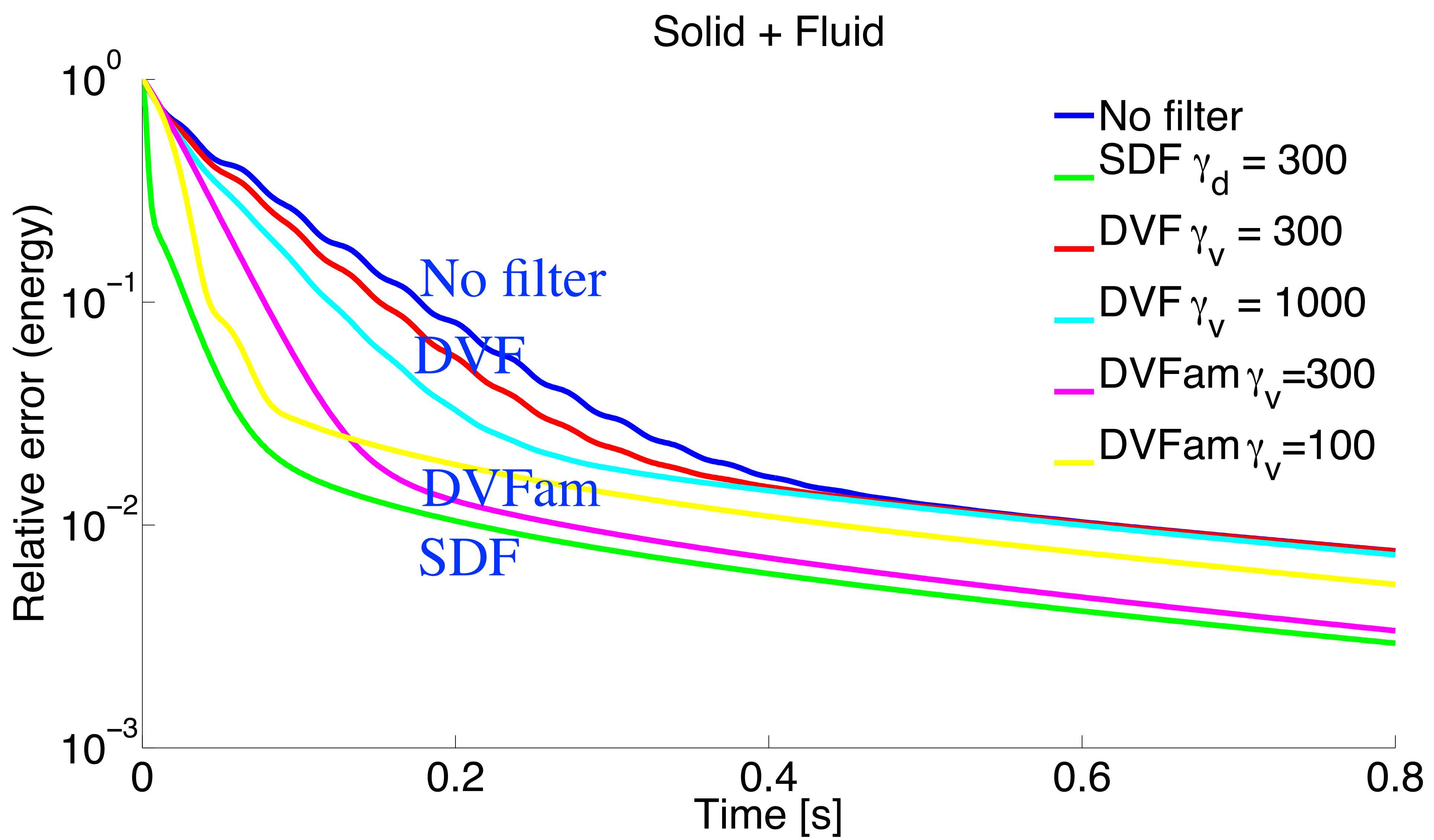
Standard DVF filter



“DVFam” filter
(norm including the added-mass)

Application to test 1





Pitfall: coupling conditions

- **Reminder SDF:** $\dfrac{\partial \boldsymbol{d}}{\partial t} = \boldsymbol{v} + \gamma_d \mathrm{Ext}(\boldsymbol{z} - \boldsymbol{d})$

  Thus $\dfrac{\partial \boldsymbol{d}}{\partial t} \neq \boldsymbol{v}$ in the solid

- At the fluid-structure interface, shall we use

$$\frac{\partial \boldsymbol{d}}{\partial t} = \boldsymbol{u} \quad \text{or} \quad \boldsymbol{v} = \boldsymbol{u} \quad ???$$

- The same analysis as before (nonlinear, spectral, sensitivity) shows that the right coupling condition is:

$$\boxed{\boldsymbol{v} = \boldsymbol{u}}$$

- Otherwise, it **kills** the efficiency of the SDF !

C Bertoglio, D Chapelle, MA Fernández, JF Gerbeau, P Moireau, State observers of a vascular fluid–structure interaction model through measurements in the solid, *Comput. Methods Appl. Mech. Eng.* 256 (2013) (2011) 149–168

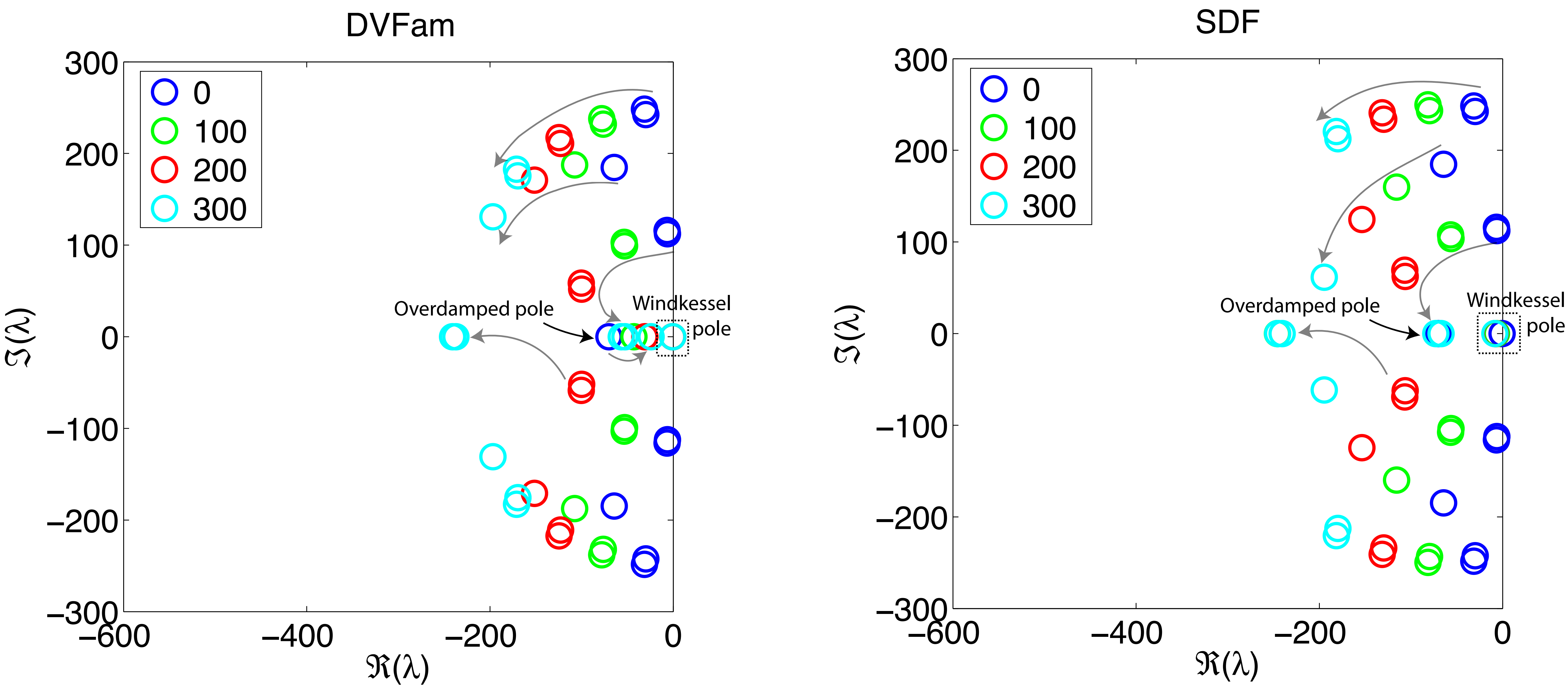
Effect of boundary conditions

Toy FSI model with a Windkessel Boundary Condition

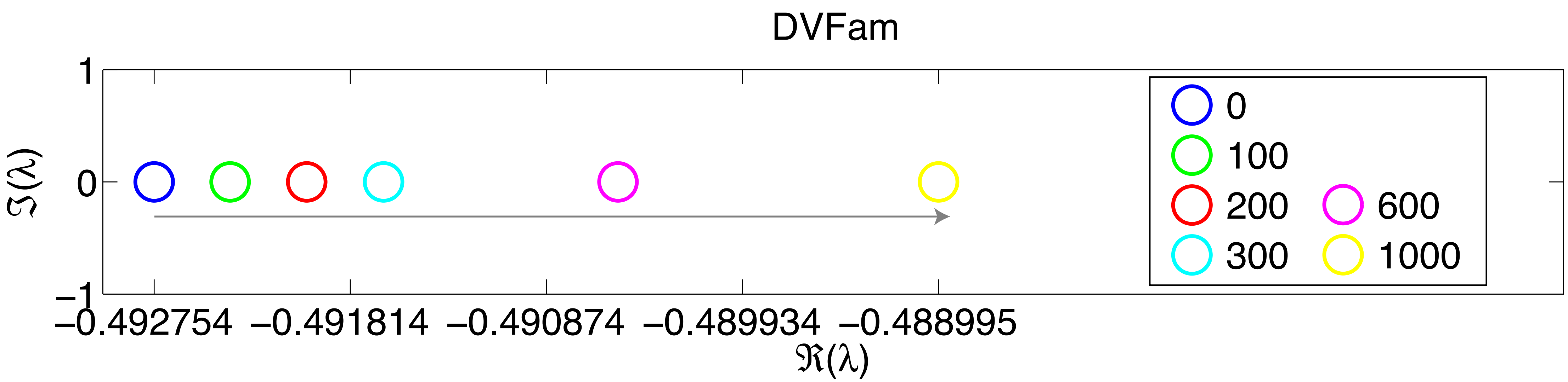
$$\begin{bmatrix} K_s & 0 & 0 \\ 0 & M_s + M_A & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} \dot{Y}_s \\ \dot{U}_s \\ \dot{\pi} \end{bmatrix} = \begin{bmatrix} 0 & K_s & 0 \\ -K_s & -C_s - R_p S \cdot S^\intercal & S \\ 0 & -S^\intercal & -\frac{1}{R_d} \end{bmatrix} \begin{bmatrix} Y_s \\ U_s \\ \pi \end{bmatrix}$$

$$C \frac{d\pi}{dt} + \frac{\pi}{R_d} = Q$$

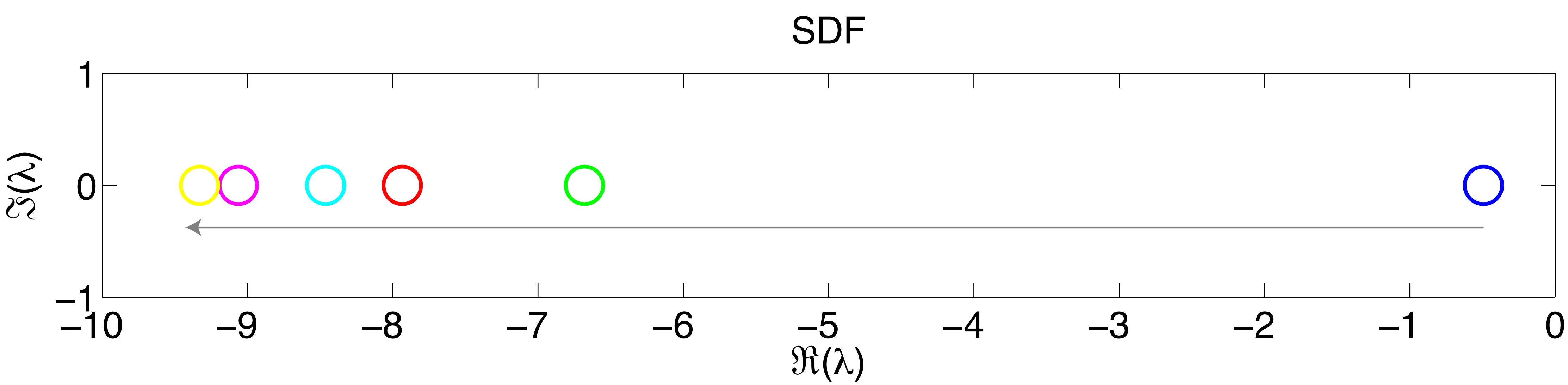
- Analytical sensitivity analysis still possible
- It confirms the observations of the numerical spectral analysis



DVFam
SDF
0
100
200
300
Overdamped pole
Windkessel pole
ℜ(λ)
ℑ(λ)



➡ DVFam (slightly) **destabilizes** the Windkessel pole



➡ SDF reasonably good at improving the Windkessel pole

Effect of fluid dissipation

In the toy FSI model, replace the potential fluid by Stokes:

$$\left\{ \begin{aligned} \rho_\mathrm{f}\partial_t \boldsymbol{u}_\mathrm{f} - \boldsymbol{\nabla}\cdot\boldsymbol{\sigma}_\mathrm{f}(\boldsymbol{u}_\mathrm{f}, p) &= \boldsymbol{0}, \quad \text{in } \Omega_0^\mathrm{f} \\ \boldsymbol{\nabla}\cdot\boldsymbol{u}_\mathrm{f} &= 0, \quad \text{in } \Omega_0^\mathrm{f} \end{aligned} \right.$$

- First 100 smallest eigenvalues in module :
  - Real
  - Almost the same with Stokes or with Stokes + Structure
  - Almost unaffected by any filter

Summary for Luenberger observer in FSI

	Displacement measurements		Velocity measurements	
	SDF with $u=v$	SDF with $u=\dot{d}$	DVF	DVFam
Added-mass	☺	☹	☹	☺
Dissipative BC	☺	☹	☹	☹
Fluid viscosity	☹	☹	☹	☹

Possible remedies

- Add pressure measurements and consider Windkessel observer like:

$$R_{\mathrm{d}} C \dot{\hat{\pi}} + \hat{\pi} = R_{\mathrm{d}} \hat{Q} + \gamma_\pi (z_\pi - \hat{\pi}),$$

- Add fluid measurements and devise a filter for the fluid

Outline

- Reminders on Data Assimilation
- Luenberger filters and FSI
- Applications to blood flow

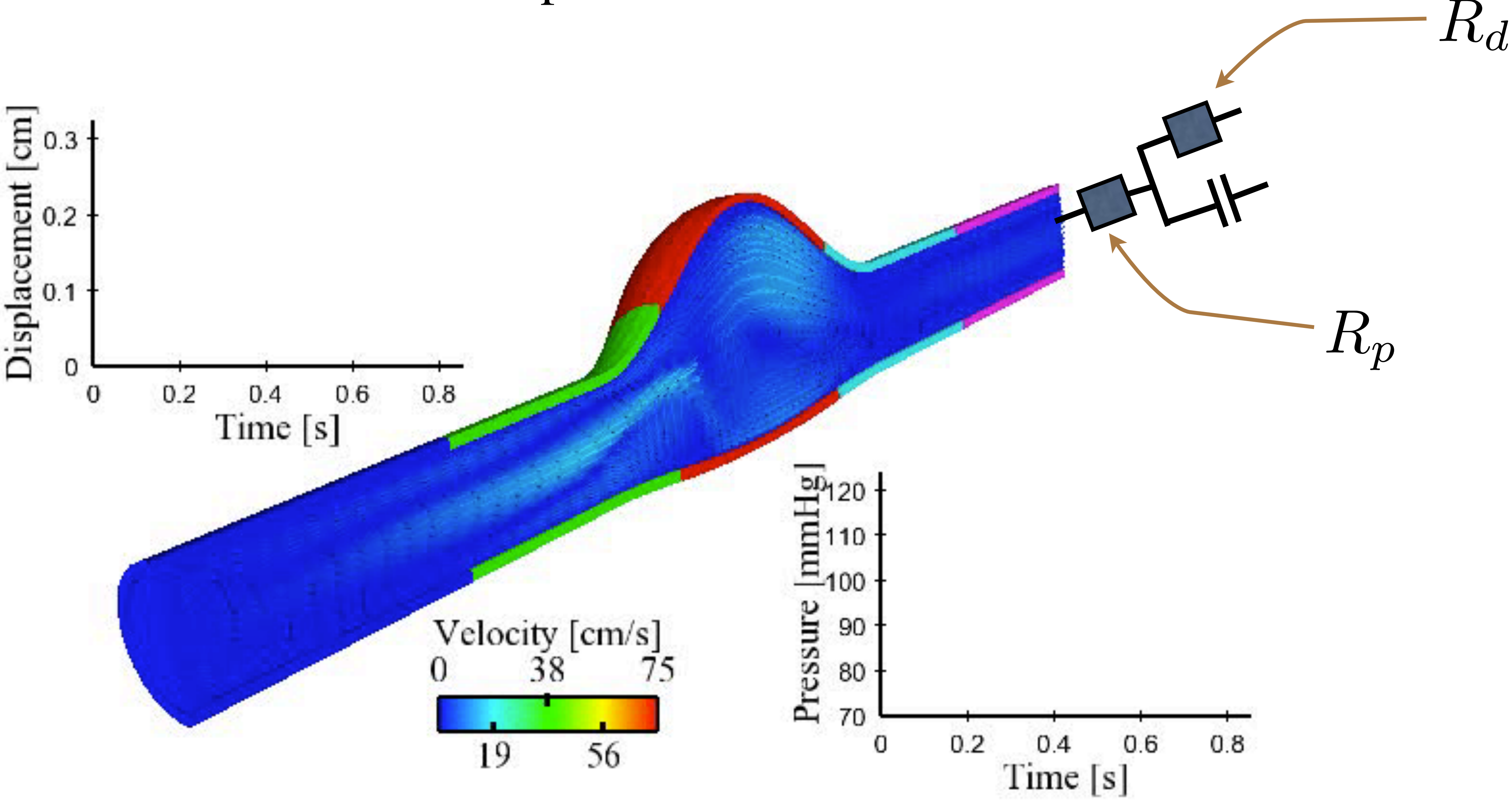
C Bertoglio, P Moireau, JF Gerbeau. Sequential parameter estimation for fluid–structure problems: Application to hemodynamics. *International Journal for Numerical Methods in Biomedical Engineering* (2012), 28(4), 434-455.

P Moireau, C Bertoglio, N Xiao, CA Figueroa, CA Taylor, D Chapelle, JF Gerbeau. Sequential identification of boundary support parameters in a fluid-structure vascular model using patient image data. *Biomechanics and modeling in mechanobiology (2013)*, 12(3), 475-496.

Parameter estimation

Stiffness estimation

- Parameter estimation: Young modulus $E_i = 2^{\theta_i}$ in 5 regions
- Observations: wall displacement on fluid-solid interface

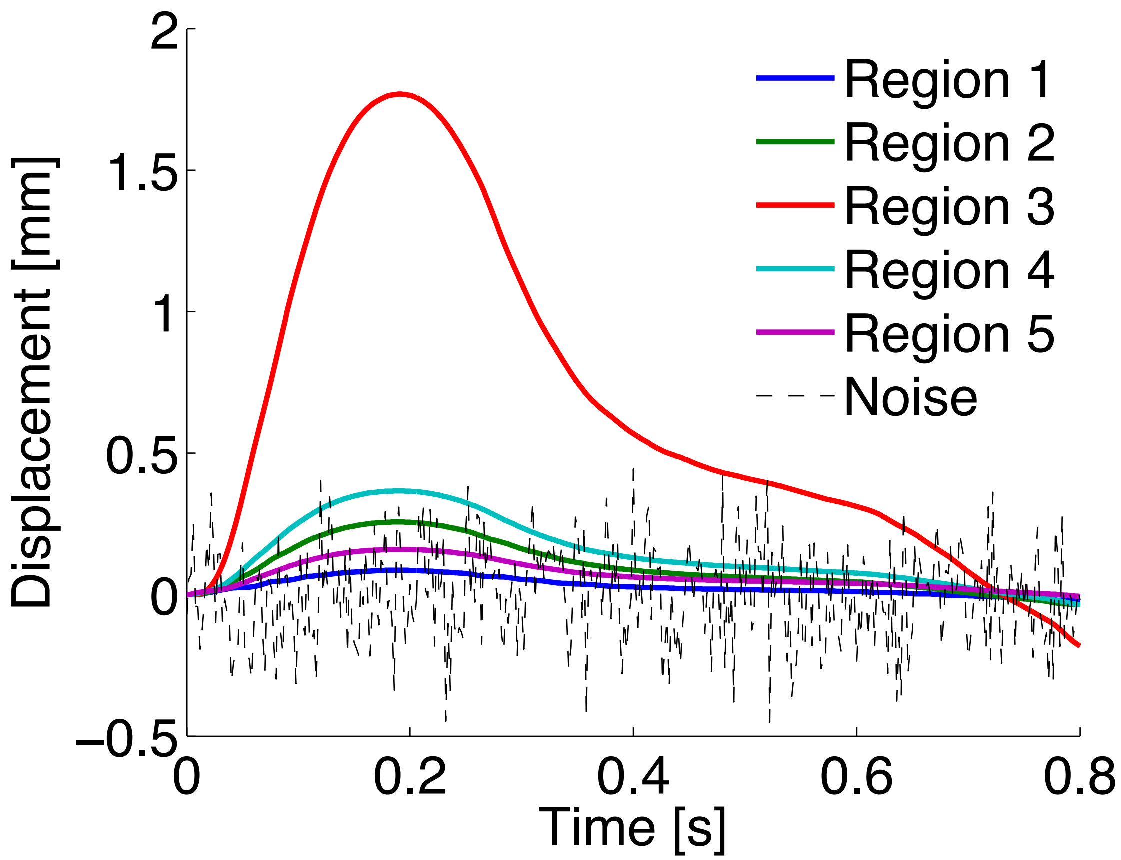


C Bertoglio, P Moireau, JF Gerbeau. "Sequential parameter estimation for fluid–structure problems: Application to hemodynamics." *International Journal for Numerical Methods in Biomedical Engineering* 28.4 (2012): 434-455.

Synthetic data

Stiffness estimation

- Synthetic data
- Gaussian noise (10%) and time resampling $\delta t_{obs} = 10\delta t$

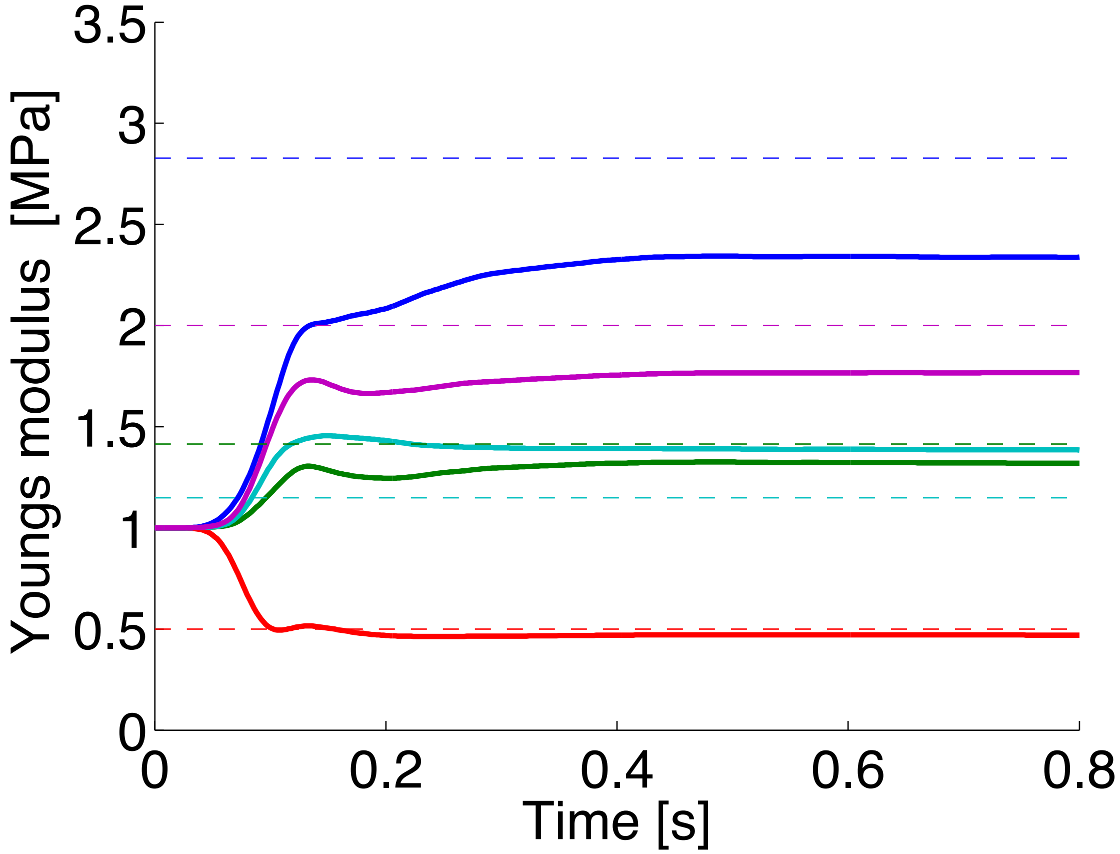


Region	SNR
1	0.43
2	1.25
3	8.8
4	1.8
5	0.75

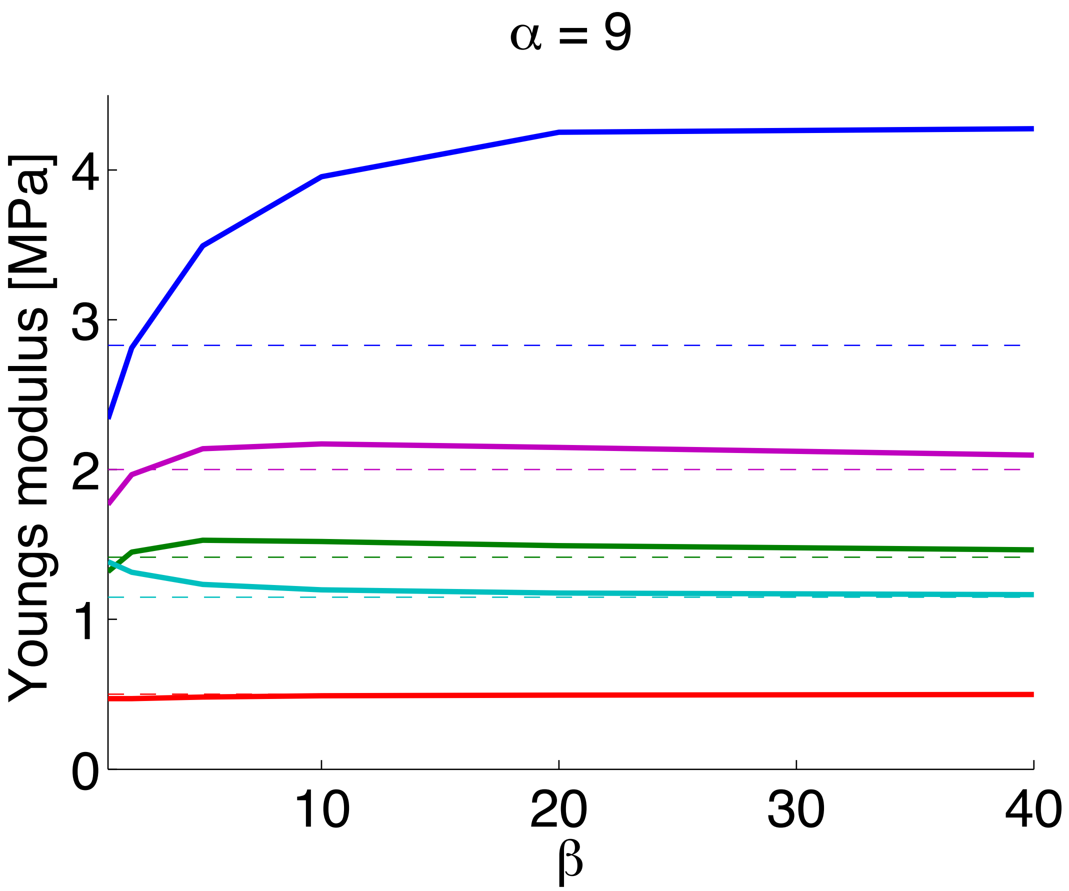
- Two tuning coefficients:
  - A priori covariance on parameters: $\alpha\mathbb{I}$
  - Gain on the observations: β

Synthetic data

Stiffness estimation



$$\alpha = 9, \beta = 1$$

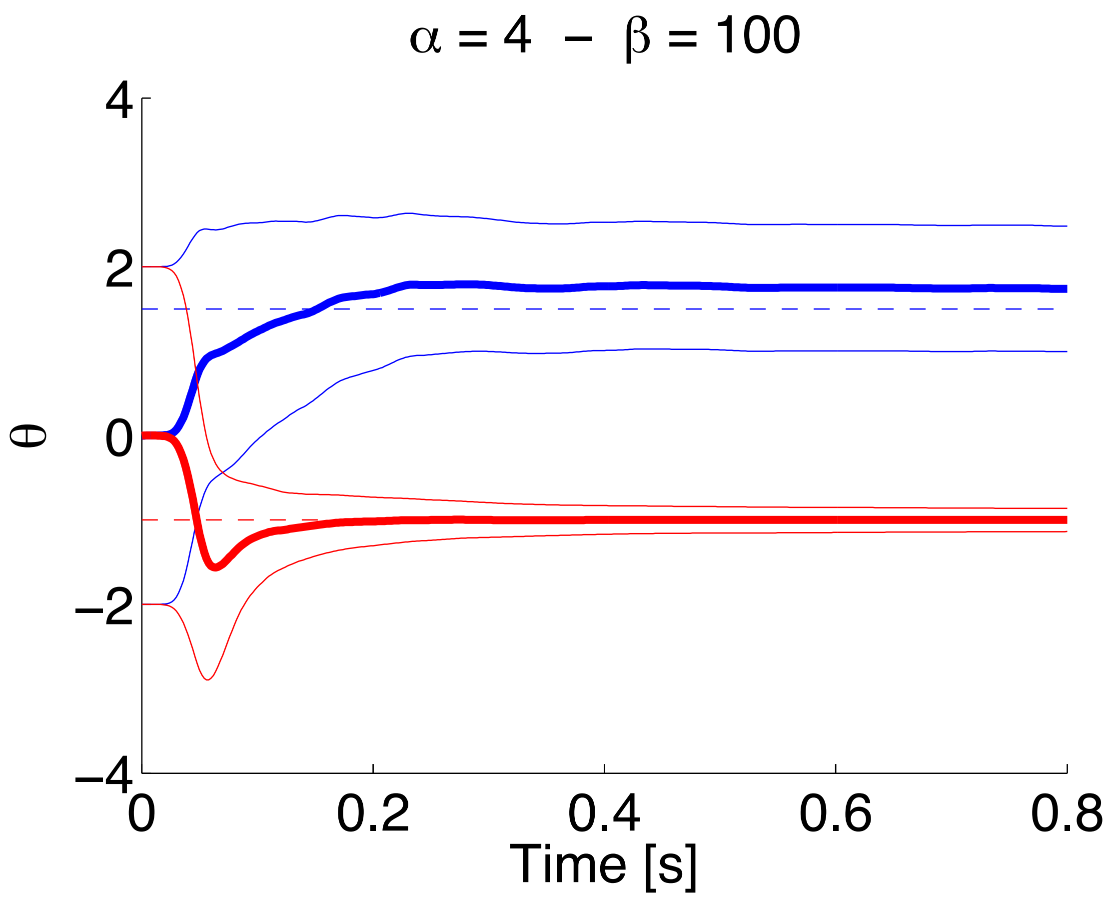


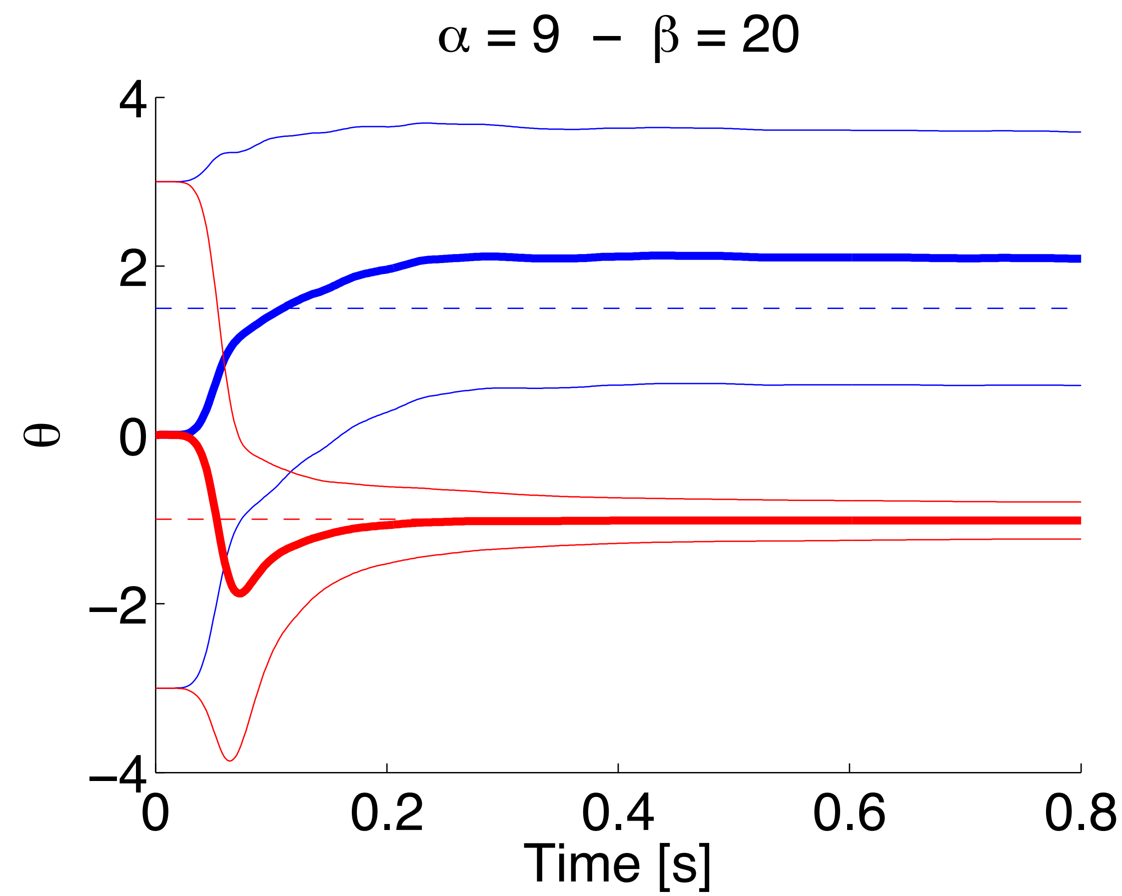
Simulation : C.Bertoglio

Synthetic data

Stiffness estimation

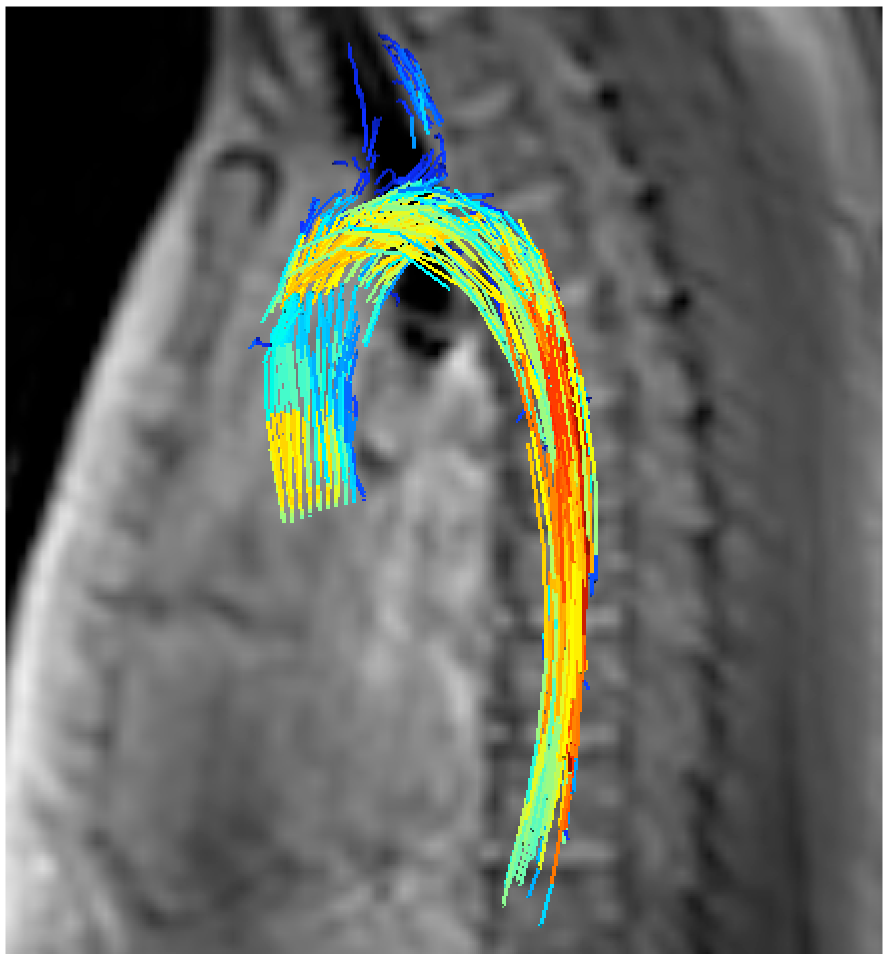
- The filtering algorithm provides an *a posteriori covariance* P_n^θ
- We can plot $\hat{\theta}_n \pm \sqrt{\mathrm{diag}(P_n^\theta)}$

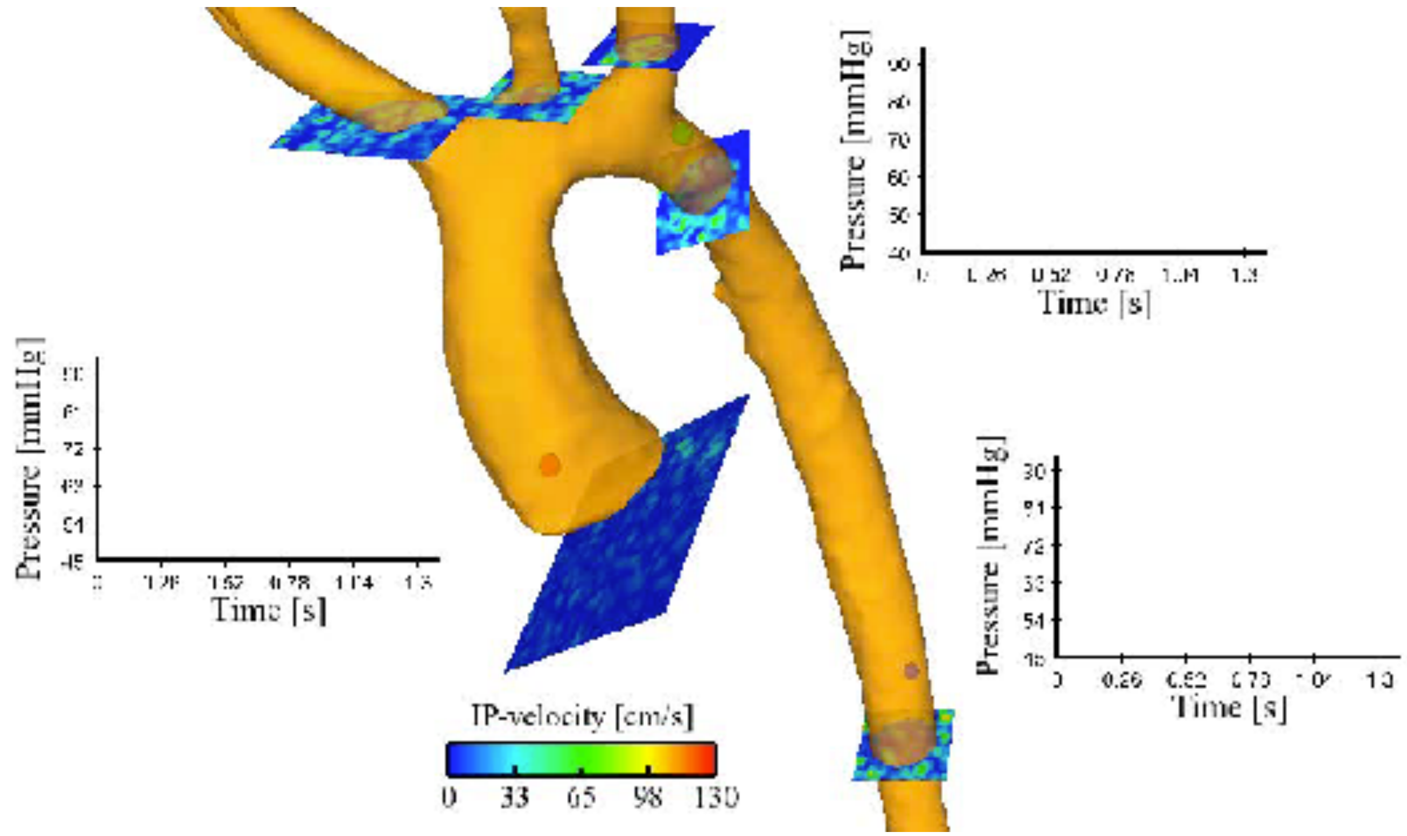




Simulation : C.Bertoglio

FSI assimilation with real data





Data: Gaddum, Rutten, Beerbaum (KCL).
Segmentation and processing: Hose, Barber (Sheffield)

FSI assimilation with real data

1st difficulty: the artery wall is pre-stressed

- solve an inverse problem to obtain the reference configuration

2d difficulty: the *displacement* is usually not available

- The pseudo-displacement of the registered surfaces cannot be used (*"image-driven FSI"*):
  - Normal displacement : OK
  - **Tangential displacement : False !**
- The discrepancy between the state and the measurements is evaluated through the signed distance:

$$\mathrm{dist}_{\mathcal{S}_k} : \left| \begin{array}{r} (L^2(\Sigma))^3 \mapsto (L^2(\Sigma))^3 \\ \mathrm{x}(\boldsymbol{\xi}) \to \mathrm{dist}_{\mathcal{S}_k}(\mathrm{x}(\boldsymbol{\xi}))\boldsymbol{n}_{\mathcal{S}_k}(\mathrm{x}(\boldsymbol{\xi})) \end{array} \right.$$

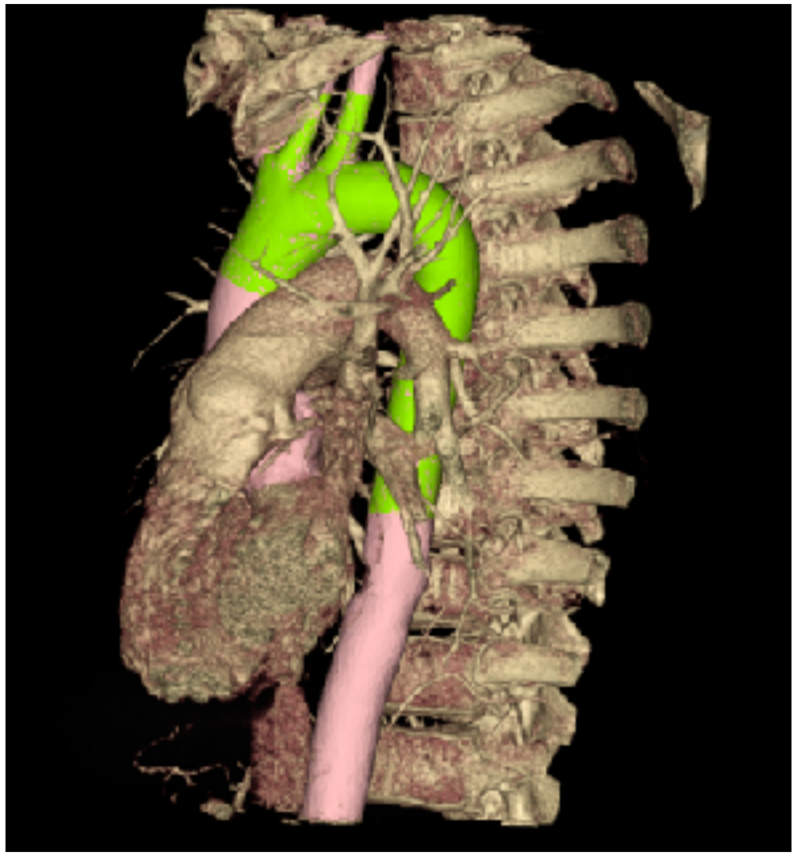
3rd difficulty: external tissues

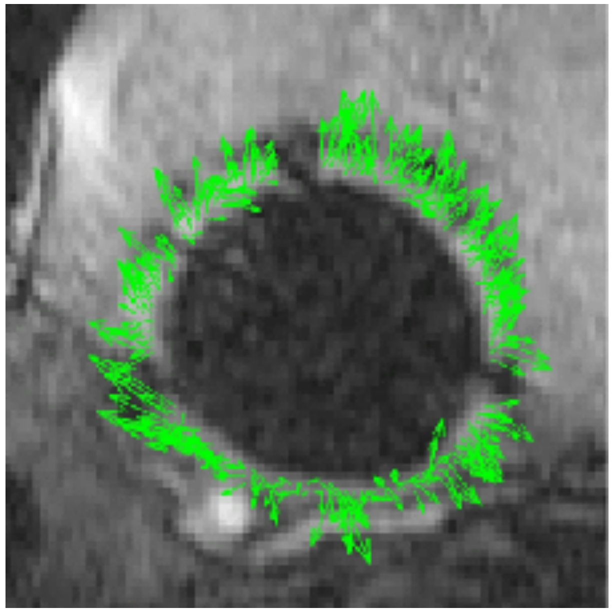
- The artery *is not* isolated !

External tissues model

External tissue support

- Typical b.c. on the external part of the vessel : $\boldsymbol{\sigma}_s \boldsymbol{n} = p_0 \boldsymbol{n}$





Abdominal aorta

Courtesy of C. Taylor

- A simple and affordable way to model the external tissues :

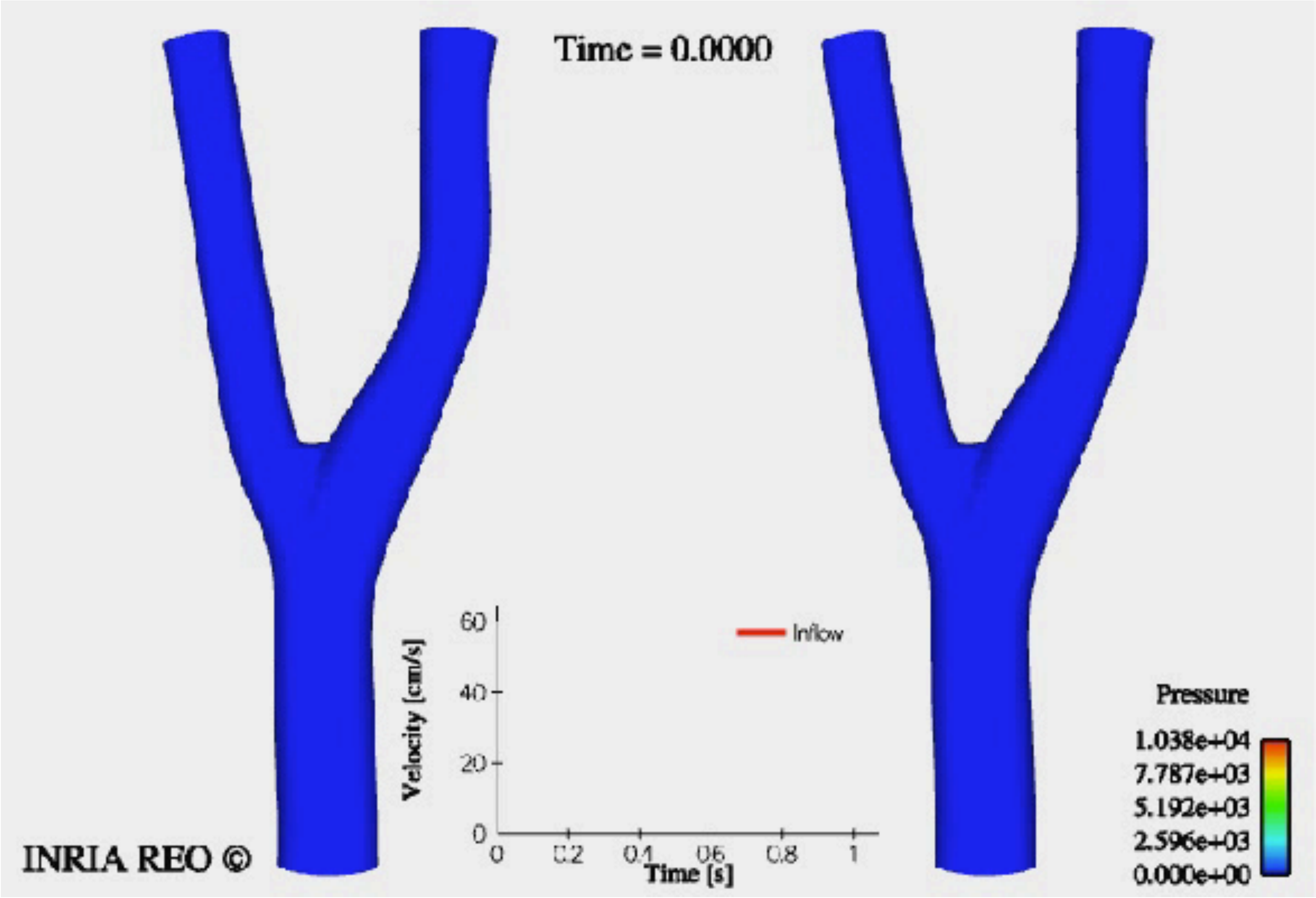
$$\boldsymbol{\sigma}_s \boldsymbol{n} = -k_s \boldsymbol{d} - c_s \frac{\partial \boldsymbol{d}}{\partial t}$$

P Moireau, N Xiao, M Astorino, CA Figueroa, D Chapelle, CA Taylor, JF Gerbeau, (2012). "External tissue support and fluid–structure simulation in blood flows". *Biomechanics and modeling in mechanobiology,* 11(1-2), 1-18.

External tissue support

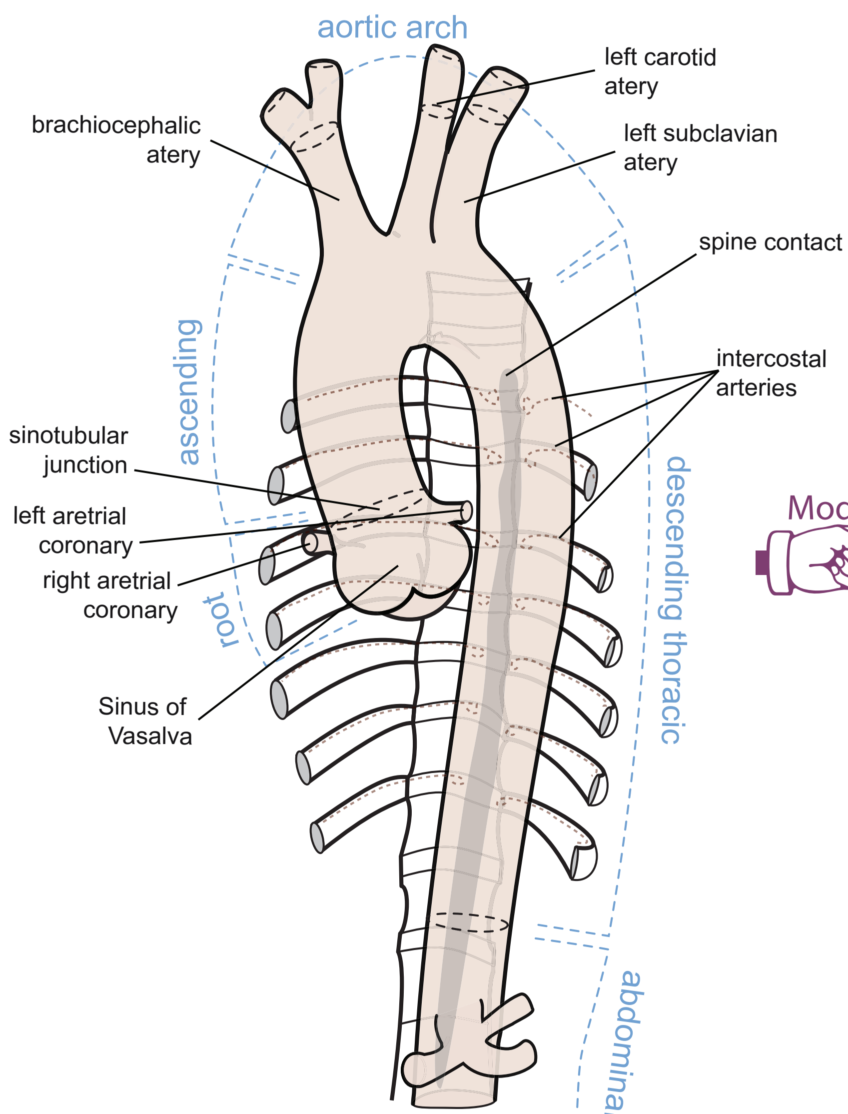
$$\boldsymbol{\sigma}_s \boldsymbol{n} = -k_s \boldsymbol{d} - c_s \frac{\partial \boldsymbol{d}}{\partial t}$$

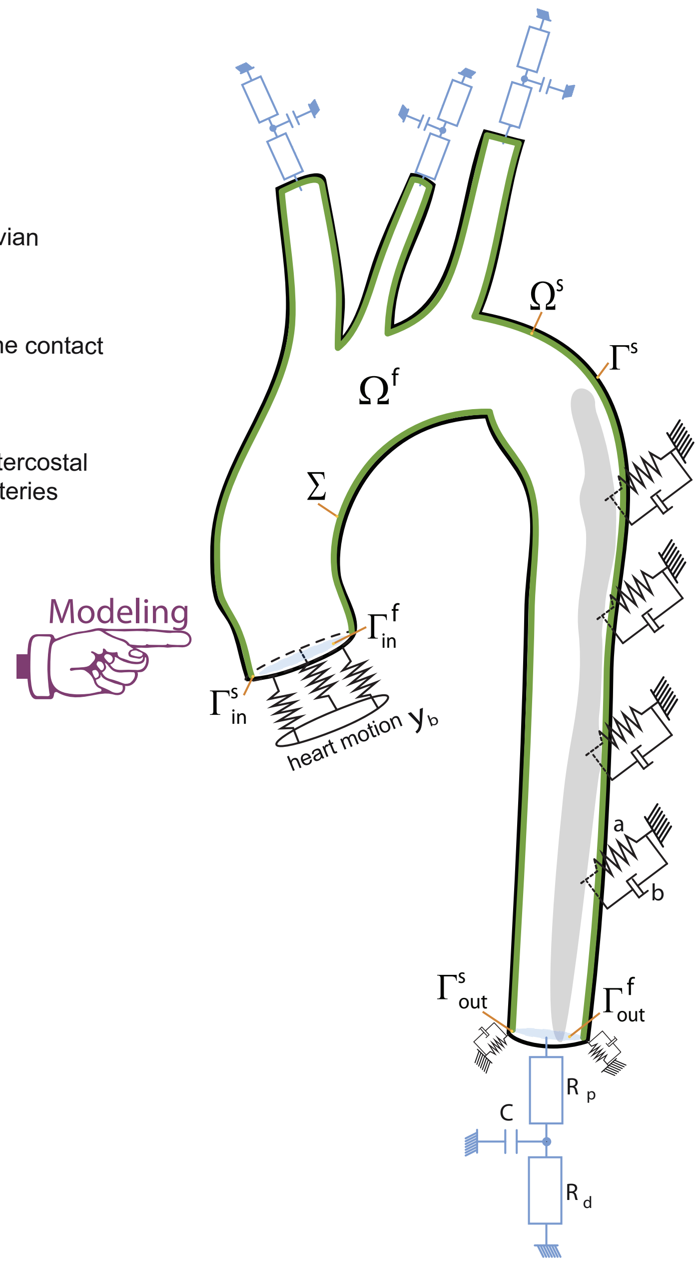
$$\boldsymbol{\sigma}_s \boldsymbol{n} = -p_{ext} \boldsymbol{n}$$

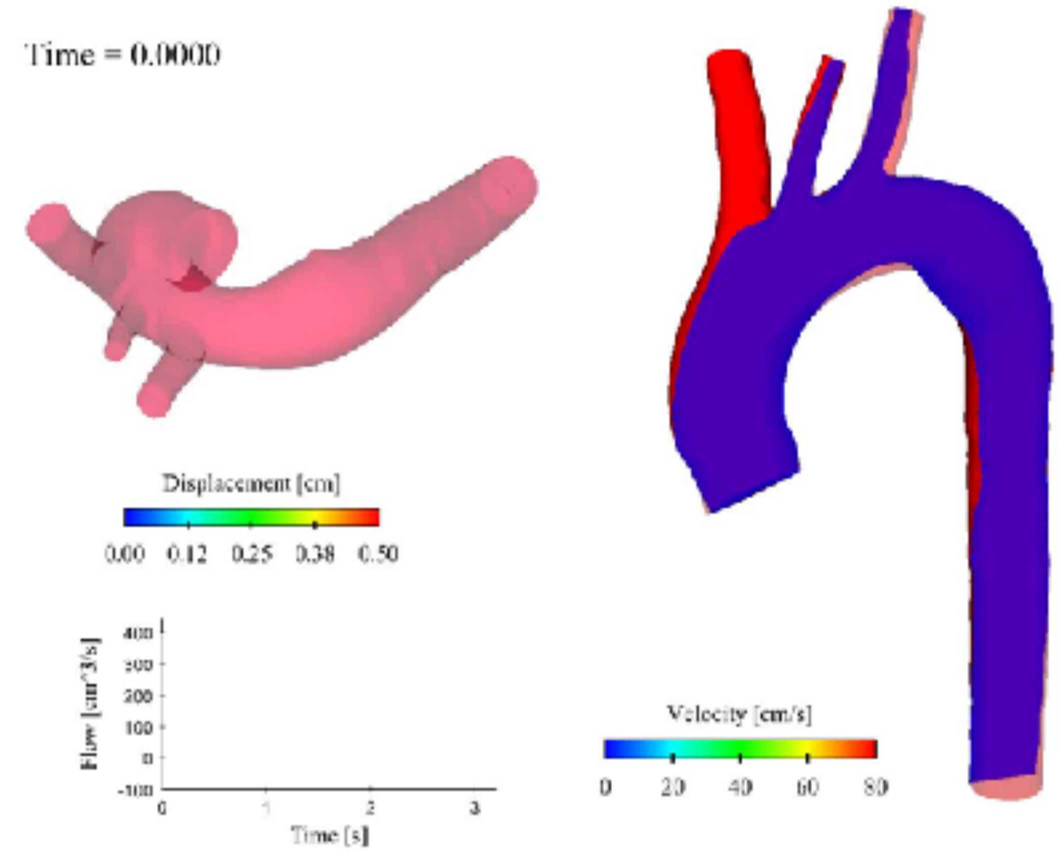


Simulation: M. Astorino

Estimation of the external tissues model





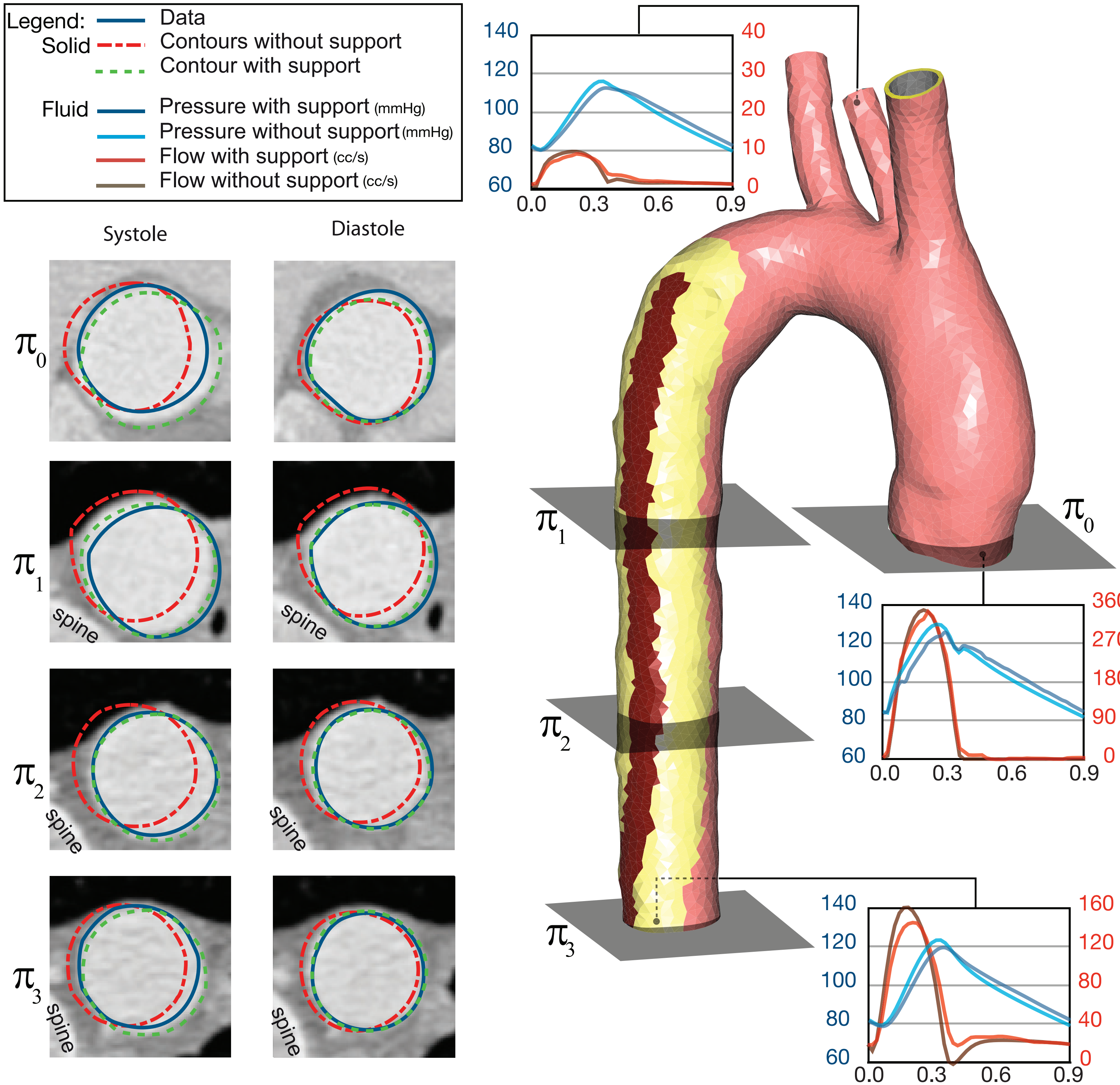


$$\boldsymbol{\sigma}_s \boldsymbol{n} = -k_s \boldsymbol{d} - c_s \frac{\partial \boldsymbol{d}}{\partial t}$$

with heterogeneous coefficients

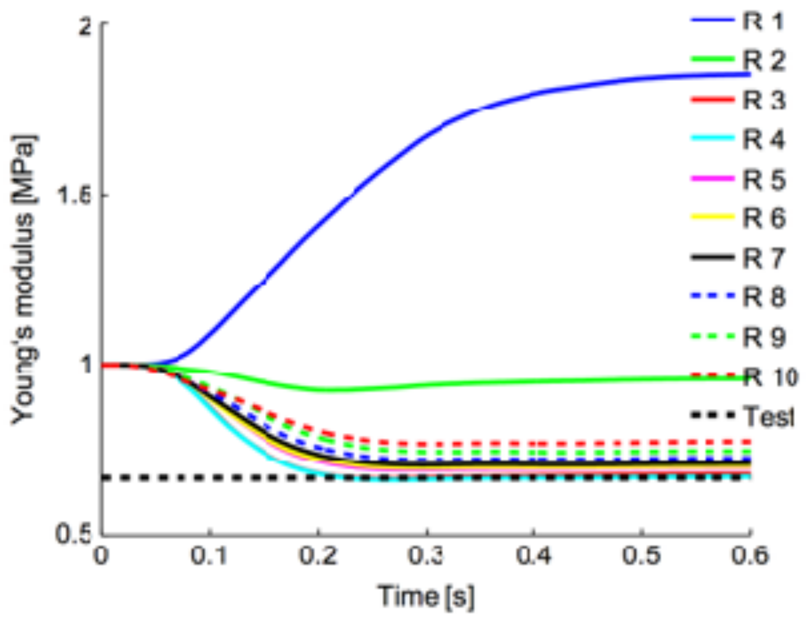
P Moireau, C Bertoglio, N Xiao, CA Figueroa, CA Taylor, D Chapelle, JF Gerbeau (2013). "Sequential identification of boundary support parameters in a fluid-structure vascular model using patient image data". Biomechanics and modeling in mechanobiology, 12(3), 475-496.

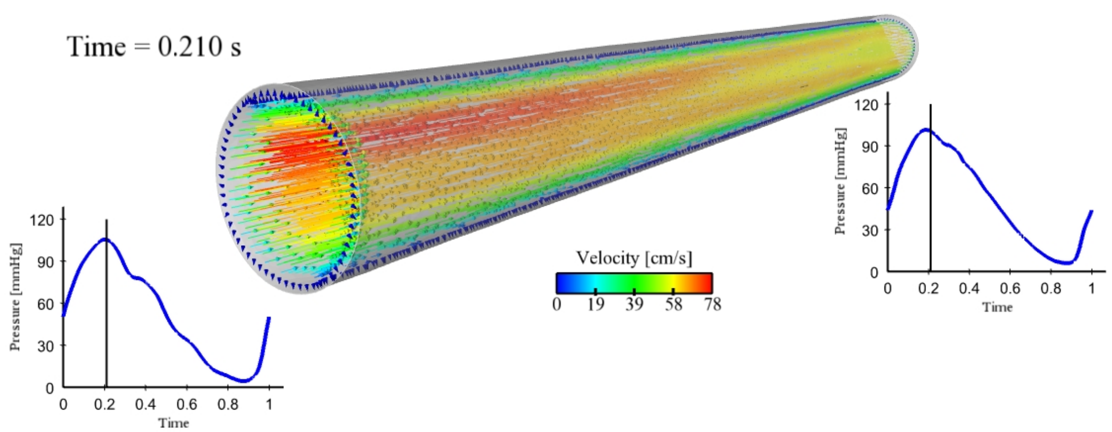
External tissues model

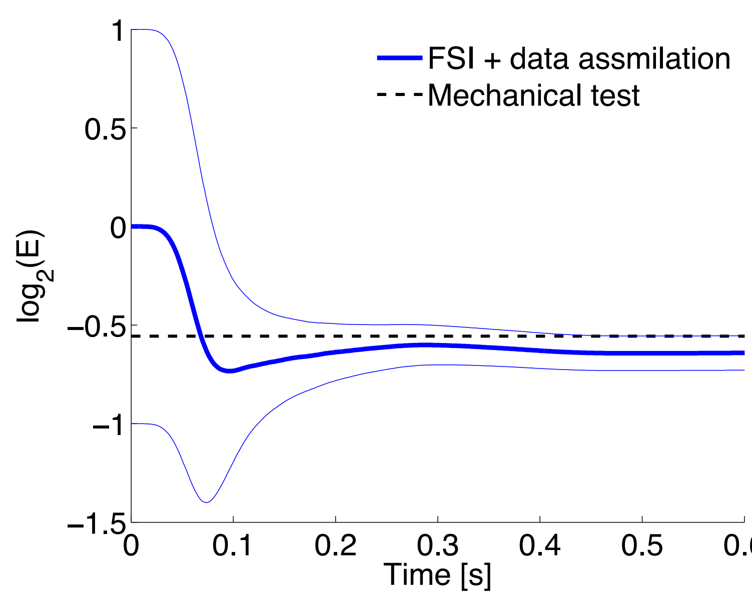


Estimation of the stiffness: experiment

- Silicon rubber aortic phantom
- Measurements: MRI (3T)
- Subdivision in 10 regions
- An elastic strap makes region 1 and 2 stiffer



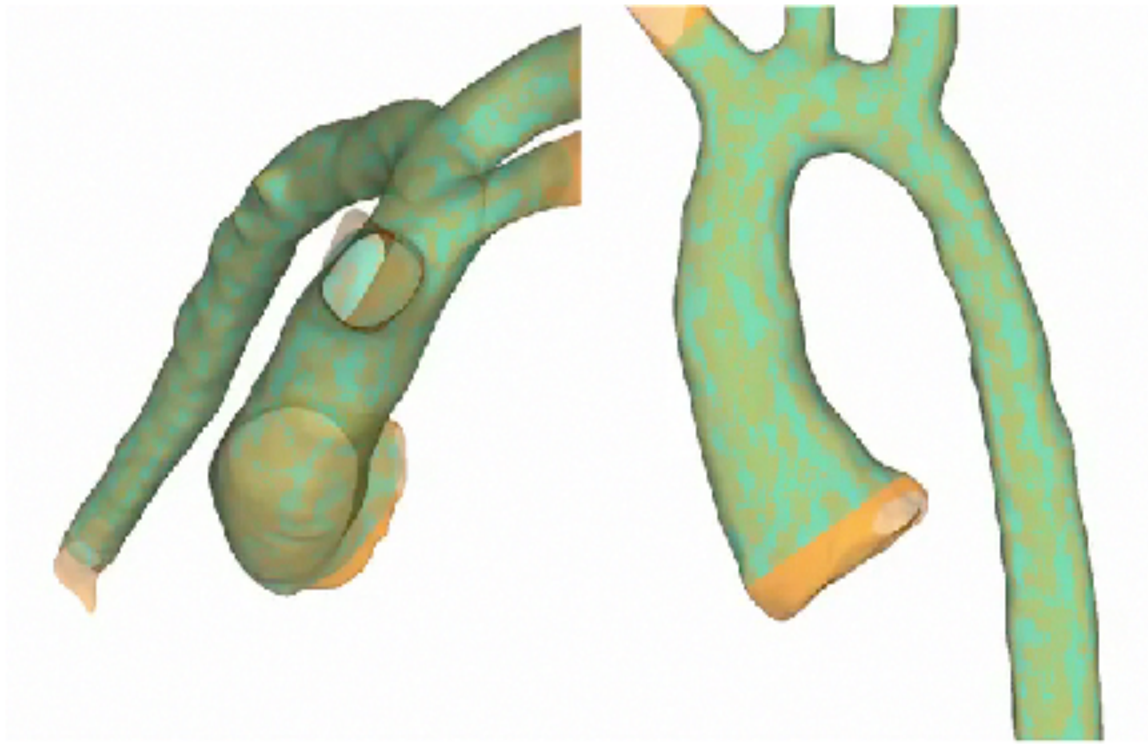




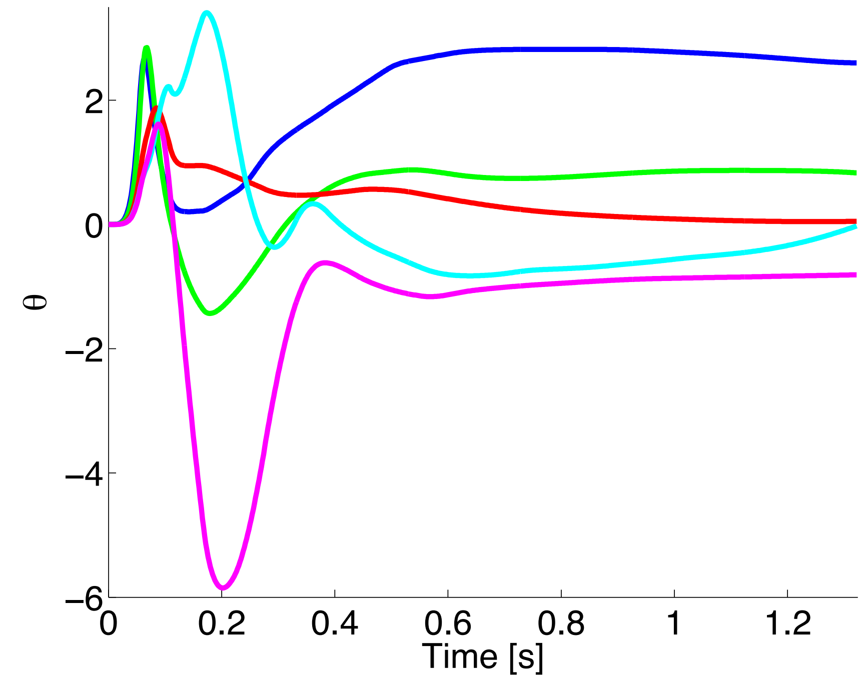
C Bertoglio, D Barber, N Gaddum, I Valverde, M Rutten, P Beerbaum, P Moireau, R Hose, JF Gerbeau. "Identification of artery wall stiffness: In vitro validation and in vivo results of a data assimilation procedure applied to a 3D fluid–structure interaction model". *Journal of biomechanics (2014)*, 47(5), pp.1027-1034.

Estimation of the stiffness: clinical data

- MRI data:



- Stiffness estimation

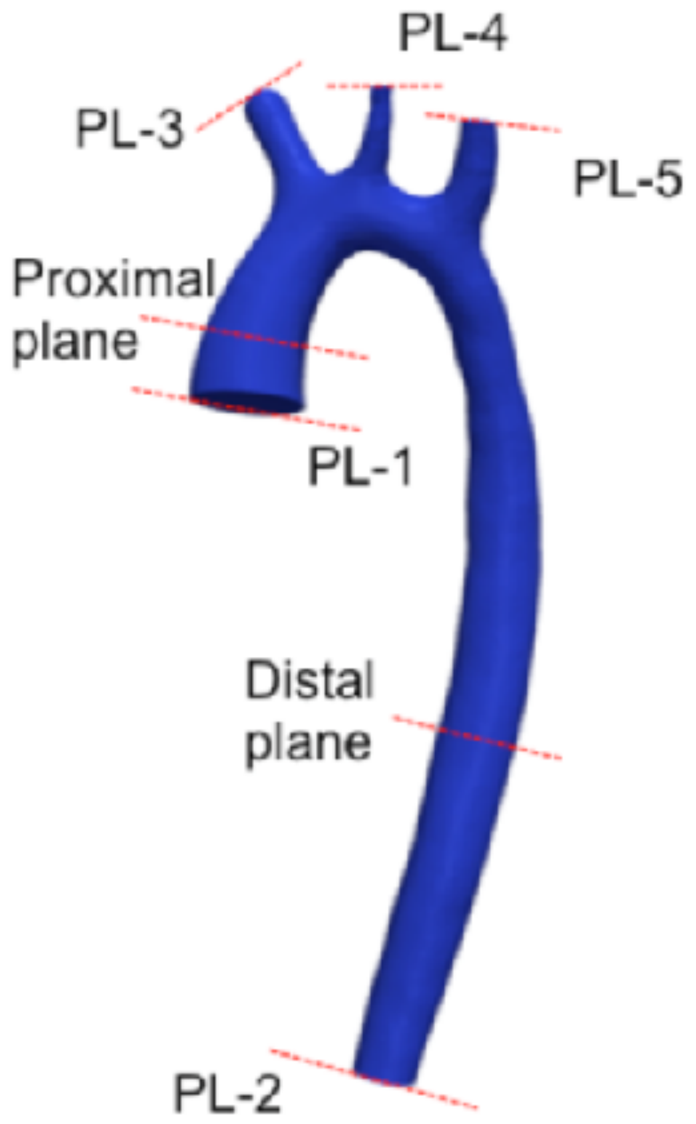


C Bertoglio, D Barber, N Gaddum, I Valverde, M Rutten, P Beerbaum, P Moireau, R Hose, JF Gerbeau. "Identification of artery wall stiffness: In vitro validation and in vivo results of a data assimilation procedure applied to a 3D fluid–structure interaction model". *Journal of biomechanics (2014)*, 47(5), pp.1027-1034.

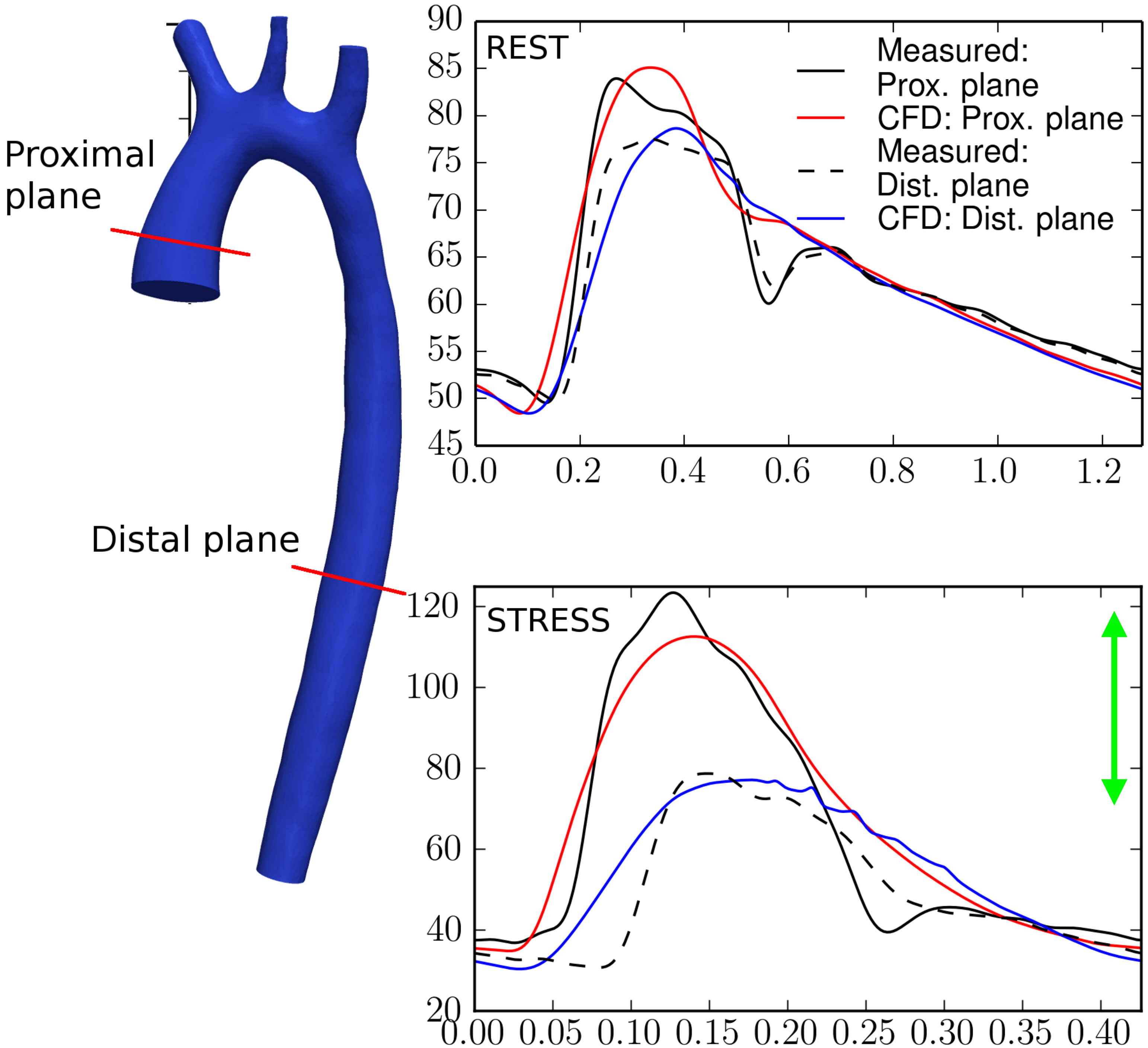
A validation example

- Stacom/Miccai 2013 Challenge:
  - Predict the pressures the pressure drop due to a coarctation while matching given features of experimentally measured data.

- Measured data in the patient (rest and stress cases)
  - Flow waveform in ascending aorta (PL-1)
  - Flow waveform in descending aorta (PL-1)
  - Mean flow rates in the supra aortic vessels:
    innominate artery (PL-3),
    left carotid artery (PL-4),
    and left subclavian artery (PL-5)
  - Pressure waveform in the
    ascending aorta (Proximal plane).



A validation example



- Method

- 0D abstraction of 3D model
- 14 parameters estimated with UKF (4 RCR + time shift + one capacitance to mimic FSI)
- Observation: pressure in the ascending aorta, flow at the outlets (flow split)

S Pant, B Fabrèges, JF Gerbeau, I Vignon-Clementel, A methodological paradigm for patient-specific multi-scale CFD simulations. *Int. J. Num. Meth. Biomed. Engng*, 30(12) :1614–1648, 2014.

Concluding remarks

- Data assimilation allows us to
  - reduce the uncertainties of the models
  - enrich the data with physical-based models
- Two families of methods
  - variational: based on gradient, and adjoint equations
  - sequential: modified dynamics accounting for the measurements
- Benefits of sequential algorithms
  - simplicity of implementation, efficiency, versatility
  - but intractable for large systems…
- Possible strategies for large systems, as in FSI:
  - Luenberger observers (nudging) for the state
  - nonlinear Kalman filter for the parameters
- Research topics:
  - adapt the assimilation algorithms to the new kinds of medical data
  - leverage the information of multi-physics systems