Geometry description and mesh construction from medical imaging

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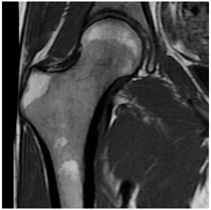


Cemracs 2018 - Marseille August 23, 2018

Motivation

- Simulation of patient-specific biophysical phenomena.
- Requiring efficient algorithms for defining and meshing a patient specific domain in which the simulation will take place.

Our project aims at providing a geometry description and a good quality mesh of the patient specific domain.

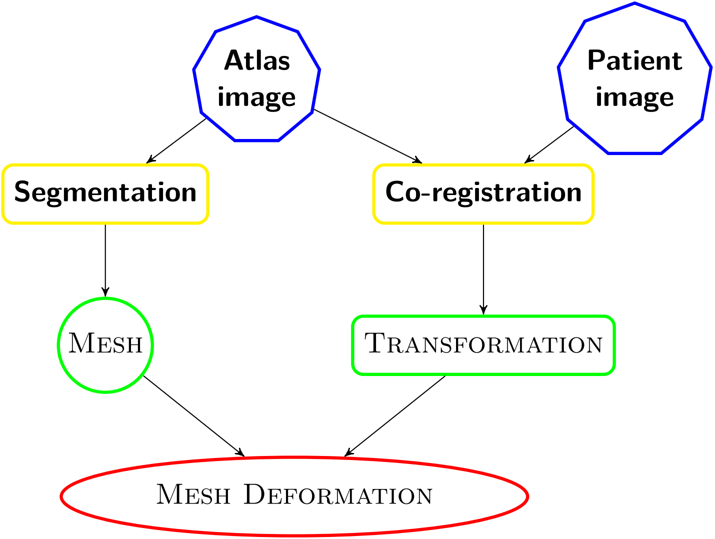
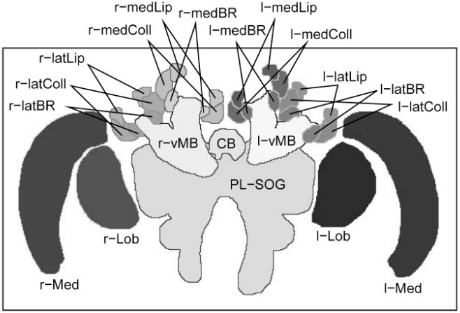

Atlas image
Patient image
Segmentation
Co-registration
Mesh
Transformation
Mesh Deformation

Image segmentation

From ***Quo vadis*, Atlas-based segmentation ?** of T. Rohlfing, R. Brandt, R. Menzel, D.B. Russakoff, and C.R. Maurer, Jr. 2007.



Segment an image = tag each pixel/voxel with a semantic label.

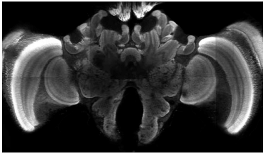
Atlas image. Atlas mesh

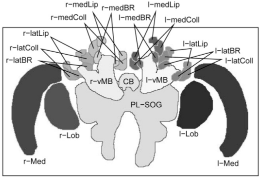
- Image $=$ map from a domain $\Omega \subset \mathbb{R}^N$ to $\mathbb{R}^+$, i.e. $A : \Omega \rightarrow \mathbb{R}^+$,
- label set $L = \{e_1, e_2, ..., e_K\}$,
- assigning label map $\boldsymbol{\mathcal{A}} : \Omega \rightarrow L$.

A is segmented by superposing $\mathcal{A}$, then $(A, \boldsymbol{\mathcal{A}})$ is called **atlas image**.

$\rightsquigarrow$ An atlas image is an image consisting of geometry, topology descriptions of specific interested domains.

$\rightsquigarrow$ Meshes are constructed on $(A, \boldsymbol{\mathcal{A}})$ and naturally inherit the geometry description from $(A, \boldsymbol{\mathcal{A}})$.





Mesh construction. Geometry description

(1) Create an atlas.
(2) Define contours of the atlas.
(3) Extract sample points on the contours without compromising the geometry of the atlas.
(4) Generate triangle mesh by Delaunay's algorithm.

L={Calcaneus, Cuboid, Navicular, Talus}

Image co-registration - Transformation

- Grey scale images $\longleftrightarrow$ real valued functions. We denote by **Im** the space of grey images.
- Given grey images $A : \Omega_A \to \mathbb{R}^+$, $P : \Omega_P \to \mathbb{R}^+$.

Co-registration of A and P reduces to finding a mapping $\theta : \Omega_A \to \Omega_P$ such that $P \circ \theta$ and A are as "close" as possible.

- A parametric transformation is denoted by $\theta_\alpha(x) := \Theta(x; \alpha)$, e.g. $\theta_\alpha(x) = \sum_{i=1}^N \alpha_i e_i$ in $\mathbb{R}^N$.
- The closeness between the two images is evaluated by the so-called distance $d : \mathbf{Im} \times \mathbf{Im} \to \mathbb{R}$, hence, $d(P \circ \theta_\alpha, A)$.

Let $c(\alpha) := d(P \circ \theta_\alpha, A)$.

$$\text{Co-registration} \quad \rightsquigarrow \quad \min_{\alpha \in \mathbb{R}^N} c(\alpha)$$

Optimization problem

Minimize $\alpha \mapsto c(\alpha) = \delta(P \circ \theta_\alpha) := d(P \circ \theta_\alpha, A)$ among $\alpha \in \mathbb{R}^N$.

- Unconstrained optimization problem.
- There are several available algorithms (a good choice may depend on the characteristics of the transformation θ_α and the cost function $c(\alpha)$).

Gradient of $c(\alpha)$ is computed as

$$\nabla c(\alpha) = \int_\Omega D\delta(P \circ \theta_\alpha; x) \left[\nabla P(\theta_\alpha(x)) \, J_\alpha \Theta(x; \alpha)\right] dx$$

which is given by the three following modules:

distance module + image module + transformation module.

Each module is independent $\rightsquigarrow$ This facilitates the construction of a toolbox code.

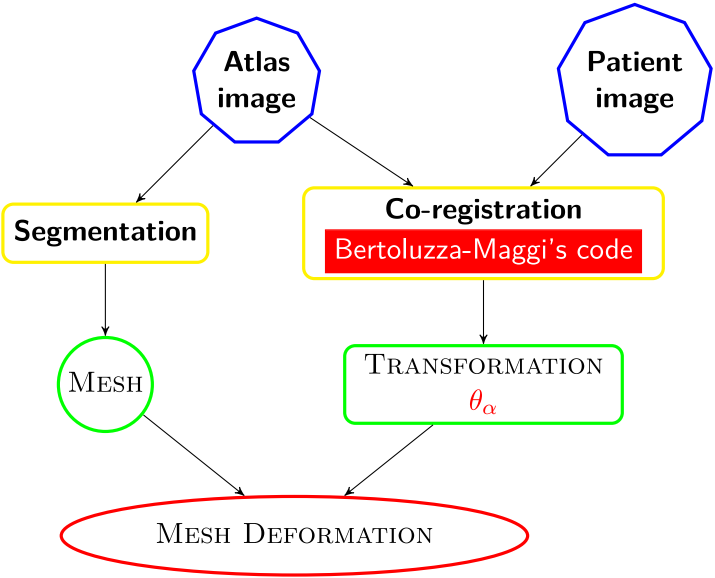
Matlab code [Bertoluzza, Maggi]

- Several choices of image distances.
- Interpolating wavelet transformation.
- Several ways to compute image gradient.

[1] **Registration algorithms for medical images**, S. Bertoluzza, G. Maggi, 2013.

[2] **A New Class of Wavelet-Based Metrics for Image Similarity Assessment**, M.G. Albanesi, R. Amadeo, S. Bertoluzza, G. Maggi, 2018.

Our project points at exploiting the image registration technique in [1] for constructing high quality mesh of patient specific domain from medical images.


Atlas image
Patient image
Segmentation
Co-registration
Bertoluzza-Maggi's code
Mesh
Transformation
θ_α
Mesh Deformation

2D medical images

Ulna & Radius

Femur

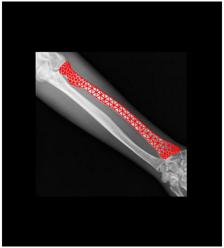
Tibia & Fibula

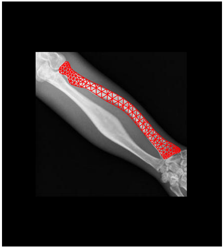
Atlas

Patient

2D meshes

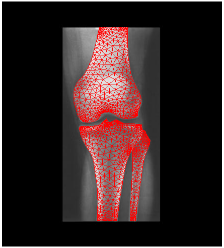
Ulna & Radius

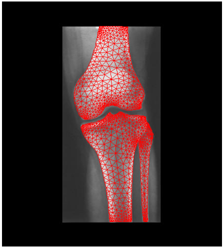




Femur

Tibia & Fibula





Atlas mesh

Deformed (patient) mesh

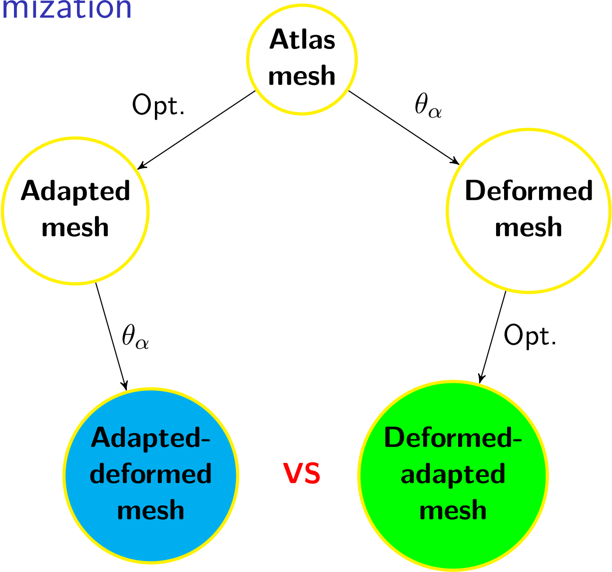
Mesh adaptation

Iterate:

(1) Get a solution in a starting mesh.

(2) Use the just computed solution for an error indicator and build an adapted mesh.

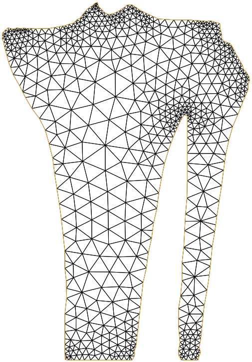
Mesh optimization



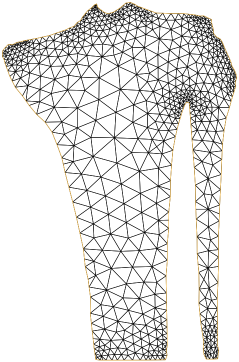
AD = Adapted-Deformed mesh, DA = Deformed-Adapted mesh

Tibia and fibula

Non optimized meshes



Atlas

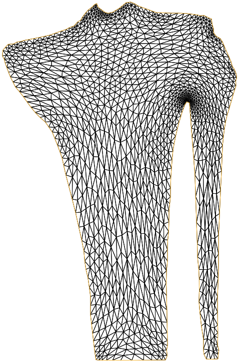


Patient

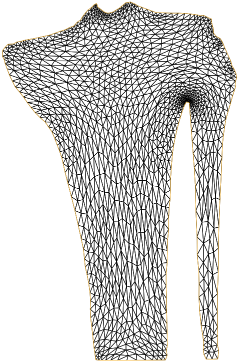
Poisson equation

Optimized meshes on patient image

$$\begin{cases} -\Delta u = 10^{-2} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$



AD

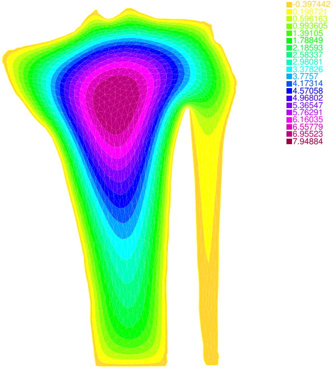


DA

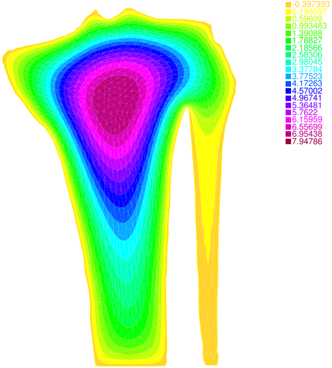
Poisson equation

Solutions on optimized (patient) meshes

$$\begin{cases} -\Delta u = 10^{-2} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$



AD



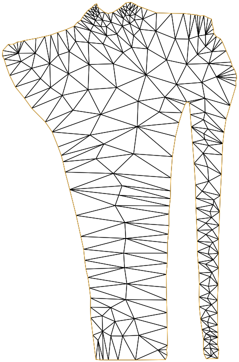
DA

ADR equation

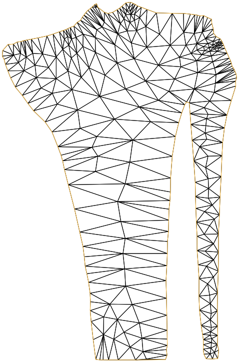
Optimized meshes on patient image

$$\begin{cases} -k\Delta u + \beta \cdot \nabla u + u = f & \text{in } \Omega \\ u = u_{ex} & \text{on } \partial\Omega \end{cases}$$

$$k = 10^{-5}, \quad \beta = [0\ 1]^T, \quad u_{ex} = kxy^2$$



AD



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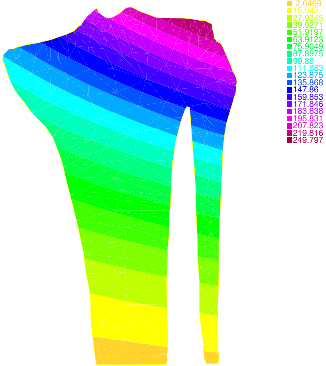
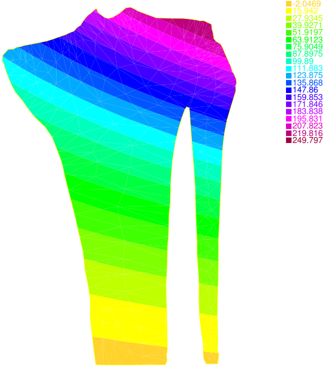


AD



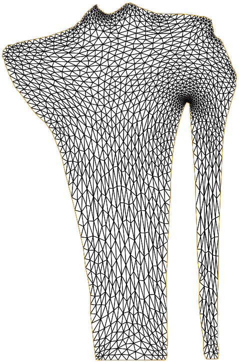
DA

Elastic equation

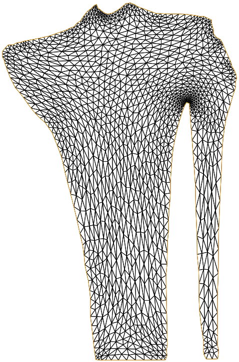
Optimized meshes on patient image

$$\begin{cases} -\nabla \cdot \boldsymbol{\varepsilon}(\boldsymbol{u}) = \rho \boldsymbol{g} & \text{in } \Omega \\ \boldsymbol{u} = \boldsymbol{0} & \text{on } \partial\Omega \end{cases}$$

$$E = 14.8 \; 10^6 \text{ Pa}, \quad \nu = 0.19, \quad \rho = 1.9 \; 10^3 \text{ kg m}^{-2}$$



AD



DA

Elastic equation

Solutions on optimized (patient) meshes

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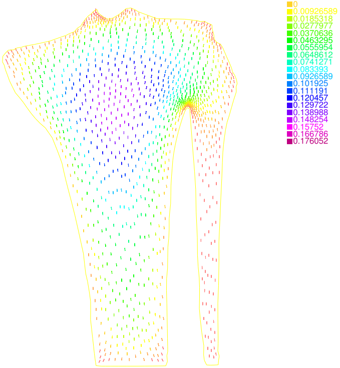
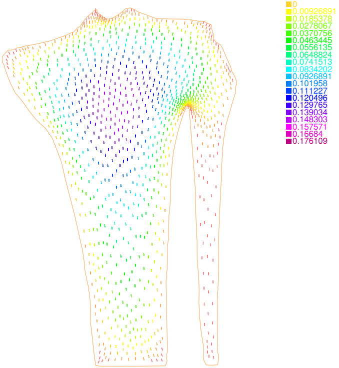


AD



DA

Comparison results

$$\text{Relative error} = \frac{\|u - u^*\|_2}{\|u^*\|_2}$$

where u^* is either the exact solution or the best solution we have.

Equation	AD	DA
Poisson	1.95 E-3	1.91 E-3
ADR	4.09 E-4	2.69 E-4
Elastic	2.44 E-3	2.48 E-3

Distances

We consider the two following cost functionals

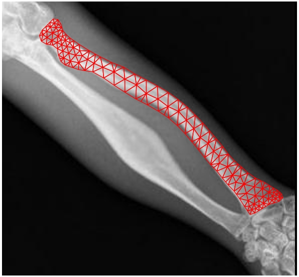
$$\delta_0(P \circ \theta_\alpha) := \frac{1}{2}\|P \circ \theta_\alpha - A\|_2^2$$
$$\delta_\lambda(P \circ \theta_\alpha) := \frac{1}{2}\left(\|P \circ \theta_\alpha - A\|_2^2 + \lambda\|\nabla(P \circ \theta_\alpha - A)\|_2^2\right)$$

whose Frechet derivatives are:

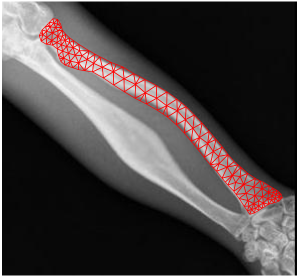
$$D\delta_0(P \circ \theta_\alpha)[h] = \langle P \circ \theta_\alpha - A, h\rangle$$
$$D\delta_\lambda(P \circ \theta_\alpha)[h] = \langle P \circ \theta_\alpha - A, h\rangle + \lambda\langle\nabla(P \circ \theta_\alpha - A), \nabla h\rangle$$

for any $h \in H^1(\Omega)$.

Distances



L2



H1

A practical experiment

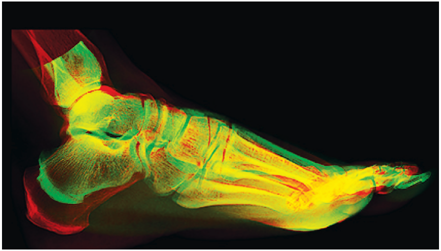

Atlas


Patient

What if we don't get a good transformation ?

Atlas

Patient



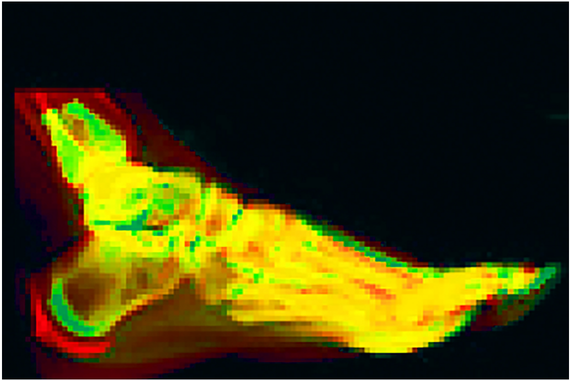
Atlas vs Transformed Patient

Multiscale approach

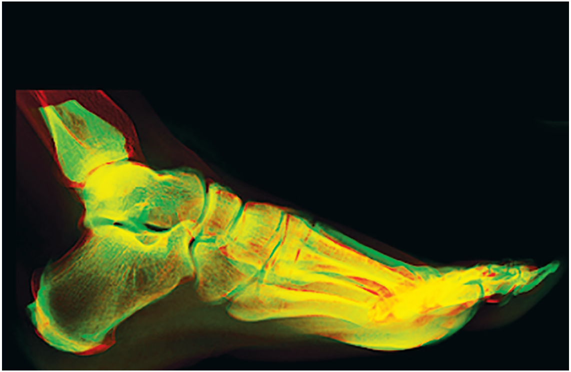
We speed up the co-registration procedure by using multiscale approach:

- First co-register two images with low resolution.
- Then repeat the co-registration with the real resolution of the two images while the starting point of this step is set up with the output of the first step.

Multiscale approach. 1st step: 128x128 resolution



Multiscale approach. 2nd step: 512x512 resolution



Singlescale vs Multiscale

Atlas mesh

Singlescale vs Multiscale

Further works

- Control the distortion of the transformation by adding some regularization term to the cost functional:

$$\tilde{c}(\alpha) = d(P \circ \theta_\alpha, A) + E(\theta_\alpha)$$

  where $E(\cdot)$ is some measure of distortion.
- Develop the Bertoluzza-Maggi's code to register 3D medical images.
- Create 3D meshes for 3D atlas images.
- Transform and adapt these atlas meshes to the patient images.

This 3D extension is currently being implemented. BUT:
$\rightsquigarrow$ Limitation: memory requirement and computational time.
$\rightsquigarrow$ Need high performance computational platform.

Thank you for your attention !