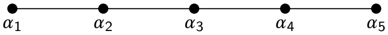
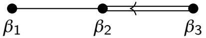
An example: (X, G) of type (C_3, A_5)

X as the fixed points subgroup of a graph automorphism of G





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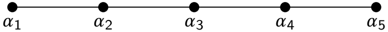

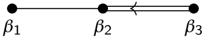

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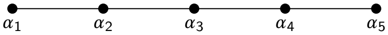

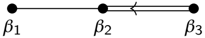

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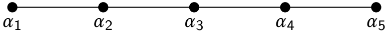

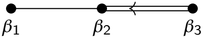

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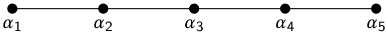

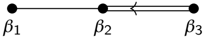

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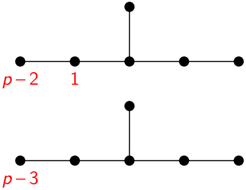
Same arguments works for (X,G) of type (C_n, A_{2n-1}).

Comparing irreducible and two composition factors $p \geq 13$

Irreducible action
(Testerman '88)
$(X, G) = (F_4, E_6)$



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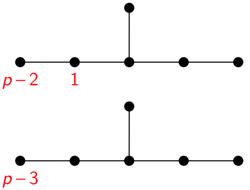
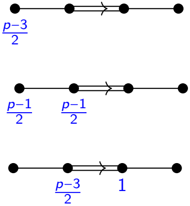


$(X, G) = (B_4, F_4)$



Two composition factors action
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