REPRESENTATIONS IN LIE THEORY AND INTERACTIONS

CIRM, LUMINY, 5 - 9 NOVEMBER, 2018

Mini-courses:

Chris Bowman

The partition algebra and the Kronecker coefficients

Abstract: A central problem in algebraic combinatorics is to provide an algorithm for calculating the coefficients arising in the decomposition of a tensor product of two simple representations of the symmetric group. The coefficients in such a decomposition are known as the ”Kronecker coefficients”; these coefficients include the Littlewood-Richardson coefficients as a special case. The ultimate goal in this area is to generalise the Littlewood-Richardson rule to provide a general ”Kronecker-tableaux” counting algorithm for computing all Kronecker coefficients.

We shall review the combinatorics of the Littlewood-Richardson rule and see how this can be generalised in the setting of the partition algebra. We introduce the notion of a Kronecker tableaux and generalise one half of the Littlewood-Richardson rule and hence provide an algorithm for computing one of the largest families of Kronecker coefficients considered to date.

Ivan Marin

Hecke algebras and invariants of links

Abstract: The Iwahori-Hecke algebras are the Hecke algebras of finite reductive groups with respect to their Borel subgroups. It has been known for 30 years now that the Iwahori-Hecke algebras provide a way to construct invariants of knots and links. In this minicourse, made of three lectures of one hour each, we will start by recalling this classical construction and then we will present several variations on this that have been unveiled and/or understood only recently. One of them involves a rather mysterious extension of the Iwahori-Hecke algebra, another one is the Hecke algebra of a finite reductive group with respect to a torus, also known as the Yokonuma-Hecke algebra. Finally we will show how complex reflection groups and (partly proved) conjectures of Broué, Malle and Rouquier on generalized Iwahori-Hecke algebras can help understand the more complicated algebras involved in the computation of classical knot invariants.

Talks (1 hour)

Stephen Donkin

Taking tensor products with the Steinberg module

Abstract: Let G be a connected, semisimple, simply connected, algebraic group over a field of positive characteristic. I shall discuss various conjectures about taking tensor products with the Steinberg module and in particular the recent work of Bendel, Nakano, Pillen and Sobaje. I will then discuss the question, raised by Sobaje, of whether the tensor product of the Steinberg module and the coordinate algebra of the first infinitesimal subgroup of G (regarded as a G-module via the conjugation action) has a good filtration, and give a proof in good characteristic.

Jérémie Guilhot

Balanced system of cell representations in affine Hecke algebras

Abstract: The aim of this talk is to introduce the notion of a balanced system of cell representations in Hecke algebras and to explain how the existence of such a system can be used to compute Lusztig a-function. We will focus on the affine Hecke algebra of type G2 where it is known that such a system exists and we will show that, in this case, Lusztig conjectures P1-P15 hold for arbitrary parameters.
This is a work in collaboration with James Parkinson (Sidney).

Steffen Koenig

Homological invariants of blocks of Schur algebras

Abstract: Let $S(n, r)$ be a classical or quantised Schur algebra with $n \geqslant r$. Totaro determined the global dimension of classical $S(n, r)$ and Donkin extended this result to quantum $S(n, r)$. Ming Fang and I have determined the dominant dimension (which in some sense measures the strength of Schur-Weyl duality) in both cases. Schur algebras are, however, not indecomposable as algebras, and thus we should ask about the homological invariants of their blocks. This has been answered in recent joint work with Ming Fang and Wei Hu. We have been proving that - unlike in the general case - global dimension is invariant under derived equivalences between finite dimensional algebras with simple preserving dualities, and under an additional assumption satisfied by Schur algebras, dominant dimension is invariant, too. Using the derived equivalences constructed by Chuang and Rouquier, we then get explicit formulae for global and dominant dimensions of blocks.

Emmanuel Letellier

Exotic Fourier transforms on connected reductive groups

Abstract:

The use of Fourier transforms in representation theory was initiated by Springer to establish his well-known correspondence, it was later used by Kawanaka and Lusztig to investigate the generalized Gelfand-Graev characters. More recently it was used by Juteau to construct the modular Springer correspondence. However the connection between Fourier transforms and representations of finite Lie groups is rather indirect precisely because Fourier transforms are defined on Lie algebras. The only exception concerns $\mathrm{GL}n$ as it is naturally embedded in its Lie algebra. In this special case Fourier transforms have very powerful applications : cohomological interpretation of coefficients structure of the character ring of $\mathrm{GL}n(\mathbb{F}q)$, cohomology of quiver varieties, Kac conjectures on quiver representations. In fact all these applications come from

the fact that we can construct the unipotent characters of $GLn(Fq)$ as the Fourier transform of nilpotent orbits of gln. In this talk we will discuss the case of other reductive groups motivated by the work of Braverman-Kazhdan and then Lafforgue. This is joint work with Laumon.

Vanessa Miemietz

Analogues of centraliser subalgebras for fiat $2-categories$

Abstract: I will motivate and explain the concept of flat $2-categories$ and their $2-representations$. I will then describe a reduction method for the classification of their simple $2-representations$.

Sophie Morier-Genoud

Symplectic friezes and moduli spaces

Abstract: Frieze patterns of numbers are combinatorial objects introduced by Coxeter in the early 70's. The recent revival of friezes is due to connections with the theory of cluster algebras. After a short and elementary introduction to Coxeter's friezes and their generalizations, I will explain how the spaces of friezes are identified with the moduli spaces of points in projective spaces. In particular I will focus on the case of symplectic friezes which are related to Langrangian configurations of points in the 3-dimensional space and to cluster algebras of Dynkin type C_2.

Raphaël Rouquier

Finite groups of Lie type and double affine Lie theory

Abstract: I will discuss a conjectural relation between modular representations of finite groups of Lie type and deformations of structures related to double affine Hecke algebras, with coherent and constructible incarnations.

Talks (35 min.)

Floriana Amicone

Modular invariants of graded Lie algebras

Abstract: Consider the Lie algebra g of a reductive algebraic group G, defined over an algebraically closed field k. In many instances g admits a $Z/mZ - grading$, for m a nonnegative integer, and there exists a connected reductive subgroup $G(0) of G$ acting algebraically on each graded component. The orbit structure and invariant theory for the action of $G(0)$ on a fixed graded component has been extensively studied over the past decades. Vinberg proved that, if k is of characteristic 0, the ring of invariant functions for this action is a polynomial ring. Levy generalised this result to fields of characteristic $p > 0$ in the case in which the integers p and m are coprime, obtaining again polynomiality of invariants. My talk will be devoted to present some results relative to the yet untreated case of a $Z/pZ - grading$ in characteristic $p > 0$.

Tristan Bozec

Irreducible components of the global nilpotent cone

Abstract: Given a curve X of genus g, the moduli space of Higgs bundles of rank r and degree d is known to be of dimension $2(g-1)r^2$. It can be viewed as the cotangent space of the moduli space of vector bundles of class (r,d) over X, and Laumon proved that the subvariety of nilpotent Higgs pairs is Lagrangian. This subvariety is a global analog of the nilpotent cone, and is nothing but the 0-fiber of the Hitchin map. It is highly singular, and one first interesting step toward its comprehension is the study of its irreducible components. It is motivated by the fact that the number of these components which are stable (with respect to the usual slope) is given by the value at 1 of the Kac polynomial associated with the quiver with one vertex and g loops (conjectured by Hausel, Letellier and Rodriguez Villegas, proved by Mellit). I will give a natural combinatorial description of this set of components, in terms of lattice points of explicit polytopes.

Marion Chommaux

Distinction of the Steinberg representation and a conjecture of Prasad and Takloo-Bighash

Abstract: Let F be a non-archimedean local field of odd characteristic, E/F a quadratic extension and D a central division $F - algeabra$. We set $G = GL(n, D)$ and $H = (C_{M_n(D)}(E))^{\times}$ where $C_{M_n(D)}(E)$ is the centralizer of $E in M_n(D)$. We will see in which case the Steinberg representation $St(\chi)$ of G (for χ a character of F^*) is H-distinguished, that is to say we will study in which case $Hom_H(St(\chi), 1)$ is non-trivial. This study allows us to check a conjecture of Prasad and Takloo-Bighash for the Steinberg representation.

Thorsten Heidersdorf

Orthogonal Deligne categories and Representations of $OSp(m|2n)$

Abstract: I will talk about the connection between the interpolating categories $Rep(O_t)$, $t \in \mathbb{C}$, of Deligne for the orthogonal groups and finite-dimensional algebraic representations of the orthosymplectic supergroup $OSp(m|2n)$. The Deligne categories allow to understand the decomposition of $V^{\otimes r}$ for the natural representation $V = \mathbb{C}^{m|2n}$. This in turn can be used to study the Duflo-Serganova cohomology functor $DS : Rep(OSp(m|2n)) \to Rep(OSp(m-2r, |2n-2r))$.

Katerina Hristova
Representations of Complete Kac-Moody Groups

Abstract: Complete Kac-Moody groups are locally compact totally disconnected groups which admit a BN-pair. Using their structure of topological groups one can look at their so-called smooth representations - these are representations with an extra continuity condition. We discuss some interesting properties of the category of smooth representations of Kac-Moody groups, in particular, its cohomological dimension and localisation theory. If time allows we explain how our results apply to a more general class of groups called topological groups of Kac-Moody type. Joint work with Dmitriy Rumynin.

Abel Lacabane
Categorification of Z-modular data associated to complex reflection groups

Abstract: In the classification of characters of finite reductive groups Lusztig has constructed N-modular data, which depends only on the Weyl group, which he interprets in terms of Drinfeld double of finite groups. A generalization of these modular data has been constructed by Malle for the complex reflection groups $G(d, 1, n)$. We will explain a categorification of some of these data with a slightly degenerate category of representations of a central extension of $U_q(sl_n)$ at an $2d - th$ root of unity.

Stacey Law
Linear characters of Sylow subgroups of the symmetric group

Abstract: Let p be an odd prime and n a natural number. We determine the irreducible constituents of the permutation module induced by the action of the symmetric group Sn on the cosets of a Sylow p-subgroup Pn . In the course of this work, we also prove a symmetric group analogue of a well-known result of Navarro for p-solvable groups on a conjugacy action of N G (P). Before describing some consequences of these results, we will give an overview of the background and recent related results in the area.

César Lecoutre
Enveloping Skewfields in dimension 3

Abstract: The well-known Gelfand-Kirillov conjecture asserts that the enveloping skewfield of a complex finite dimensional algebraic Lie algebra is isomorphic to a Weyl skewfield. We study in arbitrary characteristic a family of three-dimensional solvable Lie algebras, which are not algebraic for certain values of the parameter when seen as complex Lie algebras. In particular we show that in positive characteristic there are only two isomorphism classes of enveloping skewfield of theses algebras.

Emily Norton

BGG resolutions, character formulas, and bases of simple modules

Abstract: I will describe a homological construction of unitary simple modules for Cherednik algebras of type A and quiver Hecke algebras, via BGG resolutions. These resolutions lift character formulas to a categorical level, and their combinatorics comes from affine type A alcove geometry.
This is joint work with Chris Bowman and José Simental.

Salim Rostam

Stuttering blocks of Ariki-Koike algebras

Abstract: There is a natural shift action defined on multipartitions and residue multisets of their Young diagrams. Two multipartitions with the same residue multiset may have different orbit cardinalities, but the orbit cardinality of a given residue multiset is the maximum of the orbit cardinalities of all associated multipartitions. Using the abacus representation of a partition, we will see how the proof reduces to a quadratic integer minimisation problem.

Nathan Scheinmann

Restricting irreducible representations to maximal subgroup

Abstract: Let G be a simply connected simple algebraic group over an algebraically closed field and consider a maximal closed connected simple subgroup X of G. The study of the action of X on irreducible representations of G goes back to Dynkin in characteristic 0 and was extended by Seitz and Testerman in positive characteristic. In this talk, we present the classification of all the p-restricted irreducible representations of G of exceptional type on which X acts with two composition factors and give an overview of the methods used to obtain this result.

Jacinta Torres

Kostka-Foulkes polynomials in type C

Abstract: I will recall a conjecture of Lecouvey on a closed, combinatorial formula for Kostka-Foulkes polynomials in type C, and present work in progress towards a proof in weight zero, using recent results of Lecouvey-Lenart.
This is joint work with Thomas Gerber.

Jonathan Wilson

Cluster structures on laminated surfaces

Abstract: Using laminations of surfaces we link two constructions which arose as seemingly unrelated generalisations of cluster algebras. One of the generalisations (quasi-cluster algebras) being based on triangulated surfaces, and the other (Laurent phenomenon algebras) based on the Laurent phenomenon.