Stochastic differential equations with singular perturbations

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Liéanard Oscillator

We consider the Liéanard Oscillator driven by random force given by the second order stochastic differential equation

$$\varepsilon \ddot{z}_t + \dot{z}_t - h(z_t) = \delta\, \dot{w}_t\,,$$

where $0 < \varepsilon < 1$, $\delta > 0$ are some parameters, $\dot{w}_t$ is the white noise. This model arises in the description of the motion of a small particle in a viscous media. Setting $x^\varepsilon_t = z_t$ and $y^\varepsilon_t = \dot{z}_t$ we can represent this physical model in the stochastic calculus form

$$\begin{cases} \dfrac{\mathrm{d}x^\varepsilon_t}{\mathrm{d}t} = y^\varepsilon_t\,, \quad x^\varepsilon_0 = x^0\,; \\ \\ \varepsilon \mathrm{d}y^\varepsilon_t = -y^\varepsilon_t + h(x^\varepsilon_t) + \delta\, \mathrm{d}w_t\,, \quad y^\varepsilon_0 = y^0\,, \end{cases}$$

where $(w_t)_{0\le t\le T}$ is the Wiener process.

Stochastic volatility models

Fouque, Papanicolaou and Sicar (2000) considered the financial markets (B, S) in which the bond $B = (B_t^\varepsilon)_{0\le t\le T}$ and the stock $S = (S_t^\varepsilon)_{0\le t\le T}$ are defined as

$$\begin{cases} \mathrm{d}B_t^\varepsilon = r(y_t^\varepsilon) B_t^\varepsilon \mathrm{d}t\,, \quad B_0^\varepsilon = 1\,; \\ \mathrm{d}S_t^\varepsilon = \mu(y_t^\varepsilon) S_t^\varepsilon \mathrm{d}t + \sigma(y_t^\varepsilon) S_t^\varepsilon \mathrm{d}w_t^s\,, \quad S_0^\varepsilon = S^0\,; \\ \varepsilon \mathrm{d}y_t^\varepsilon = F(y_t^\varepsilon)\mathrm{d}t + \beta\sqrt{\varepsilon}G(y_t^\varepsilon)\mathrm{d}w_t^y\,, \qquad y_0^\varepsilon = y^0\,, \end{cases}$$

where the market coefficients $r(\cdot)$, $\mu(\cdot)$, $\sigma(\cdot)$ are some functions, $(w_t^s)_{0\le t\le T}$ and $(w_t^y)_{0\le t\le T}$ are two Wiener processes, $F(\cdot)$ and $G(\cdot)$ satisfy some conditions for which one can study the behavior of the process $(y_t^\varepsilon)_{0\le t\le T}$.

Singular perturbations for ODE

The system

$$\begin{cases} \dfrac{\mathrm{d}x_t^\varepsilon}{\mathrm{d}t} = f(t, x_t^\varepsilon, y_t^\varepsilon), \quad x_0^\varepsilon = x^0 ; \\ \varepsilon \dfrac{\mathrm{d}y_t^\varepsilon}{\mathrm{d}t} = F(t, x_t^\varepsilon, y_t^\varepsilon), \quad y_0^\varepsilon = y^0 . \end{cases}$$

The "slow" variable $x_t^\varepsilon \in \mathbb{R}^k$ and the "fast" variable $y_t^\varepsilon \in \mathbb{R}^n$, $0 < \varepsilon < 1$ is a small parameter.

The problem is to study the limit behavior for the solutions $(x_t^\varepsilon)_{0 \le t \le T}$ and $(y_t^\varepsilon)_{0 \le t \le T}$ when $\varepsilon \to 0$?

Limit form

We define now the following system

$$\frac{\mathrm{d}x_t}{\mathrm{d}t} = f(t, x_t, \varphi(t, x_t)) , \quad x_0 = x^0 ,$$

where the $[0, T] \times \mathbb{R}^k \to \mathbb{R}^n$ function φ is the stable root

$$F(t, x, \varphi(t, x)) = 0 .$$

Theorem (Tikhonov, 1952)

$$\lim_{\varepsilon \to 0} \sup_{0 \le t \le T} |x_t^\varepsilon - x_t| = 0 \quad \text{and} \quad \lim_{\varepsilon \to 0} \sup_{t_0 \le t \le T} |y_t^\varepsilon - \varphi(t, x_t)| = 0$$

for any fixed $0 < t_0 < T$.

Stochastic systems

The system

$$\begin{cases} \mathrm{d}x_t^\varepsilon = f(t, x_t^\varepsilon, y_t^\varepsilon)\mathrm{d}t + g(t, x_t^\varepsilon, y_t^\varepsilon)\,\mathrm{d}w_t^x\,, \quad x_0^\varepsilon = x^0\,; \\ \\ \varepsilon\mathrm{d}y_t^\varepsilon = F(t, x_t^\varepsilon, y_t^\varepsilon)\mathrm{d}t + \beta\sqrt{\varepsilon}\,G(t, x_t^\varepsilon, y_t^\varepsilon)\mathrm{d}w_t^y\,, \quad y_0^\varepsilon = y^0\,, \end{cases}$$

where $\beta = \beta_\varepsilon = \mathrm{o}(1/|\ln\varepsilon|)$ as $\varepsilon \to 0$.

The coefficients satisfy some technical conditions under which this system has an unique strong solution for any $\varepsilon > 0$.

The problem is to study the asymptotic behavior of the solution when $\varepsilon \to 0$.

Limit form

To describe the limit form for the slow variables we set

$$\mathrm{d}x_t = f(t, x_t, \varphi(t, x_t))\mathrm{d}t + g(t, x_t, \varphi(t, x_t))\,\mathrm{d}w^x_t\,, \quad x_0 = x^0\,.$$

Theorem (Kabanov and Pergamenchtchikov, 1990)

$$\mathbf{P} - \lim_{\varepsilon \to 0} \sup_{0 \le t \le T} |x^\varepsilon_t - x_t| = 0 \quad \text{and} \quad \mathbf{P} - \lim_{\varepsilon \to 0} \sup_{t_0 \le t \le T} |y^\varepsilon_t - \varphi(t, x_t)| = 0$$

for any fixed $0 < t_0 < T$.

Fast processes

We consider the fast process

$$\varepsilon \mathrm{d}y_t^\varepsilon = F(y_t^\varepsilon)\mathrm{d}t + \beta\sqrt{\varepsilon}G(y_t^\varepsilon)\mathrm{d}w_t\,, \quad y_0^\varepsilon = y^0\,.$$

Main condition:

There exists a stable root φ_∞ of the equation $F(y) = 0$, i.e. such that the continuous derivative matrix $F'(\cdot)$ exists and the real parts of all eigenvalues $A = F'(\varphi_\infty)$ are strictly negative

$$\mathbf{P} - \lim_{\varepsilon\to 0} \sup_{t_0\le t\le T} |y_t^\varepsilon - \varphi_\infty| = 0$$

for any fixed $0 < t_0 < T$.

Boundary layer

To study the correction we need to consider the following system

$$\frac{\mathrm{d}\widetilde{y}_t}{\mathrm{d}t} = F(\widetilde{y}_t)\,, \quad \widetilde{y}_0 = y^0\,.$$

Vasil'eva and Butusov (1973)

$$\widetilde{y}_t = \varphi_\infty + \Pi(t)$$

the boundary layer function $\Pi(\cdot)$ is such that $\Pi(0) = y^0 - \varphi_\infty$ and for some positive constants $\mathbf{c} > 0$ and $\gamma > 0$

$$|\Pi(t)| \leq \mathbf{c}\, e^{-\gamma t}$$

for any $t \geq 0$.

First order correction

In the neighborhood of zero:

$$y_t^\varepsilon = \widetilde{y}_r + \beta \widetilde{\eta}_r + \beta \widetilde{\Delta}_t^\varepsilon \,, \quad r = t/\varepsilon \,,$$

where

$$\mathrm{d}\widetilde{\eta}_s = F'(\widetilde{y}_s)\widetilde{\eta}_s \mathrm{d}s + G(\widetilde{y}_s)\mathrm{d}\widetilde{w}_s \,, \quad \widetilde{\eta}_0 = 0$$

and

$$\widetilde{w}_s = \frac{1}{\sqrt{\varepsilon}} \, w_{s/\varepsilon} \,.$$

The rest term

$$\mathbf{P} - \lim_{\varepsilon \to 0} \max_{0 \le t \le T} |\widetilde{\Delta}_t^\varepsilon| = 0 \,.$$

First order correction

In the outside of any neighborhood of zero:

$$y_t^{\varepsilon} = \varphi_{\infty} + \beta\,\eta_r + \beta\Delta_t^{\varepsilon}\,, \quad r = t/\varepsilon\,,$$

where

$$\mathrm{d}\eta_s = \ A\eta_s \mathrm{d}s + B\mathrm{d}\widetilde{w}_s\,, \quad \eta_0 = 0$$

$A = F'(\varphi_{\infty})$, $B = G(\varphi_{\infty})$ and the rest term

$$\mathbf{P} - \lim_{\varepsilon\to 0}\ \max_{t_0 \le t \le T}\ |\Delta_t^{\varepsilon}| = 0$$

for any fixed $0 < t_0 < T$.

Uniform metric

To describe the large deviation in the uniform metric we need to introduce the following $\mathcal{C}(\mathbb{R}_+) \to \mathbb{R}_+$ functional. For any $y \in \mathcal{C}^1(\mathbb{R}_+)$ this functional is defined as

$$S(y) := \frac{1}{2}\int_0^\infty |\Sigma^{-1/2}(y_s)\,(\dot{y}_s - F(y_s))\,|^2\,\mathrm{d}s\,,$$

where $\Sigma(y) = G(y)G'(y)$. Moreover, we set $S(y) = +\infty$ for $y \notin \mathcal{C}^1(\mathbb{R}_+)$.
This is the well-known Freidlin - Wentzell functional on the interval $[0, +\infty]$.

Uniform metric

We remind that for the process $\widetilde{y}_r^\varepsilon = y_{r\varepsilon}^\varepsilon$

$$\mathrm{d}\widetilde{y}_r^\varepsilon = F(\widetilde{y}_r^\varepsilon)\mathrm{d}r + \beta G(\widetilde{y}_r^\varepsilon)\mathrm{d}\widetilde{w}_r\,, \quad \widetilde{w}_r = \frac{1}{\sqrt{\varepsilon}}\, w_{r/\varepsilon}$$

and $y_0^\varepsilon = y^0$.
In this case for $0 \le r \le T/\varepsilon$

$$\widetilde{y}_r^\varepsilon = \widetilde{y}_r + \beta\widetilde{\eta}_r + \dots$$

where

$$\frac{\mathrm{d}\widetilde{y}_r}{\mathrm{d}r} = F(\widetilde{y}_r)\,, \quad \widetilde{y}_0 = y^0\,.$$

Uniform metric

Theorem

For any $\nu > 0$

$$\lim_{\varepsilon\to 0} \beta^2 \ln \mathbf{P}\Big(\max_{0\le r\le T/\epsilon} \big|\widetilde{y}^{\varepsilon}_r - \widetilde{y}_r\big| > \nu \Big) = - \inf_{y\in \mathbf{B}(\nu)} S(y)\,,$$

where

$$\mathbf{B}(\nu) = \Big\{ y \in \mathbf{C}(\mathbb{R}_+)\ :\ \sup_{r\ge 0} |y_r - \widetilde{y}_r| > \nu\,,\ y_0 = y^0 \Big\}$$

Deviation in the metric $\mathbf{L}_2[0,T]$

To describe the deviation in the integral metric we need to define $\mathbf{C}[0,T] \to \mathbb{R}_+$ as

$$\check{S}_T(y) := \frac{1}{2}\int_0^T |\Sigma^{-1/2}(y_t)\,F(y_t)|^2 \mathrm{d}t\,,$$

where $\Sigma(y) = G(y)G'(y)$. Remind that the usually for regular perturbations Freidlin - Wentzell functional is defined as

$$S_T(y) := \frac{1}{2}\int_0^T |\Sigma^{-1/2}(y_t)\,(\dot{y}_t - F(y_t)|^2 \mathrm{d}t\,.$$

Deviation in the metric $\mathbf{L}_2[0, T]$

Theorem

For any $\nu > 0$

$$\lim_{\varepsilon \to 0} \beta^2 \varepsilon \ln \mathbf{P} \left(\int_0^T |y_t^\varepsilon - \varphi_\infty|^2 \mathrm{d}t > \nu \right) = - \inf_{y \in \check{\mathbf{B}}(\nu)} \check{S}_T(y) \,,$$

where

$$\check{\mathbf{B}}(\nu) = \left\{ y \in \mathbf{C}[0, T] \, : \, \int_0^T |y_t - \varphi_\infty|^2 \mathrm{d}t > \nu \right\} .$$

Applications PDEs

Let us consider the following Cauchy problem:

$$
\begin{cases}
\dfrac{\partial}{\partial t}\, v^{\varepsilon}(t,x) & = \dfrac{\beta^2}{\varepsilon}\, G^2(x)\dfrac{\partial^2}{(\partial x)^2} v^{\varepsilon}(t,x) + \dfrac{1}{\varepsilon}\, F(x)\dfrac{\partial}{\partial x} v^{\varepsilon}(t,x) \\
& \quad + r(x)\, v^{\varepsilon}(t,x) + h(x)\,; \\
v^{\varepsilon}(0,x) = \mathbf{b}(x)\,, & \quad x \in \mathbb{R}\,.
\end{cases}
$$

The limit equation for $t > 0$

$$
\frac{\partial}{\partial x}\, v(t,x) = 0\,.
$$

Applications PDEs

Using the "fast" process

$$\varepsilon \mathrm{d} y_t^{x,\varepsilon} = F(y_t^{x,\varepsilon})\mathrm{d}t + \beta\sqrt{\varepsilon}G(y_t^{x,\varepsilon})\mathrm{d}w_t\,, \quad y_0^{x,\varepsilon} = x\,,$$

we can represent the PDE solution as

$$v^{\varepsilon}(t,x) = \mathbf{E}\,\mathbf{b}(y_t^{x,\varepsilon})\, e^{\int_0^t r(y_u^{x,\varepsilon})\mathrm{d}u} + \mathbf{E}\int_0^t h(y_u^{x,\varepsilon})\, e^{\int_0^u r(y_v^{x,\varepsilon})\mathrm{d}v}\mathrm{d}u\,.$$

If the equation $F(x) = 0$ has unique stable root φ_∞ then for any $t > 0$ and any $x \in \mathbb{R}$

$$\lim_{\varepsilon \to 0} v^{\varepsilon}(t,x) = \mathbf{b}_\infty e^{r_\infty t} + h_\infty \frac{e^{r_\infty t} - 1}{r_\infty}\,,$$

where $\mathbf{b}_\infty = \mathbf{b}(\varphi_\infty)$, $r_\infty = r(\varphi_\infty)$ and $h_\infty = h(\varphi_\infty)$.

Applications PDEs

Let us consider now the following problem:

$$\begin{cases} \frac{\partial}{\partial t} v^{\varepsilon}(t,z) & = \mathcal{L}^{\varepsilon} v^{\varepsilon}(t,z)\,; \\ v^{\varepsilon}(0,z) & = \mathbf{b}(z)\,, \end{cases}$$

where $z = (x, y) \in \mathbb{R}^2$ and

$$\mathcal{L}^{\varepsilon} := g^2(t,z)\frac{\partial^2}{\partial x^2} + \frac{\beta^2}{\varepsilon} G^2(t,z)\frac{\partial^2}{\partial y^2}$$

$$+ f(t,z)\frac{\partial}{\partial x} + \frac{1}{\varepsilon} F(t,z)\frac{\partial}{\partial y}\,.$$

Applications PDEs

By the probability representation we can write that

$$v^{\varepsilon}(t,z) = v^{\varepsilon}(t,x,y) = \mathbf{E}\,\mathbf{b}(X_t^{x,\varepsilon}, Y_t^{y,\varepsilon})\,,$$

where

$$\begin{cases} \mathrm{d}X_t^{\varepsilon} = f(t, X_t^{x,\varepsilon}, Y_t^{y,\varepsilon})\mathrm{d}t + g(t, X_t^{x,\varepsilon}, Y_t^{y,\varepsilon})\,\mathrm{d}w_t^x\,, \quad X_0^{x,\varepsilon} = x\,; \\ \varepsilon\mathrm{d}Y_t^{\varepsilon} = F(t, X_t^{x,\varepsilon}, Y_t^{y,\varepsilon})\mathrm{d}t + \beta\sqrt{\varepsilon}\,G(t, X_t^{x,\varepsilon}, Y_t^{y,\varepsilon})\mathrm{d}w_t^y\,, \quad Y_0^{y,\varepsilon} = y\,. \end{cases}$$

Applications PDEs

The stochastic Tikhonov theorem implies that

$$\lim_{\varepsilon\to 0} v^{\varepsilon}(t,x,y) = v_0(t,x)\,,$$

where

$$v_0(t,x) = \mathbf{E}\,\mathbf{b}(t,X_t^x,\varphi(t,X_t^x))$$

and

$$\mathrm{d}X_t^x = f(t,X_t^x,\varphi(t,X_t^x))\mathrm{d}t + g(t,X_t^x,\varphi(t,X_t^x))\,\mathrm{d}w_t^x\,, \quad X_0^x = x\,.$$

Applications PDEs

Using the probability representation we obtain that

$$\begin{cases} \frac{\partial}{\partial t} v_0(t,x) & = g^2(t,\varphi(t,x)) \frac{\partial^2}{\partial x^2} v_0(t,x) + f(t,\varphi(t,x)) \frac{\partial}{\partial x} v_0(t,x) \,; \\ v_0(0,x) & = \mathbf{b}(0,x,\varphi(0,x)) \,. \end{cases}$$

Example. Let consider the stochastic volatility model, i.e. $B \equiv 1$ and

$$\begin{cases} \mathrm{d}S_t^\varepsilon = \sigma(y_t^\varepsilon) S_t^\varepsilon \mathrm{d}w_t^s \,, \quad S_0^\varepsilon = S^0 \,; \\ \varepsilon \mathrm{d}y_t^\varepsilon = F(y_t^\varepsilon)\mathrm{d}t + \beta\sqrt{\varepsilon} G(y_t^\varepsilon)\mathrm{d}w_t^y \,, \qquad y_0^\varepsilon = y^0 \end{cases}$$

Stable root: $F(\varphi_\infty) = 0$.

Pricing problem

Consider now the european option with the payoff h

$$\frac{\partial}{\partial t} v_\varepsilon(t,x,y) + \mathcal{L}^\varepsilon v_\varepsilon(t,x,y) = 0\,, \quad v_\varepsilon(T,x) = h(x)$$

and

$$\mathcal{L}^\varepsilon := \sigma^2(y)\frac{\partial^2}{\partial x^2} + \frac{1}{\varepsilon}F(y)\frac{\partial}{\partial y} + \frac{\beta^2}{\varepsilon}G^2(y)\frac{\partial^2}{\partial y^2}$$

If we change the variable variable $s = T - t$ we obtain the initial Cauchy problem and by making use the previous convergence we obtain that

$$\lim_{\varepsilon\to 0} v_\varepsilon(t,x,y) = v_0(t,x)\,,$$

where $v_0(t,x)$ is the option price for the Black-Scholes market with the volatility $\sigma(\varphi_\infty)$.

Filtering of Nearly Observed processes

Let us consider the model described by two processes $x_t \in \mathbb{R}^n$ (unobservable signal) and $y^\varepsilon_t \in \mathbb{R}^n$ (observations), both given by

$$\begin{cases} \mathrm{d}x_t = f_t\,\mathrm{d}t + g\,\mathrm{d}w^x_t\,, & x^\varepsilon_0 = x^0\,; \\ \mathrm{d}y^\varepsilon_t = x_t\mathrm{d}t + \varepsilon\,\mathrm{d}w^y_t\,, & y^\varepsilon_0 = y^0\,, \end{cases}$$

where $w^x_t \in \mathbb{R}^m$ and $w^y_t \in \mathbb{R}^n$ are independent Wiener processes and g is $n \times m$ nonrandom known. We use the filter $\widehat{x}^\varepsilon$ defined as

$$\mathrm{d}\widehat{x}^\varepsilon_t = \widehat{f}^\varepsilon_t\mathrm{d}t - \varepsilon^{-1}A\left(\mathrm{d}y^\varepsilon_t - \widehat{x}^\varepsilon_t\mathrm{d}t\right),$$

where $\widehat{f}^\varepsilon_t$ is measurable with respect to $\sigma\{y_u\,, 0 \le u \le t\}$ and A is symmetric negative defined matrix. For example, we can take $A = -\kappa I_n$ (I_n is the identity matrix of order n) or $A = -\left(gg'\right)^{1/2}$.

Filtering accuracy

We study the deviation $\Delta_t^\varepsilon = \widehat{x}_t^\varepsilon - x_t$. We obtain that

$$\mathrm{d}\Delta_t^\varepsilon = \varepsilon^{-1} A \Delta_t^\varepsilon \mathrm{d}t + (\widehat{f}_t^\varepsilon - f_t)\mathrm{d}t + G\mathrm{d}\tilde{w}_t \,,$$

where the matrix $G = (AA' + gg')^{1/2}$. Assume that the function f_t is linear, i.e.

$$f_t = b_t + D_t x_t \,,$$

where b_t is unknown and D_t is known. In this case we chose $\widehat{f}_t = D_t \widehat{x}_t$ and, therefore,

$$\mathrm{d}\widehat{x}_t^\varepsilon = D_t \widehat{x}_t \mathrm{d}t - \varepsilon^{-1} A \left(\mathrm{d}y_t^\varepsilon - \widehat{x}_t^\varepsilon \mathrm{d}t \right) \,.$$

Filtering accuracy

Therefore, in the linear case

$$\mathrm{d}\Delta_t^{\varepsilon} = \varepsilon^{-1} A \Delta_t^{\varepsilon} \mathrm{d}t + (D_t \Delta_t^{\varepsilon} - b_t)\mathrm{d}t + G\mathrm{d}\tilde{w}_t \,.$$

Asymptotically, as $\varepsilon \to 0$, the accuracy $\Delta_t^{\varepsilon} \approx \eta_t^{\varepsilon}$, where

$$\mathrm{d}\eta_t^{\varepsilon} = \varepsilon^{-1} A \eta_t^{\varepsilon} \mathrm{d}t + G\mathrm{d}\tilde{w}_t \,.$$

In this case $\varphi_{\infty} = 0$, $\beta = \sqrt{\varepsilon}$ and

$$\check{S}_T(y) = \frac{1}{2} \int_0^T y_t' A (A^2 + gg')^{-1} A y_t \mathrm{d}t \,.$$

Filtering accuracy

For any $\nu > 0$

$$\lim_{\varepsilon\to 0} \varepsilon^2 \ln \mathbf{P}\left(\int_0^T |\Delta_t^\varepsilon|^2 \mathrm{d}t > \nu\right) = -\inf_{y\in B(\eta)} \check{S}_T(y)$$

where

$$\check{\mathbf{B}}(\nu) = \left\{ y \in \mathbf{C}[0,T] \, : \, \int_0^T |y_t|^2 \mathrm{d}t > \nu \right\}.$$

In this case

$$\inf_{y\in B(\eta)} \check{S}_T(y) = \frac{\nu}{2}\lambda_{min}(A(A^2+gg')^{-1}A),$$

and $\lambda_{min}(B)$ is minimal eigenvalue of the matrix B.

Filtering accuracy

For any $\nu > 0$

$$\lim_{\varepsilon \to 0} \varepsilon^2 \ln \mathbf{P} \left(\int_0^T |\Delta_t^\varepsilon|^2 \mathrm{d}t > \nu \right) = -\frac{\nu}{2} \lambda_{min} (A(A^2 + gg')^{-1} A) \,.$$

Note that if $n = m = 1$ and $A = -\kappa$ we obtain that for any $\nu > 0$

$$\lim_{\varepsilon \to 0} \varepsilon^2 \ln \mathbf{P} \left(\int_0^T |\Delta_t^\varepsilon|^2 \mathrm{d}t > \nu \right) = -\frac{\kappa^2 \nu}{2(\kappa^2 + g^2)} \,.$$

Signal estimation

If in the model $g = 0$ we obtain that for any $\nu > 0$

$$\lim_{\varepsilon\to 0} \varepsilon^2 \sup_{x\in\Theta} \ln \mathbf{P}\left(\int_0^T |\widehat{x}^{\varepsilon}_t - x_t|^2 \mathrm{d}t > \nu\right) = -\frac{\nu}{2}\,.$$

This is the best estimator in the sense that for any estimator $(\widetilde{x}_t)_{0\le t\le T}$

$$\lim_{\varepsilon\to 0} \varepsilon^2 \sup_{x\in\Theta} \ln \mathbf{P}\left(\int_0^T |\widetilde{x}^{\varepsilon}_t - x_t|^2 \mathrm{d}t > \nu\right) \ge -\frac{\nu}{2}\,.$$

Conclusion

- The stochastic version of the Tikhonov theorem is shown. The boundary layer and the asymptotic expansions are studied.
- The large deviations methods for the fast variables in the uniform metric and in the metric $\mathbf{L}_2[0, T]$ are developed. The rate functions are found.
- Applications of the stochastic singular perturbation method: statistic of stochastic processes, financial markets, optimal control, PDE analysis.

Thanks

THANK YOU VERY MUCH

FOR YOUR ATTENTION

HAPPY BIRTHDAY YOURI !!!