# Optimal make-take fees for market making regulation.

Thibaut Mastrolia
CMAP, École Polytechnique
Joint with Omar El Euch, Mathieu Rosenbaum and Nizar Touzi.

Innovative Research in Mathematical Finance
(in honour of Yuri Kabanov)
CIRM September 2018.

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Here, we take the position of an exchange how aims at attracting liquidity by giving incentives to one market maker.

# Actors of the market

A market-maker (MM) controls the bid and ask price processes of an asset.

A platform/exchange aims at motivating the MM to increase the liquidity by giving a compensation.

The platform observes but not controlled market-maker's activities.

$\hookrightarrow$ Incentive theory (Principal/Agent framework).

# Related literature

Optimal market making (without incentive policy):

- Avellaneda and Stoikov, *Quantitative Finance*, 2008. A market maker aims at optimizing the utility of her payoff.
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$\hookrightarrow$ No restriction on the set of proposed contracts *via* (2)BSDE.

$\hookrightarrow$ Problem of the Principal is a stochastic control problem with state variables the output and the continuation utility of the Agent (see Sannikov, *The Review of Economic Studies*, 2008).

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  Ask price process $P_t^a := S_t + \delta_t^a$
- Inventory of the Market maker: $Q_t = N_t^b - N_t^a$.

We assume that there exists a critical $\bar{q} \in \mathbb{N}$ such that

$$\lambda_t^a = 0 \text{ if } Q_t \leqslant -\bar{q}, \quad \lambda_t^b = 0 \text{ if } Q_t \geqslant \bar{q}.$$

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$$\lambda_t^a := \lambda\left(\delta_t^a\right)\mathbf{1}_{Q_t > -\overline{q}},\ \lambda_t^b := \lambda\left(\delta_t^b\right)\mathbf{1}_{Q_t < \overline{q}},$$

with $\lambda(x) := Ae^{-\frac{k}{\sigma}(x+c)},\ A, k > 0$.
Dependance on the ratio spread/volatility: see Madhaval, Richardson and Roomans (1994), Wyart, Bouchaud, Kockelkoren, Potters, and M. Vettorazzo (2008) or Dayri and Rosenbaum (2013).

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Let $\mathbb{P}^\delta$ be the probability such that $\widetilde{N}_t^{\delta,a} := N_t^a - \int_0^t \lambda(\delta_s^a)ds$, $\widetilde{N}_t^{\delta,b} := N_t^b - \int_0^t \lambda(\delta_s^b)ds$, are $\mathbb{P}^\delta$ martingales. Moreover, there exists a (true) martingale $\Phi^\delta$ such that $\frac{d\mathbb{P}^\delta}{d\mathbb{P}^0}_{|\mathcal{F}_t} = \Phi_t^\delta$.

# Improvement of the quality of the market

Our work: investigate the impact of an incentive policy from an exchange to a market maker on the quality of the market.

# Market making without incentives

Avellaneda Stoikov. The P&L of the market maker given a strategy $\delta$

$$PL_t^{\delta} := X_t^{\delta} + S_t Q_t,$$

with

- $X_t^{\delta} := \int_0^t P_u^a dN_u^a - \int_0^t P_u^b dN_u^b$
- $S_t Q_t$ is the inventory risk.

**Problem of the market maker:**

$$V_{MM}(0) = \sup_{\delta} \mathbb{E}^{\delta}\left[-\exp\left(-\gamma(PL_T^{\delta} - PL_0^{\delta} + 0)\right)\right].$$

# Market making under incentive policy

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Problem of the exchange:

$$V_0^E = \sup_\xi \mathbb{E}^{\hat{\delta}(\xi)} \left[ -e^{-\eta\left(c(N_T^a - N_0^a + N_T^b - N_0^b) - \xi\right)} \right]$$

# Admissible contracts and characterization

- **Admissible contracts.** For some $\eta' > \eta, \ \gamma' > \gamma$

$$\mathcal{C} = \left\{ \xi, \quad \sup_{\delta \in \mathcal{A}} \mathbb{E}^{\delta}\left[ e^{\eta' \xi} \right] < +\infty, \quad \sup_{\delta \in \mathcal{A}} \mathbb{E}^{\delta}\left[ e^{-\gamma' \xi} \right] < +\infty, \quad V_{MM}(\xi) \geqslant R \right\},$$

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- **Smooth contracts.** Let $Y_0 \in \mathbb{R}$, and $Z = (Z^S, Z^a, Z^b)$,

$$Y_t^{Y_0,Z} = Y_0 + \int_0^t \left(Z_r^a dN_r^a + Z_r^b dN_r^b + Z_r^S dS_r - H(Z_r, Q_r)\right) dr,$$

where $Z$ is such that conditions holds with $\xi = Y_T^{0,Z}$.

$$\Xi = \left\{Y_T^{Y_0,Z} : \ Y_0 \in \mathbb{R},\ Z, \text{ and } V_{\mathrm{MM}}(Y_T^{Y_0,Z}) \geqslant R\right\}.$$

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$\hookrightarrow$ Choice of $H$ is such that we have an explicit solution of the MM's problem in the class $\Xi$.

# Problem of the market maker for a fixed compensation

$$h(\delta, z, q) \quad = \quad \frac{1 - e^{-\gamma(z^a + \delta^a)}}{\gamma} \lambda(\delta^a) \mathbf{1}_{\{q > -\bar{q}\}} + \frac{1 - e^{-\gamma(z^b + \delta^b)}}{\gamma} \lambda(\delta^b) 1_{\{q < \bar{q}\}},$$

and

$$H(z, q) \quad = \quad \sup_{|\delta^a| \vee |\delta^b| \leqslant \delta_\infty} h(\delta, z, q) - \frac{1}{2}\gamma\sigma^2(z^S + q)^2,$$

## Theorem

(ii) *For any $\xi \in \Xi$, the market maker utility value is*

$$V_{\mathrm{MM}}(\xi) = -e^{-\gamma Y_0}, \; \Xi \; = \; \left\{ Y_T^{Y_0, Z} : \; Y_0 \geqslant \frac{-1}{\gamma} \log\left(-R\right) \right\},$$

*with optimal bid-ask policy ($i \in \{a, b\}$)*

$$\hat{\delta}_t^i(\xi) = \Delta(Z_t^i), \; \textit{where } \Delta(z) = (-\delta_\infty) \vee \left\{ -z + \frac{1}{\gamma} \log\left(1 + \frac{\sigma\gamma}{k}\right) \right\} \wedge \delta_\infty.$$

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## Theorem

(i) *Any contract $\xi \in \mathcal{C}$ has a unique representation as $\xi = Y_T^{Y_0,Z}$, for some $(Y_0, Z)$.* ***In particular, $\mathcal{C} = \Xi$.***

(ii) *Under this representation, the market maker utility value is*

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# Exchange's problem

**We now turn to the problem of the exchange given the optimal spread of the market maker.**

# Exchange's problem

**Problem of the Exchange:**

$$V_0^E := \sup_{\xi} \mathbb{E}^{\hat{\delta}} \left[ -e^{-\eta\left(c(N_T^a - N_{\mathbf{0}}^a + N_T^b - N_{\mathbf{0}}^b) - \xi\right)} \right]$$

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becomes

$$V_0^E = e^{\eta Y_0^\star} \sup_{Z} \mathbb{E}^{\delta^\star} \left[ -e^{-\eta\left(c(N_T^a - N_0^a + N_T^b - N_0^b) - Y_T^Z\right)} \right]$$

with

$$Y_t^Z = \int_0^t Z_u^S dS_u + \int_0^t Z_u^a dN_u^a + \int_0^t Z_u^b dN_u^b - \int_0^t H(Z_u, q_u) du.$$

# Intuitive HJB equation

Applying the standard dynamic programming approach, we are led to the HJB equation

$$\begin{cases} \partial_t v(t,q) + H_E\big(q, v(t,q), v(t,q+1), v(t,q-1)\big) \; = \; 0, \; t \in (0,T], \\ v(T,q) = -1, \\ q \in \{-\bar{q}, \cdots \bar{q}\}, \end{cases}$$

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With a change of variable, we obtain a linear differential equation that can be solved explicitly (linearization inspired by Guéant et al.)

More exactly...

# HJB equation and linearization

... by setting $u := (-v)^{-\frac{k}{\sigma\eta}}$, we have to solve

$$\begin{cases} \partial_t u(t,q) - C_1 q^2 u(t,q) + C_2 \left( u(t,q+1)\mathbf{1}_{\{q<\bar{q}\}} + u(t,q-1)\mathbf{1}_{\{q>-\bar{q}\}} \right) \; = \; 0, \\ u(T,q) = 1, \end{cases}$$

with $C_1 = \frac{k\gamma\eta\sigma}{2(\gamma+\eta)}$ and $C_2 = C\frac{k}{\sigma\eta}$.

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By considering $\mathbf{u}(t) = \big(u(t,q)\big)_{q\in\{-\bar{q},\dots,\bar{q}\}}$, it remains to solve the linear ODE

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It can be solve explicitly and we notice in particular that

$$e^{-C_1\bar{q}^2 T} \leqslant u \leqslant e^{2C_2 T}.$$

# The exchange's problem *via* verification

**Theorem**

*The optimal contract for the problem of the exchange is given by*

$$\hat{\xi} = Y_0^{\star} + \int_0^T \hat{Z}_r^a dN_r^a + \hat{Z}_r^b dN_r^b + \hat{Z}_r^S dS_r + \Big(\frac{1}{2}\gamma\sigma^2\big(\hat{Z}_r^S + Q_r\big)^2 - H\big(\hat{Z}_r, Q_r\big)\Big) dr,$$

*with* $\hat{Z}^s = -\frac{\gamma}{\gamma+\eta} Q$, *and*

$$\hat{Z}^a = \zeta_0 + \underbrace{\frac{1}{\eta}\log\left(\frac{v(\cdot, Q)}{v(\cdot, Q-1)}\right)}_{\text{bounded}}, \text{ and } \hat{Z}^b = \zeta_0 + \underbrace{\frac{1}{\eta}\log\left(\frac{v(\cdot, Q)}{v(\cdot, Q+1)}\right)}_{\text{bounded}},$$

*where*

$$\zeta_0 = c + \frac{1}{\eta}\log\left(1 - \frac{\sigma^2\gamma\eta}{(k+\sigma\gamma)(k+\sigma\eta)}\right).$$

*The market maker's optimal effort is given by*

$$\hat{\delta}_t^a = \hat{\delta}_t^a(\hat{\xi}) = -\hat{Z}_t^a + \frac{1}{\gamma}\log(1+\frac{\sigma\gamma}{k}), \quad \hat{\delta}_t^b = \hat{\delta}_t^b(\hat{\xi}) = -\hat{Z}_t^b + \frac{1}{\gamma}\log(1+\frac{\sigma\gamma}{k}).$$

# Discussion on the incentive contract

From the (probabilistic) expression of $v$ from the linearization, roughly speaking:

- For $\hat{Z}^a$: $\log\left(\frac{v(t,q_t)}{v(t,q_t-1)}\right) \sim Q_t$, incentive to attract buy market orders for large inventory.

- For $\hat{Z}^b$: $\log\left(\frac{v(t,q_t)}{v(t,q_t+1)}\right) \sim -Q_t$, converse effect.

- $\hat{Z}^S$: risk sharing w.r.t. the inventory.

# Fixing $c$

From numerical computations

$$\frac{u(t,q)^2}{u(t,q-1,u(t,q+1)} \approx 1,\ \forall (t,q).$$

We take $c$ so that the optimal spread $\hat{\delta}^a + \hat{\delta}^b$ is close to 1 Tick.
We get for $\frac{\sigma\gamma}{k}$ small enough,

$$c \approx \frac{\sigma}{k} - \frac{1}{2}\text{Tick}.$$

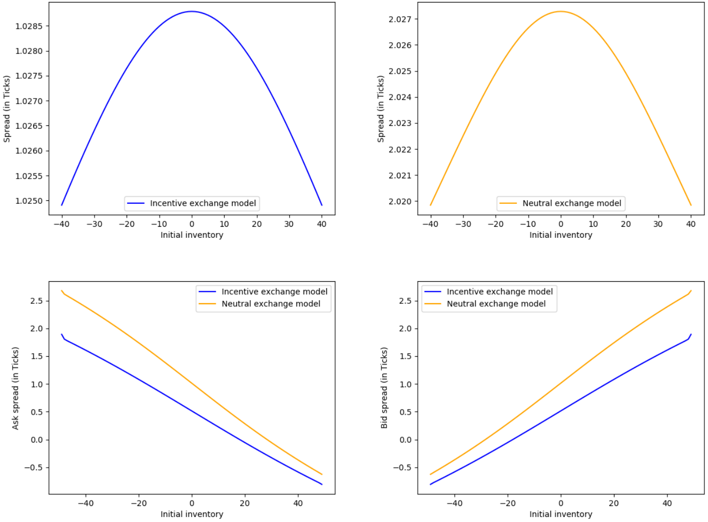
$\hookrightarrow$ can be used in practice.

# Comparison of models with/without incentive policy

**We investigate numerically the impact of the incentive policy on the quality of the market.**

# Comparison of models with/without incentive policy

Optimal initial spreads w.r.t. initial inventory.



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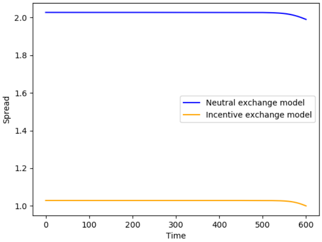
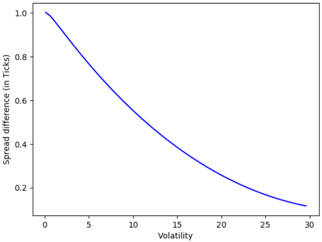
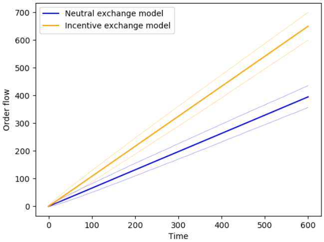
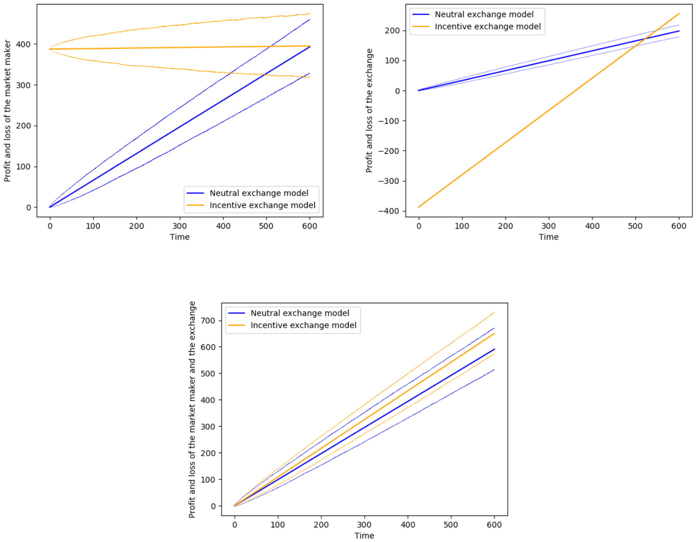


# Comparison of models with/without incentive policy

Impact of the incentive policy on the P&L of the platform and market maker.



Thank you and happy birthday Yuri.