When Capital is a Funding Source: The XVA Anticipated BSDEs

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Albanese, C., S. Caenazzo, and S. Crépey (2017). Credit, funding, margin, and capital valuation adjustments for bilateral portfolios. *Probability, Uncertainty and Quantitative Risk 2*(7), 26 pages.

Introduction

- XVAs: Add-ons (or rebates) with respect to the counterparty-risk-free value of financial derivatives, meant to account for counterparty risk and its capital and funding implications.
- VA stands for valuation adjustment and X is a catch-all letter to be replaced by C for credit, D for debt, F for funding, M for margin, and K for capital.

Outline

1. The XVA equations are well posed, including in the realistic case where capital can be used for funding variation margin

2. XVA nested Monte Carlo computational strategies

- Consider a bank engaged into bilateral trading with a client, with final maturity of the portfolio T.
- Let MtM (mark-to-market) denote the counterparty risk free value of the client portfolio of the bank, counted positively when positive to the bank
- Let R_c denote the recovery rate of the client in case it defaults at time τ_c.
- Let VM denote the variation margin from the client to the bank
    - collateral guarantee tracking the mark-to-market of the portfolio, assumed remunerated at the risk-free rate by the receiving party
- Let λ denote the funding spread of the bank

CVA and FVA in the base case without capital at risk

- Pricing stochastic basis $(\mathbb{F}, \mathbb{P})$ with risk-neutral discount factor β
- "Pricing cash flows by risk-neutral discounted expectation" yields, for $0 \le t \le T$:

$$
\begin{aligned}
\mathrm{CVA}_t &= \mathbb{E}_t\Big[\mathbb{1}_{\{t<\tau_c\le T\}}\beta_t^{-1}\beta_{\tau_c}\times \\
&\qquad (1-R_c)\big(\mathrm{MtM}_{\tau_c}-\mathrm{VM}_{\tau_c}\big)^+\Big], \\
\mathrm{FVA}_t &= \mathbb{E}_t\int_t^{\tau_c\wedge T}\beta_t^{-1}\beta_s\times \\
&\qquad \lambda_s\Big(\mathrm{MtM}_s-\mathrm{VM}_s-\mathrm{CVA}_s-\mathrm{FVA}_s\Big)^+ ds.
\end{aligned}
$$

- The above (and what follows) can be readily extended to several clients, initial margin, positive liquidation times, other (such as centrally cleared) trading setups, etc.

Capital at risk and capital valuation adjustment

- The capital at risk (CR) of the bank is dynamically modeled as the conditional expected shortfall ($\mathbb{ES}_t$) at some quantile level a ($= 97.5\%$) of the one-year-ahead trading loss L of the bank, i.e., also accounting for discounting:

$$\mathrm{CR}_t = \mathbb{ES}_t^a\Big(\int_t^{t+1} \beta_t^{-1}\beta_s dL_s\Big).$$

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Remark 1

Assuming a constant hurdle rate h, the amount needed by the bank to remunerate its shareholders for their capital at risk in the future is

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^T e^{-\int_t^s (r_u+h)du} \mathrm{CR}_s ds\,, \quad t \in [0, T]. \tag{1}$$

Proof.

- A constant hurdle rate h means (as the KVA itself is loss-absorbing)

$$-\, d\mathsf{KVA}_t + r_t \mathsf{KVA}_t dt = h(\mathrm{CR}_t - \mathsf{KVA}_t)dt - d\mathsf{Mart}_t, \tag{2}$$

  i.e., setting $\bar{\beta}_t = e^{-\int_0^t (r_s+h)ds}$

$$-d(\bar{\beta}_t \mathsf{KVA}_t) = h\bar{\beta}_t \mathrm{CR}_t dt - \bar{\beta}_t d\mathsf{Mart}_t$$

- Added to a terminal condition $\mathsf{KVA}_T = 0$, this is equivalent to

$$\bar{\beta}_t \mathsf{KVA}_t = \mathbb{E}_t \int_t^T \bar{\beta}_s h \mathrm{CR}_s ds,$$

  which is (1). ∎

What is L?

- Assume market risk replicated by the bank but no XVA hedge
- Setting $Q = \mathrm{MtM} - \mathrm{VM}$ and $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$, we obtain

$$
\begin{aligned}
&L_0 = z \text{ (arbitrary, set to } \mathrm{CA}_0 \text{ henceforth) and, for } t \in (0, T],\\
&dL_t = d\mathrm{CA}_t - r_t \mathrm{CA}_t dt + (1 - R_c) Q^+_{\tau_c} \boldsymbol{\delta}_{\tau_c}(dt)\\
&\qquad\qquad + \lambda_t \big(\mathbb{1}_{\{t \le \tau_c\}} Q_t - \mathrm{CA}_t\big)^+ dt.
\end{aligned} \tag{3}
$$

$\rightarrow$ $L =$ martingale part of $(\mathrm{CA} + (1 - R_c) Q^+ \boldsymbol{.} \boldsymbol{\delta}_{\tau_c})$, where $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$

Using capital at risk as variation margin

- Accounting for the possibility for a bank to use capital at risk as variation margin (VM), the variation margin funding needs, i.e. the drift of the FVA BSDE (essentially), are diminished from $\left(\mathbb{1}_{\{t \leq \tau_c\}} Q_t - \mathrm{CA}_t\right)^+$ to

$$\left(\mathbb{1}_{\{t \leq \tau_c\}} Q_t - \mathrm{CA}_t - \mathrm{CR}_t\right)^+,$$

where

$$\mathrm{CR}_t = \mathbb{ES}_t^a\left(\int_t^{t+1} \beta_t^{-1} \beta_s dL_s\right),$$

- But "$L =$ martingale part of $(\mathrm{CA} + (1 - R_c) Q^+ \bullet \boldsymbol{\delta}_{\tau_c})$, where $\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA}$"

$\rightarrow$ FVA anticipated BSDE "of the McKean type" in the line of

Peng, S. and Z. Yang (2009). Anticipated backward stochastic differential equations. *The Annals of Probability 37*(3), 877–902.

- See also Agarwal, Marco, Gobet, López-Salas, Noubiagain, and Zhou (2018) in relation with their modeling of the so-called initial margin.

- And even $(\mathrm{FVA}, \mathrm{KVA})$ ABSDE system, as the KVA is actually part of capital at risk (CR), hence CR_t needs in fact be replaced by $\mathrm{CR}_t = \max\big(\mathbb{ES}^a_t(\int_t^{t+1} \beta_t^{-1}\beta_s dL_s), \mathrm{KVA}_t\big)$ everywhere in the above.

Theorem 1

The XVA equations are well-posed, including in the realistic case where capital is fungible with variation margin.

Proof. We extend the main result in Peng and Yang (2009) to jumps (for client defaults), monotone coefficients (as interest rates are only bounded from below in most models), and conditional expected shortfall (instead of conditional expectation) anticipated dependence. ∎

What is λ?

- "Credit and/or liquidity"
- But liquidity spreads are typically in the order of a handful of basis points while banks funding spreads can run into the hundreds of basis points
- If "credit mainly", it seems we forgot (at least) half of the story
- Banks are themselves risky and this is precisely the reason why we saw all these regulatory changes
- What then about DVA etc..?

Adding bank default

- In the realistic case of a defaultable bank, we need to account for the discrepancy between the so called clean pricing model used by the base traders of the bank, who ignore the default of the bank itself, and the pricing model of the XVA traders, that have the default of the bank in mind
- We also need to stop all equations 'before the bank default' in order to be aligned with the interest of bank shareholders, who have the decision power as long as the bank is nondefault

Unilateral versus bilateral XVAs

Theorem 2

Ultimately same equations (hence well posed) as before after reduction of all XVA equations 'stopped before the bank default' to the clean pricing model (filtration and pricing measure).

- Hence, even though our setup includes the default of the bank itself, we end up with unilateral CVA, FVA and KVA (pre-default) formulas [and DVA is irrelevant] pricing the related cash flows until the final maturity T of the portfolio
  - As opposed to $\bar{\tau} = \tau \wedge T$
  - And these equations only involve the risk-free discount factor, without any credit spread.
- In particular, our approach is therefore consistent with the regulatory requirement that the reserve capital $(\mathrm{CA} = \mathrm{CVA} + \mathrm{FVA})$ of a bank should not diminish simply because of a deterioration of the bank credit spread.

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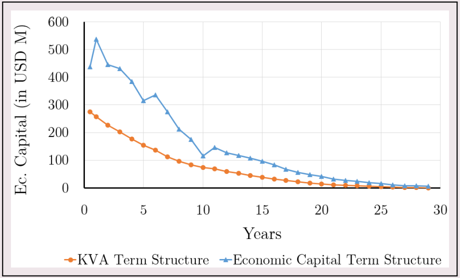
Real portfolio study (Albanese, Caenazzo, and Crépey (2017))

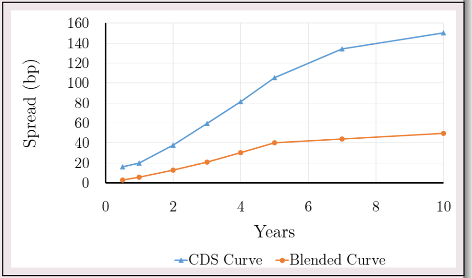
- Representative banking portfolio with about 2,000 counterparties, 100,000 fixed income trades including swaps, swaptions, FX options, inflation swaps and CDS trades.
- $\mathrm{VM} = 0$.

Representative banking portfolio XVA values.

XVA	$Value
CVA_0	242 M
$FVA_0^{(0)}$	126 M
FVA_0	62 M
KVA_0	275 M
FTDCVA	194 M
FTDDVA	166 M

Left: Term structure of economic capital compared with the term structure of KVA.
Right:FVA blended funding curve computed from the ground up based on capital projections.





Thanks for your attention!

Agarwal, A., S. D. Marco, E. Gobet, J. López-Salas, F. Noubiagain, and A. Zhou (2018). Numerical approximations of McKean anticipative backward stochastic differential equations arising in initial margin requirements. https://hal.archives-ouvertes.fr/hal-01686952v2.

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