On Overconfidence, Bubbles and the Stochastic Discount Factor

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Innovative Research in Mathematical Finance
in honor of the 70th birthday of Yuri Kabanov,
The Cirm Center, Luminy.

When Yuri meets Louis...

- x random variable.
  1. Δx limit of a discrete-time random walk (Bachelier, 1900).
  2. A discrete-time ordered-random-walk specified in solvency cone (Kabanov-Lepinette, 2014).
- Mathematical expectation: dx/dt does not exist. We cannot compute $E(dx/dt)$. We can compute $E(dx)$ and $(\frac{1}{dt})E(dx)$.
- (Ito) Continuous but not differentiable: Written as taylor series expansion. $dx = \alpha(x,t)dt + \beta(x,t)dz$ where $\alpha(x,t)$ and $\beta(x,t)$ are not randomed, dz is the wiener process.
- The convexity approach, portfolio dynamics, super-hedging ...

Overpricing .

Undefined Financial Object (UFO) in financial crisis.

- Financial crises: a loss of paper wealth,
  not necessarily result in changes in the real economy.

> A speculative bubble exists in the event of large, sustained overpricing of stock and other assets .

- Induction: a base and steps climbing to measure fundamentals.
  (metaphor: fundamentals as a mountain)
  (another example: falling dominos).
- Overpricing: which is not equivalent to fundamentals.

Literature

- No Arbitrage in frictionless markets (Ross '49, Harrison and Krep '34): Non-negative payoffs, strictly positive price.
- The law of one price (Hansen '16): Non-negative portfolio payoffs (violation of No Arbitrage).
- ex. 2 players, same payoffs but the price of portfolio is different.
- The intermediate trading of (Hansen-Scheinkman '09).

A model.

- Consider a continuous-time Markov process, filteration by its histories,
- A representation of pricing without an operator in a differential form.
- Any admissible payoff: Dividends.
- Zero prices from intermediate trading by a bounded random variable available to the investor at the intermediate date.
- One of two groups with overconfidence.

Highlight

- Volatility: showing mean-reverting, implying non-zero valued fundamental prices.
  1. Mixed induction:
     - (1st-order) prior info, mean-reverting,
     - (2nd-order) an interval of overconfidence.
  2. Overconfidence: $\phi = 0$ or $\phi = 1$ (extended result from Scheinkman-Xiong (2003).
  3. Two groups: who can be speculators? Investors α vs. Traders β ? $\alpha \succ \beta$.
- Contraction: in belief, coordination of expectation implying the analysis about "economic drivers", ($\neq$ perfect prediction implying "no errors of data" with several scenarios).

Conjectures

1. The value of bubbles cannot be zero? overpricing
2. High trading volume driven by investors (a 2-player game) eventually makes the end of bubbles?
3. Effects of the crash can be measured by mixed induction?

Time, Crash and Displacement.

A model in the continuous *TIME*

- Dividends as fundamentals
  - Instantaneous total return: $\frac{dp_t}{p_t} + \frac{D_t}{p_t} dt$,
    where p_t is price and D_t is instantaneous rate of dividend.
  - No Div. at the deterministic rate where $p = 1, D_t = r_t^f$:
    $\frac{dp_t}{p_t} = r_t^f dt$.

Cumulative dividend process (Scheinkman Xiong (2003)

$$dD_t = Q f_t dt + \sigma_D dZ_t^D$$

where

1. f is the fundamental random variable not depending on dividend noise of the standard Brownian motion Z^D of a constant volatility parameter σ_D,

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CRASH

Inducing the specific model from Ornstein–Uhlenbeck process (AR): $df_t = \eta - \lambda(f_t - \overline{f}))e^{\eta t} f_t dt + \sigma f_t dZ$ with mean reversion where the trend-line parameter η equals zero.

1. taking the value $df_t = -\lambda(f_t - \overline{f})dt + \sigma_f dZ_t^f$,
2. given mean reversion (moving to average) $\lambda \geq 0$, $\overline{f}$ denotes long-run mean of f.

- Extrapolation (on past returns) and Displacement (at bullish news (Barberis et al, 2018).

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Stationarity

Definition

(Stationarity γ)

$$\gamma \equiv \frac{\sqrt{(Q\lambda + \phi\frac{\sigma_f}{\sigma_s})^2 + (1-\phi^2)(2\frac{\sigma_f^2}{\sigma_s^2} + \frac{\sigma_f^2}{\sigma_D^2})} - (Q\lambda + \phi\frac{\sigma_f}{\sigma_s})}{\frac{1}{\sigma_D^2} + \frac{2}{\sigma_s^2}}$$

Mean reversion is equivalent to stationarity γ in mean is re-written with *mutually independant* four standard Brownian motions

$$Z^D, Z^f, Z^A, Z^B$$

denoted as cumulative dividend process D, Fundamental f, Signal of group A, Signal of group B.

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Stationarity

Lemma

(Variance of Stationary)
The variance of Stationarity γ *decreases with* ϕ.

- Agents attribute to their own forecast of the current level of fundamentals as a larger overconfidence ϕ increases.

(stationary of Rogers and Williams (1987),
Lipster and Shiryaev (1977).

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DISPLACEMENT, An operator approach

Theorem 1: Bounded Rationality of fundamental traders

1. Mean reversion λ doesn't work to maintain the variance of stationarity γ (implying "burst").
2. Threshold of fundamental oscillation $Q\lambda$ is captured by the noise movement between $B(= \phi\sigma_f/\sigma_s)$ and $-(\sigma_f/\sigma_s)$.

$$\text{hence, } -\sigma_f/\sigma_s < Q\lambda < B(= \phi\sigma_f/\sigma_s),$$

(A similar result to expected demand function approach of Barberis et al (2018)

(1) Volatility

- The variance of 10-year returns should be ten times the variance of 1-year return?
- The variance of short-horizon returns will be proportionally higher (Daniel, 2001).

(2) Belief Contraction

- **Perfect prediction**: mean-reversion, temporary shocks in prices and dividends are predictable by random walks of prices.
- Expectation of two heterogenous groups - investors having bigger assets than traders, and traders.
- **Coordination** of expectations in the one value function of dividends: linear utility? (of two heterogeneous groups).

Definition

The overconfidence parameter ϕ increases as a larger ϕ increases, agents attribute to their own forecast of the current level of fundamentals where $0 < \phi < 1$.

Aggregation of Belief, is it the same?

Assume that heterogeneous beliefs offer joint dynamics of the D, f, s^α, s^β. All agents observe a vector of signals s^α and s^β that satisfy:

$$\text{remind: } dD_t = Qf_t dt + \sigma_D dZ_t^D$$

$$\Downarrow$$

$$ds_t^\alpha = Q^\alpha f_t dt + \sigma_s dZ_t^\alpha$$

$$ds_t^\beta = Q^\beta f_t dt + \sigma_s dZ_t^\beta$$

Agents of group $\alpha(\beta)$ believe that innovations dZ^α (dZ^β) in the signal $s^\alpha(s^\beta)$ are correlated with the innovation dZ^f in the fundamental process, with ϕ $(0 < \phi < 1)$ as the correlation parameter

Overconfidence with two heterogeneous groups

Lemma

(Two Groups) Agents of group $\alpha(\beta)$ *believe that innovations* $dZ^{\alpha}(dZ^{\beta})$ *in the signal* $s^{\alpha}(s^{\beta})$ *are correlated with the innovations* dZ^{f} *in the fundamental process, with* $\phi(0 < \phi < 1)$ *as the correlation parameter. Assumed that if* $dZ_t^{\alpha} = dZ_t^{f}$, *then* $\phi = \{0, 1\}$ *such that*

$$(1 - \sqrt{1 - \phi^2})dZ_t^{\alpha} \equiv \phi \cdot dZ_t^{f} \quad (1)$$

- Each group reacts to the value of fundamentals.

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Overconfidence with two heterogeneous groups

Lemma

(Disagreements) The overconfidence parameter ϕ generates disagreements among two heterogeneous groups regarding asset fundamentals $dS_t^\alpha - \sigma_s dZ_t^\alpha$ such that

$$\phi \equiv \frac{dS_t^\alpha - f_t dt - \sigma_s\sqrt{1-\phi^2}dZ_t^\alpha}{\sigma_s dZ_t^f}. \quad (2)$$

- $\phi = 0$ or $\phi = 1$, what it can imply?

(2/2 pages)

A benchmark belief

Two signals from two heterogeneous groups

$$ds_t^{\alpha} = Q^{\alpha} f_t d_t + \sigma_s \phi dZ_t^{f_t} + \sigma_s \sqrt{1-\phi^2} dZ_t^{\alpha}$$
$$ds_t^{\beta} = Q^{\beta} f_t d_t + \sigma_s \phi dZ_t^{f_t} + \sigma_s \sqrt{1-\phi^2} dZ_t^{\beta}$$

Regardless of "overconfidence =0"

The result in detail

Theorem

Let ds_t^α and ds_t^β be two signal processes according to each belief of the group α and the group β. If overconfidence $\phi = 1$, then the bubble at the equilibrium exists at the fundamental value given prior information.

Proof.

$$g^\alpha = Q^\beta \hat{f}^\beta - Q^\alpha \hat{f}^\alpha,\ g^\beta = Q^\alpha \hat{f}^\alpha - Q^\beta \hat{f}^\beta = -g^\alpha$$
$$ds_t^\alpha = Q^\alpha f_t dt + \sigma_s dZ_t^f,\ ds_t^\beta = Q^\beta f_t dt + \sigma_s dZ_t^f$$
$$ds_t^\alpha - ds_t^\beta = (Q^\alpha - Q^\beta) f_t dt$$

□

How can we prove one overconfident group among two groups

Theorem

The overconfident group α: If the expected operator denoted as Q is in the interval $[0,1]$, then the group α is overconfident.

Proof.

We assume that two heterogeneous groups are involved in trading such that $\alpha \neq \beta$: $\frac{ds_t^\alpha - ds_t^\beta}{f_t dt} = Q^\alpha - Q^\beta$ where the fundamental payoff f_t at t is not revealed. If $f_t dt \to \infty$ and $ds_t^\alpha - ds_t^\beta \to \infty$, then $Q^\alpha - Q^\beta = 1$
by the assumption of $0 \leq Q \leq 1$, $Q^\beta = 0 \Rightarrow Q^\alpha = 1$. □

Happy Birthday, Yuri.