IFRENCH-GERMAN MEETING ON COMPLEX ALGEBRAIC GEOMETRY

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Abstracts

Ana-Maria Brecan

Defomation theory of twistor spaces of K3 surfaces

Abstract: Twistor spaces of K3 surfaces are non-Kähler compact complex manifolds which play a fundamental role in the moduli theory of K3 surfaces. They come equipped with a holomorphic submersion to the complex projective line which under the period map corresponds to a twistor line in the K3-period domain. In this talk I will explain how one can view a twistor line as a certain base point in the linear cycle space of the period domain. Then, based on joint work in progress with Daniel Greb, Tim Kirschner and Martin Schwald I will present new results concerning the deformations of twistor spaces of K3 surfaces and their relation to the cycle space of the period domain.

Frédéric Campana

Orbifold Rational Connectedness

Abstract: We define the notion of 'slope rational connectedness' (sRC for short) for smooth complex projective orbifold pairs (X,D) (and thus in particular for quasi-projective manifolds) by requiring the negativity, in a suitable sense, of their cotangent bundles. When D=0, we recover the classical notion of rational connectedness. Conjecturally, sRC for (X,D) should still coincide with the connectedness of X by 'D-rational curves', naturally defined. We also show, among other things, the existence of a 'rational quotient' (ie: MRC fibration) in this orbifold context, as well as the fact that klt Fano orbifolds are sRC, this being in general false in the lc case.

Olivier Debarre

Periods of polarized hyperkähler manifolds

Abstract: Hyperkähler manifolds are higher-dimensional analogs of K3 surfaces. Verbitsky and Markmann recently proved that their period map is an open embedding. In a joint work with E. Macri, we explicitly determine the image of this map in some cases. I will explain this result together with a nice application (found by Bayer and Mongardi) to the (almost complete) determination of the image of the period map for cubic fourfolds, hereby partially recovering a result of Laza.

Jean-Pierre Demailly
Improved bounds for the Kobayashi conjecture on generic hyperbolicity

Abstract: A famous conjecture of Kobayashi from the 1970s asserts that a generic algebraic hypersurface of sufficiently large degree $d \geqslant d_n$ in the complex projective space of dimension $n+1$ is hyperbolic. Yum-Tong Siu introduced several fundamental ideas that led recently to a proof of the conjecture. In 2016, Damian Brotbek gave a new geometric argument based on the use of Wronskian operators and on an analysis of the geometry of Semple jet bundles. Shortly afterwards, Ya Deng obtained effective degree bounds by means of a refined technique. Our goal here will be to explain a drastically simpler proof that yields an improved (though still non optimal) degree bound, e.g. $d_n = [(en)^{2n+2}/5]$. We will also present a more general approach that could possibly lead to optimal bounds.

Stéphane Druel
Codimension one foliations with numerically trivial canonical class on singular spaces

Abstract: In this talk, I will describe the structure of codimension foliations with canonical singularities and numerically trivial canonical class on varieties with terminal singularities, extending a result of Loray, Pereira and Touzet to this context.

Gavril Farkas
Fundamental groups, Chow forms and syzygies of canonical curves

Abstract: I will discuss an algebraic statement concerning the vanishing of the Koszul modules associated to any subspace K inside the second exterior product of a complex vector space. This statement, which turns out to be equivalent to Mark Green's Conjecture on syzygies of canonical curves (proven by Claire Voisin), has many interesting topological applications of which I will discuss (1) a universal upper bound on the nilpotence index of the fundamental group of any compact Kaehler manifold and (2) a bound on the length of the nilpotence index on the Torelli groups associated to the moduli space of curves and (3) an explicit description of the Cayely-Chow form of the Grassmannian G(2,n).
This is joint work with Aprodu, Papadima, Raicu and Weyman.

Patrick Graf
The Lipman–Zariski conjecture for low-genus surface singularities

Abstract: The Lipman–Zariski conjecture asserts that a complex algebraic variety with locally free tangent sheaf (i.e. locally free module of derivations) is necessarily smooth. I will explain why it suffices to consider normal surface singularities, and then prove the conjecture for surfaces whose singularities are "not too far" from being rational, in the sense that their geometric genus is low. As an application, I will give several global statements. For example, a normal compact complex surface whose smooth locus has trivial tangent bundle is already smooth (and hence it is a complex 2-torus).

Vincent Guedj

Geometry and topology of the space of Kaehler metrics on singular varieties

Abstract: Let Y be a compact Kaehler normal space. We study metric properties of the space of Kaehler metrics in a fixed Kaehler class using Mabuchi geodesics. We extend several results by Calabi, Chen, Darvas and Tian previously established when the underlying space is smooth. As an application we analytically characterize the existence of Kaehler-Einstein metrics on Q-Fano varieties.
This is a joint work with E. Di Nezza.

Victoria Hoskins

Group actions on quiver varieties and applications

Abstract: We study two types of actions on King's moduli spaces of quiver representations over a field k, and we decompose their fixed loci using group cohomology in order to give modular interpretations of the components. The first type of action arises by considering finite groups of quiver automorphisms. The second is the absolute Galois group of a perfect field k acting on the points of this quiver moduli space valued in an algebraic closure of k; the fixed locus is the set of k-rational points, which we decompose using the Brauer group of k, and we describe the rational points as quiver representations over central division algebras over k. Over the real numbers, we have two types of rational points arising from real and quaternionic quiver representations. Over the complex numbers, we describe the symplectic and holomorphic geometry of these fixed loci in hyperkaehler quiver varieties using the language of branes.
This is joint work with Florent Schaffhauser.

Klaus Hulek

The Picard numbers of abelian varieties

Abstract: If A is a g-dimensional abelian variety then its Picard number $\rho(A)$ lies in the interval $[1, g^2]$. In this talk I will discuss the question what one can say about the distribution of the possible Picard numbers within this interval for given genus g and how this behaves asymptotically as g goes to infinity.
This is joint work with R. Laface.

Stefan Kebekus

Nonabelian Hodge correspondences for klt varieties and quasi-étale uniformisation

Abstract: Simpson's classic nonabelian Hodge correspondence establishes an equivalence of categories between local systems on a projective manifold, and certain Higgs sheaves on that manifold. This talk surveys recent generalisations of Simpson's correspondence to the context of projective varieties with klt singularities. Perhaps somewhat surprisingly, these spaces exhibit two correspondences: one pertaining to local systems on the whole space, and one to local systems on its smooth locus. As one application, we resolve the quasi-étale uniformisation problem for minimal varieties of general type, and to obtain a complete numerical characterisation of singular quotients of the unit ball by discrete, co-compact groups of automorphisms that act freely in codimension one.

Bruno Klingler

o-minimal geometry and period maps

Abstract: Any variation of Hodge structures on a smooth complex quasi-projective variety S is fully described by its associated period map. In this talk, I will explain that the geometry of this map is tame, in the precise sense of o-minimal geometry. As an immediate corollary one recovers the result of Cattani-Deligne-Kaplan that Hodge loci are algebraic.

Alex Küronya

Newton-Okounkov bodies, singular divisors, and syzygies of low-dimensional abelian varieties

Abstract: Syzygies of projective varieties are invariants coming from minimal free resolutions of homogeneous coordinate rings, which reveal a lot about the geometry of the variety in question. Here we study syzygies of abelian surfaces and threefolds. By the work of Green, Inamdar, and Lazarsfeld-Pareschi-Popa, the study of syzygies of abelian varieties can be reduced to the construction of singular divisors. The existence of effective divisors with prescribed singularities and numerical behaviour is a very important question in algebraic geometry in general with many powerful applications like the classical theorems of MMP and major positivity theorems. The purpose of this talk is to show how to use Newton-Okounkov bodies, that is, convex bodies associated to divisors, to show the existence of effective divisors with a given numerical equivalence class and multiplier ideal. Such results then lead to strong statements about syzygies of abelian varieties of low dimension.

Vladimir Lazić

Generalising abundance

Abstract: I will discuss a statement which generalises known nonvanishing and semiampleness conjectures from various contexts.
This is joint work with Thomas Peternell.

Christian Lehn

A global Torelli theorem for singular symplectic varieties

Abstract: Verbitsky's Global Torelli theorem has been one of the most important advances in the theory of holomorphic symplectic manifolds in the last years. In a joint work with Ben Bakker (University of Georgia) we prove a version of the Global Torelli theorem for singular symplectic varieties and discuss applications. Symplectic varieties have interesting geometric as well as arithmetic properties, their birational geometry is particularly rich. Our results are obtained through the interplay of Hodge theory, deformation theory, and a further example of Verbitsky's technique which might go under the name "how to deduce beautiful consequences from ugly behavior of moduli spaces".

Gianluca Pacienza

Density of Noether-Lefschetz loci of polarized irreducible holomorphic symplectic varieties and applications

Abstract: We prove a density statement for Noether-Lefschetz loci inside the moduli space of marked, polarized irreducible holomorphic symplectic (IHS) varieties and present some applications.
This is joint work with Giovanni Mongardi.

Alessandra Sarti

Nikulin configurations on Kummer surfaces

Abstract: A Nikulin configuration is the data of 16 disjoint smooth rational curves on a K3 surface. According to results of Nikulin this means that the K3 surface is a Kummer surface and the abelian surface in the Kummer structure is determined by the 16 curves. An old question of Shioda is about the existence of non isomorphic Kummer structures on the same Kummer K3 surface. The question was positively answered and studied by several authors, and it was shown that the number of non-isomorphic Kummer structures is finite, but no explicit geometric construction of such structures was given. In the talk I will show how to construct explicitely non isomorphic Kummer structures on generic Kummer K3 surfaces.
This is a joint work with X. Roulleau.

Valentino Tosatti

Semipositive line bundles and $(1,1)$***-classes***

Abstract: I will discuss the general problem, first considered by Fujita, of deciding whether a holomorphic line bundle (or more generally a (1,1) cohomology class) on a compact complex manifold admits a smooth Hermitian metric with semipositive curvature (resp. a smooth semipositive representative). I will then discuss the construction of a nef R-divisor (resp. (1,1) class) on a projective (resp. non-projective) K3 surface which is not semipositive.
Joint work with S. Filip.

Frédéric Touzet

Deformations of rational curves along foliations

Abstract: Deformations of morphisms along leaves of foliations define the tangential foliation on the corresponding space of morphisms. We prove that codimension one foliations having a tangential foliation with at least one non-algebraic leaf are transversely homogeneous with structure group determined by the codimension of the non-algebraic leaf in its Zariski closure. This is particularly useful for the study of foliations on uniruled manifolds. As an application, we provide a structure theorem for degree three foliations on $\mathbb{P}^3$.

Claire Voisin

Gonality and zero-cycles of abelian varieties

Abstract: The gonality of a variety is defined as the minimal gonality of curve sitting in the variety. We prove that the gonality of a very general abelian variety of dimension g goes to infinity with g. We use for this a (straightforward) generalization of a method due to Pirola that we will describe. The method also leads to a number of other applications concerning 0-cycles modulo rational equivalence on very general abelian varieties.

Susanna Zimmermann

Signature morphisms from the Cremona group

Abstract: The plane Cremona group is the group of birational transformations of the projective plane. I would like to discuss why over algebraically closed fields there are no homomorphisms from the plane Cremona group to a finite group, but for certain non-closed fields there are (in fact there are many).
This is joint work with Stéphane Lamy.
