ABSTRACTS (alphabetical order)

Karin Baur (Universität Graz)
Friezes and cluster structures on Grassmannians

In this talk, I will explain how to obtain SL_k-friezes using Plücker coordinates by taking certain extension-closed subcategories of the Grassmannian cluster categories $C(k, n)$. These are cluster structures associated to the Grassmannian of k-dimensional subspaces in n-dimensional space. Many of these friezes arise from specialising a cluster or a cluster-tilting object to 1. We use Iyama-Yoshino reduction to reduce friezes to friezes of smaller rank. This is joint work with E. Faber, S. Gratz, G. Todorov and K. Serhiyenko

Anna Felikson (Durham University)
Quiver mutations, reflection groups and curves on a punctured disc

Mutations of acyclic quivers can be modelled using geometry of reflection groups acting in quadratic spaces. As an application of this construction, we show an easy and explicit combinatorial way to characterise c-vectors of corresponding cluster algebras. According to results of Kentaro Nagao and Alfredo Najera Chavez, c-vectors of acyclic cluster algebras correspond to real Schur roots (i.e. dimension vectors of indecomposable rigid representations of the quiver over the path algebra). In particular, we obtain a proof of a recent conjecture of Kyu-Hwan Lee and Kyungyong Lee for a large class of acyclic quivers. This work is joint with Pavel Tumarkin (`arXiv:1709.10360`).

Vladimir Fock (Université de Strasbourg)
Clusters and τ-function

Clusters has a viral nature. During twenty years they managed to spread around and contaminate a good part of mathematics. In this talk we contribute to this process and try to explain first (for those who don't know) what is a τ-function of an integrable system and then explain the statement that "a-coordinates of cluster integrable systems is a τ-function".

Sergey Fomin (University of Michigan)
Morsifications and mutations

I will discuss a connection between the topology of isolated singularities of plane curves and the mutation equivalence of the quivers associated with their morsifications. Joint work with Pavlo Pylyavskyy and Eugenii Shustin (`arXiv:1711.10598`).

David Hernandez (Université Paris-Diderot)
Quantum Grothendieck ring isomorphims, cluster algebras and Kazhdan-Lusztig algorithm

Joint work with Hironori Oya. Quantum Grothendieck rings are natural t-deformations of representations rings of quantum affine algebras. Important families of such quantum Grothendieck are known to have a structure of a quantum cluster algebra. Using these structures, we establish ring isomorphisms between certain quantum Grothendieck rings in types B and A. Combining we recent results of Kashiwara-Kim-Oh, we prove for the corresponding categories in type B a conjecture formulated by the speaker in 2002 : the multiplicities of simple modules in standard modules are given by the evaluation of certain analogues of Kazhdan-Lusztig polynomials and the coefficients of these polynomials are positive.

Rei Inoue (Chiba University)
R-**matrices in cluster algebra**

I review on the two R-matrices defined as sequences of mutations:

(1) the "cluster R-matrix" acting on a triangular grid quiver on a cylinder, introduced by T. Lam, P. Pylyavskyy and myself. It is related to the geometric R-matrix of symmetric power representations for $U_q'(\hat{\mathfrak{sl}}_n)$.

(2) The "Dehn twist" of quivers for a punctured disk, studied by G. Schrader and A. Shapiro. It is a realization of the quantum R-matrix via an algebraic embedding of $U_q(\mathfrak{sl}_n)$ into a quantum torus algebra.

I also mention an ongoing project on an relation between (1) and (2).

Osamu Iyama (Nagoya University)
Preprojective algebras and Cluster categories

The preprojective algebra P of a quiver Q has a family of ideals I_w parametrized by elements w in the Coxeter group W. For the factor algebra $P_w = P/I_w$, I will discuss tilting and cluster tilting theory for Cohen-Macaulay P_w-modules following works by Buan-I-Reiten-Scott, Amiot-Reiten-Todorov and Yuta Kimura.

Rinat Kedem (University of Illinois)
From Q-systems to quantum affine algebras and beyond

The theory of cluster algebras has proved useful in proving theorems about the characters of graded tensor products or Demazure modules, via the Q-system. Upon quantization, the algebra associated with this system is shown to be related to a quantum affine algebra. Graded characters are related to a polynomial representation of the quantum cluster variables. This immediately suggests a further deformation to the spherical DAHA, quantum toroidal algebras and elliptic Hall algebras.

Daniel Labardini-Fragoso (Univ. Nacional Autónoma de México)
Associative algebras arising from surfaces with orbifold points

I will sketch two constructions that assign associative algebras to triangulations of (certain) surfaces with orbifold points. The first is done via species with potential over non-algebraically closed fields and is expected to provide information about the corresponding skew-symmetrizable cluster algebras; the second one is done through quivers with potential with nilpotent loops (with Jacobian algebras that are very similar to the algebras considered recently by Geiss-Leclerc-Schröer) and realizes certain generalized cluster algebras of Chekhov-Shapiro as Caldero-Chapoton algebras. The talk will be based on joint works with Jan Geuenich and Diego Velasco.

Robert Marsh (University of Leeds)
τ-exceptional sequences

Joint work with Aslak Bakke Buan (`arXiv:1802.01169`).

We introduce the notions of tau-exceptional and signed tau-exceptional sequences for an arbitrary finite dimensional algebra. We show that, for such an algebra, there is a bijection between the set of complete signed tau-exceptional sequences and ordered support tau-tilting objects. In the hereditary case, tau-exceptional sequences coincide with exceptional sequences and support tau-tilting modules coincide with support tilting modules, so we obtain a bijection between signed exceptional sequences and ordered clusters. Signed exceptional sequences were recently introduced by Igusa and Todorov, who constructed such a bijection in the hereditary case.

Tomoki Nakanishi (Nagoya University)
Periodicities of x- and y-variables in cluster algebras

Periodicity is an interesting and important phenomenon in cluster algebras. It turns out that the periodicities of x- and y-variables coincide for any cluster algebra with skew-symmetrizable exchange matrices. This fact (and its proof) has been known to experts for some years, but it seems that it is not widely known. So, in this talk I give a concise review of how to prove this important fact.

Nathan Reading (North Carolina State University)
Dominance phenomena: mutation, scattering and cluster algebras

An exchange matrix B dominates an exchange matrix B' if they are the same size and if the signs of corresponding entries weakly agree, with the entry of B always having weakly greater absolute value. When B dominates B', interesting things often happen. For example, the mutation fan for B often refines the mutation fan for B'. (In finite type, these are g-vector fans.) The scattering (diagram) fan for B often refines the scattering fan for B'. And there is often an injective homomorphism from the principal-coefficients cluster algebra for B' to the principal-coefficients cluster algebra for B, preserving g-vectors and (less often) sending cluster variables for B' to cluster variables for B.

I call these “phenomena” because the scope of the description “often” is not settled in any of these contexts. Indeed, there are some counterexamples. In this talk, I will give background on the phenomena and present theorems that provide examples of the phenomena, with the goal of establishing that something real and nontrivial is happening.

(`arXiv:1802.10107`)

Dylan Rupel (University of Notre Dame)
Cluster Symplectic Groupoids

The connection between cluster mutations and compatible log-canonical Poisson structures was discovered by Gekhtman-Shapiro-Vainshtein. These foundations can be seen as a precursor to the naive quantization of cluster algebras by Berenstein-Zelevinsky and Fock-Goncharov.

There is in general no procedure to directly produce a geometric quantization of Poisson manifolds, however in the special case of symplectic manifolds (non-degenerate Poisson structures) the quantization procedure is well-known. Symplectic groupoids were introduced by Weinstein with the goal of extending the standard procedure and produce geometric quantizations of Poisson manifolds.

The ultimate goal of this project is to realize the Weinstein quantization program in the case of cluster manifolds with a compatible Poisson structure. In this talk, I will present joint work with Songhao Li taking the first steps in this direction by introducing various symplectic/Poisson groupoids which integrate a log-canonical Poisson structure on a cluster (A- or X-) manifold. A particular construction above yields the symplectic double of Fock-Goncharov which integrates the standard Poisson structure on a cluster X-variety.

Ralf Schiffler (University of Connecticut)
Cluster algebras and Jones polynomials

This talk is on a very concrete connection between cluster algebras and knot theory.

A special class of knots, the 2-bridge knots (or links), are parametrized by continued fractions. On the other hand, we can associate to every continued fraction a so-called snake graph and then define a cluster variable whose Laurent expansion is given as a sum over all perfect matchings of the snake graph. We thus obtain a cluster variable for every 2-bridge knot.

Knot invariants is one of the main branches in knot theory and the Jones polynomial is an important knot invariant. It is a Laurent polynomial in one variable t. We show that up to normalization by the leading term, the Jones polynomial of a 2-bridge knot (or link) is equal to the specialization of the associated cluster variable obtained by setting all initial cluster variables to 1 and specializing the initial principal coefficients of the cluster algebra as follows $y_1 = t^{-2}$ and $y_i = -t^{-1}$, for all $i > 1$.

As a consequence we obtain a direct formula for the Jones polynomial of a

2-bridge link as the numerator of a continued fraction of Laurent polynomials in $q = -t^{-1}$.

This is joint work with Kyungyong Lee (`arXiv:1710.08063`).

Jan Schröer (Universität Bonn)
Irreducible components of module varieties and cluster algebras

After recalling some decomposition results for irreducible components of module varieties, we focus on strongly reduced components. When working over Jacobian algebras, they conjecturally yield a basis of some intermediate cluster algebra. Also conjecturally, there is a close relation between strongly reduced components and Hom-orthogonal brick components. One should see this as a framework which generalizes many of the recent developments in τ-tilting theory.

Michael Shapiro (Michigan State University)
Cluster structures on Poisson-Lie groups

We use Poisson geometry approach to construct a family of non-isomorphic (generalized) cluster structures on $\mathrm{GL}(n)$. This is a joint project with M. Gekhtman and A. Vainshtein.

Sergei Tabachnikov (Penn State University)
Cross-ratio dynamics on ideal polygons

Define a relation between labeled ideal polygons in the hyperbolic space by requiring that, for all i, the complex distance (a combination of the distance and the angle) between the oriented i-th sides of the first and second polygons equals c; the complex number c is a parameter of the relation. This defines a 1-parameter family of maps on the moduli space of ideal polygons in the hyperbolic space (or, in its real version, in the hyperbolic plane). I shall discuss results and conjectures toward complete integrability of this family of maps and its relations with discrete differential geometry and the theory of cluster algebras. This is work in progress, joint with M. Arnold, D. Fuchs, and I. Izmestiev.

Hugh Thomas (Université du Québec À Montréal)
Generalized associahedra via representation theory

Suppose you are given a fan, such as the g-vector fan of a finite-type cluster algebra. If you choose a normal hyperplane to each ray arbitrarily, you get well-defined vertices corresponding to the maximal cones of the fan, but they need not be in convex position. To ensure that they are in convex position, and thus give a realization of the dual polytope, requires that one verify certain inequalities, which can be viewed as statements that certain quantities are positive. The categorificational approach to positivity is that a quantity is positive because it is the dimension of a vector space. I will discuss how to implement this strategy to produce a construction for generalized associahedra based on quiver representations.

Gordana Todorov (Northeastern University)
Homology of Picture Groups

To each Dynkin quiver, using domains of semi-invariants, we associate "spherical semi-invariant picture" $L(Q)$. To such a picture $L(Q)$ we associate the "picture group" $G(Q)$. The picture group has the same generators as the unipotent group $U(Q)$ associated to the same quiver, however it has fewer relations. In order to compute the homology of the picture group $G(Q)$, we construct the picture space $X(Q)$ and show that $X(Q)$ has only first homotopy group non-trivial, and that group is actually isomorphic to $G(Q)$, i.e. $X(Q)$ is a $K(\pi, 1)$ for $G(Q)$. Using this, we can compute homology of the picture group $G(Q)$ by computing homology of the picture space $X(Q)$. For the quiver of type A_n, we show that the homology groups are free abelian groups of ranks given by ballot numbers. Partial results for the quivers of type D will be stated. Joint work with Kiyoshi Igusa, Kent Orr and Jerzy Weyman.

Harold Williams (University of California at Davis)
The coherent Satake category

The geometric Satake equivalence identifies the Satake category of a reductive group G – that is, the category of equivariant perverse sheaves on the affine Grassmannian Gr_G – with the representation category of its Langlands dual group $G^\vee$. While the Satake category is topological in nature, it has a poorly understood algebro-geometric cousin: the category of perverse *coherent* sheaves on Gr_G. This category is not semi-simple and its monoidal product is not symmetric. We show however that it is rigid and admits renormalized r-matrices similar to those appearing in the theory of quantum loop or KLR algebras. Applying the framework developed by Kang-Kashiwara-Kim-Oh in their proof of the dual canonical basis conjecture, we use these results to show that the coherent Satake category of GL_n is a monoidal cluster categorification in the sense of Hernandez-Leclerc. This clarifies the physical meaning of the coherent Satake category: simple perverse coherent sheaves correspond to Wilson-'t Hooft operators in $\mathcal{N} = 2$ gauge theory, just as simple perverse sheaves correspond to 't Hooft operators in $\mathcal{N} = 4$ gauge theory following the work of Kapustin-Witten. Our results also explain the appearance of identical quivers in the work of Kedem-Di Francesco on Q-systems and in the context of BPS quivers. More generally, our construction of renormalized r-matrices works in any chiral E_1-category, providing a new way of understanding the ubiquity of cluster algebras in $\mathcal{N} = 2$ field theory: the existence of renormalized r-matrices, hence of iterated cluster mutation, is a *formal* feature of such theories after passing to their holomorphic-topological twists. This is joint work with Sabin Cautis (`arXiv:1801.08111`).

Lauren Williams (University of California at Berkeley)
Newton-Okounkov bodies for Grassmannians

In joint work with Konstanze Rietsch (`arXiv:1712.00447`), we use the X-cluster structure on the Grassmannian and the combinatorics of plabic graphs to associate a Newton-Okounkov body to each X-cluster. This gives, for each X-cluster, a toric degeneration of the Grassmannian. We also describe the Newton-Okounkov bodies quite explicitly: we show that their facets can be read off from A-cluster expansions of the superpotential. And we give a combinatorial formula for the lattice points of the Newton-Okounkov bodies, which has a surprising interpretation in terms of quantum Schubert calculus.

Milen Yakimov (Louisiana State University)
c-vectors of 2-Calabi-Yau categories and Borel subalgebras of $\mathfrak{sl}_\infty$

This is a report on joint work with Peter Jorgensen (`arXiv:1710.06492`). Due to the fundamental work of Fomin and Zelevinsky, c-vectors play a major role in the theory of cluster algebras in the description of the mutation of their principal coefficients and relations to root systems of simple Lie algebras. We develop a general framework for c-vectors of 2-Calabi-Yau categories with respect to arbitrary cluster tilting subcategories, based on Dehy and Keller's treatment of g-vectors via indices. This approach deals with cluster tilting subcategories which are in general unreachable from each and have infinitely many indecomposable objects. The approach does not rely on (finite or infinite) sequences of mutations. We propose a general program for decomposing sets of c-vectors and identifying each piece with a positive root system of a Levi subalgebra. In the case of the A_∞ cluster categories of Igusa and Todorov, we realize this in terms of the roots of the Borel subalgebras of $\mathfrak{sl}_\infty$ which are of substantial current interest because they are not conjugate to each other, have different full flag varieties, and different categories O.